*Journal of Interdisciplinary Mathematics*
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 29 (2026), No. 5-B, pp. 1459–1465
DOI : 10.47974/JIM-2533

# Some application fixed point in r-convex $\mathfrak{S}_{\mathscr{b}}$- metric spaces

Dhuha Abdulameer Kadhim *
*Department of Mathematics*
*Faculty of Education for Women*
*University of Kufa*
*Kufa*
*Najaf 54001*
*Iraq*

Zainab Hasan Abood †
Sada Faydh Mohammed §
*Department of Statistics*
*College of Administration and Economics*
*University of Kerbala*
*Karbala 56001*
*Iraq*

## Abstract

A notion for convex structures on metric space, first introduce by Takahashi [1] and has become an important part of the theory of fixed points and nonlinear analysis. This framework can be generalized in the present paper by introduces r-convex metric spaces notion, i.e., the metric spaces endowed with an r-convex structures. We construct and establish a fixed - points theorem on r-convex metric space on the basis of this new proposed structure. Some of the basic findings regarding the fixed points of the r-convex framework are rigorously established and addressed. We use the theoretical discoveries to prove that solutions to a set of integral equations exist. Moreover, graphic example is provide to demonstrates validity and applicability for the derived findings.

***Subject Classification:*** *47H10, 54H25.*

***Keywords:*** *Ordinary differential equations, Mathematical modeling, A duopoly economy, Market share.*

* *E-mail:* **dhuhaa.ebada@uokufa.edu.iq (Corresponding Author)**
† *E-mail:* **zainab.hasan@uokerbala.edu.iq**
§ *E-mail:* **sada.f@uokerbala.edu.iq**

## 1. Introduction

Takahashi [1] was proposed a convexities in metric space and examined several fixed - points theorem of nonresponsive mapping in this context. Since that time, convex metric spaces have attracted significant interest in fixed point theory, with several important contributions. Significant works on fixed - points theorem on the convex metric spaces were done by [2], [3], [4], [5], [6], [7], [8], [9].

Over past few year, there is an increase in researchs on generalized settings, including b - metric space so extended metric space, where multiple fixed - points theorem of different class for contraction mapping have been developed. The following are some key developments in this direction: [10-12] and [13], [14] demonstrate the increased significance of generalized metric structures in contemporary fixed-point theory.

**Definition 1.1:** [15] the map $d$: Ⅎ$^2 \longrightarrow [0,\infty)$ be defined as $\mathscr{b}$-metric is where ∃ real numbers $S \geq 1$ where Ⅎ is a nonempty set such that the following properties hold for all Ꭓ, Ч, æ ∈ Ⅎ:

(i) $d($Ꭓ, Ч$) = 0 \leftrightarrow$ Ꭓ = Ч

(ii) $d($Ꭓ, Ч$) = d($Ч, Ꭓ$)$

(iii) $d($Ꭓ, æ$) \leq S[d($Ꭓ, Ч$) + d($Ч, æ$)]$.

The structure(Ⅎ, $d$) is an $\mathscr{b}$-metric spaces.

Definition 1.2: [16] the map $S$: Ⅎ$^3 \longrightarrow [0,\infty)$ defined on Ⅎ where Ⅎ is a nonempty set if, for each Ꭓ, Ч, æ, ɷ ∈ Ⅎ: we have

(i) $S($Ꭓ, Ч, æ$) = 0$ if and only if Ꭓ = Ч = æ

(ii) $S($Ꭓ, Ч, æ$) \leq S($Ꭓ, Ꭓ, ɷ$) + S($Ч, Ч, ɷ$) + S($æ, æ, ɷ$)$

The structure (Ⅎ, $S$) is defined as S-metric spaces.

**Definition 1.3:** [17] assume Ⅎ the non - empty sets & $s \geq 1$. The mapping $\mathfrak{S}_{\mathscr{b}}$: Ⅎ × Ⅎ × Ⅎ $\longrightarrow [0,\infty)$ defined to be $\mathfrak{S}_{\mathscr{b}}$-metric⟺ ∀ Ꭓ, Ч, æ, ɷ ∈ Ⅎ: The following requirements are imposed

- $H_{\mathscr{b}}($Ꭓ, Ч, æ$) = 0 \Leftrightarrow \forall$ Ꭓ = Ч = æ
- $H_{\mathscr{b}}($Ꭓ, Ꭓ, Ч, $) = \mathfrak{S}_{\mathscr{b}}($Ч, Ч, Ꭓ$)$ ∀ Ꭓ, Ч ∈ Ⅎ
- $H_{\mathscr{b}}($Ꭓ, Ч, æ$) \leq s[H_{\mathscr{b}}($Ꭓ, Ꭓ, ɷ$) + H_{\mathscr{b}}($Ч, Ч, ɷ$) + \mathfrak{S}_{\mathscr{b}}($æ, æ, ɷ$)]$.

The structure(Ⅎ, $\mathfrak{S}_{\mathscr{b}}$) is defined as an $\mathfrak{S}_{\mathscr{b}}$-metric spaces.

**Definition 1.4:** Let (Ⅎ, $\mathfrak{S}_{\mathscr{b}}$) be an $\mathfrak{S}_{\mathscr{b}}$- metric space and {ð$_n$} be a sequence in Ⅎ. Then A sequence {Ꭓ$_j$} is defined as convergent if and only if there exists Ꭓ ∈ Ⅎ such that $\mathfrak{S}_{\mathscr{b}}($Ꭓ$_j$, Ꭓ$_j$, Ꭓ$) \longrightarrow 0$ as $j \longrightarrow \infty$. In this case, we write $\lim_{j\to\infty}$ Ꭓ$_j$ = Ꭓ.

A sequence {Ꭓ$_j$} is a Cauchy sequence ⟺ $\mathfrak{S}_{\mathscr{b}}($Ꭓ$_j$, Ꭓ$_j$, Ꭓ$_n)$ as $j, n \longrightarrow \infty$.

An S-metric space $(Ⅎ, \mathfrak{S}_{b})$ is defined as complete if each Cauchy sequence converges to some point $\mathfrak{S}_{b}$-metric space $\{Ꭓ_j\}$ Converges to a point $Ꭓ \in Ⅎ$ such that $\lim_{j,n\to\infty} \mathfrak{S}_{b}(Ꭓ_j, Ꭓ_j, Ꭓ_n) = \lim_{n\to\infty} \mathfrak{S}_{b}(Ꭓ_j, Ꭓ_j, Ꭓ) = \mathfrak{S}_{b}(Ꭓ, Ꭓ, Ꭓ)$.

## 2. Simple Model Including Quantity Demand

**Definition 2.1:** If $(Ⅎ, \mathfrak{S}_{b})$ is $S_b$metric spaces & coefficient $\geq 1$ , $I = [0,1]$, the map $\emptyset: Ⅎ \times Ⅎ \times I \times I \longrightarrow Ⅎ$ be r-convex structures on $\mathfrak{S}_{b}$-metric where $\forall\, Ꭓ, Ч, æ, ɷ \in Ⅎ$ & $\lambda, r \in I$ $\mathfrak{S}_{b}(Ꭓ, Ч, \emptyset(æ, ɷ, \lambda, r)) \leq \lambda r \mathfrak{S}_{b}(Ꭓ, Ч, æ) + (1 - \lambda r)\, \mathfrak{S}_{b}(Ꭓ, Ч, ɷ)$ holds.

Then the triplet $(Ⅎ, \mathfrak{S}_{b}, \emptyset)$ is dented $r$-convex $\mathfrak{S}_{b}$-metric space with a coefficient $\geq 1$ .

**Example 2.2:** Let $Ⅎ = R$ and the mapping $\emptyset: Ⅎ \times Ⅎ \times I \times I \to Ⅎ$ be defined by

$$\mathfrak{S}_{b}(Ꭓ, Ч, æ) = \left[\frac{1}{2}(|Ꭓ - Ч| + |Ч - æ| + |Ꭓ - æ|)\right]^2$$

For all $Ꭓ, Ч, æ \in Ⅎ$, as well as the mapping $\emptyset: Ⅎ \times Ⅎ \times I \times I \to Ⅎ$ defined by the formula $\emptyset(Ꭓ, Ч, \lambda, r) = \lambda r Ꭓ + (1 - \lambda r)Ч$.

Then $(Ⅎ, \mathfrak{S}_{b}, \Phi)$is an $r$-convex $\mathfrak{S}_{b}$-metric space with $b = 3$.

**Solution:** For any $Ꭓ, Ч, æ, ɷ \in Ⅎ$, we get

$$\mathfrak{S}_{b}(Ꭓ, Ч, \emptyset(æ, ɷ, \lambda, r)) = \frac{1}{4}[|Ꭓ - Ч| + |Ч - æ| + |Ꭓ - æ|)]^2$$

$$\leq \frac{1}{4}\,[\lambda r|Ꭓ - Ч| + (1 - \lambda r)|Ꭓ - Ч| + \lambda r|Ꭓ - æ| + (1 - \lambda r)|Ꭓ - ɷ| + \lambda r|Ꭓ - æ| + \lambda r|Ꭓ - æ| + (1 - \lambda r)|Ꭓ - ɷ|]^2$$

$$\leq \frac{1}{4}[(\lambda r)^2(|Ꭓ - Ч| + |Ч - æ| + |Ꭓ - æ|)^2 + (1 - \lambda r)^2(|Ꭓ - Ч| + |Ч - ɷ| + |Ꭓ - ɷ|)^2 + 2\lambda r(1 - \lambda r)(|Ꭓ - Ч| + |Ч - æ| + |Ꭓ - æ|)(|\Lambda - \Theta| + |\Theta - v| + |\Lambda - v|)$$

$$\leq \frac{1}{4}[\lambda r((|\Lambda - \Theta| + |\Theta - u| + |\Lambda - u|)^2 + (1 - \lambda r)(|\Lambda - \Theta| + |\Theta - v| + |\Lambda\ - v|)^2$$

$$\leq \lambda r \mathfrak{S}_{b}(\Lambda, \Theta, u) + (1 - \lambda r)\mathfrak{S}_{b}(\Lambda, \Theta, v)$$

Hence $(Ⅎ, \mathfrak{S}_{b}, \emptyset)$ is $r$-Convex $\mathfrak{S}_{b}$-metric spaces where $b = 3$.

**Definition 2.3:** Let $(Ⅎ, \mathfrak{S}_{b}, \emptyset)$ the $\mathfrak{S}_{b}$-metric spaces & $Ʊ: Ⅎ \longrightarrow Ⅎ$ is map. The sequence $\{ð_\xi\}$ be the Mann sequence where

$$\Lambda_{\xi+1} = \emptyset(\Lambda_\xi, Ʊ\Lambda_\xi, \lambda_\xi, r_\xi), \xi \in N$$

and $\lambda_\xi, r_\xi \in [0,1]$.

We now present a generalized version of Banach's Contraction Principle within the framework of r-convex S-metric spaces, thereby extending its applicability beyond classical metric settings.

**Theorem 2.4:** *For* $(\text{Ꟊ}, H_{\ell}, \emptyset)$ *is Complete r-convex* $\mathfrak{S}_{\ell}$*-metric spaces &* $\ell \geq 1$ *with* $\mho: \text{Ꟊ} \longrightarrow \text{Ꟊ}$ *is mapping such that*

$$H_{\ell}(\mho\Lambda_1, \mho\Lambda_2, \mho\Lambda_3) \leq \alpha H_{\ell}(\Lambda_1, \Lambda_2, \Lambda_3)$$

*For all* $\Lambda_1, \Lambda_2, \Lambda_3 \in \text{Ꟊ}$ *and* $\alpha \in [0,,1)$. *Suppose that the sequence* $\{\Lambda_\xi\}$ *is generated by the Mann iterative and* $\Lambda_0 \in \text{Ꟊ}$ . *If the sequence* $\{\Lambda_\xi\}$ *converges to* $\lambda < \frac{1-\ell^2\alpha r}{\ell^2-\ell^2\alpha r}$ *and* $\alpha < \frac{1}{\ell^2 r}$ *, then It P has exactly one fixed point* $\Lambda^*$ *in* Ꟊ.

***Proof:*** $\forall\ \xi \in N$, We have

$$H_{\ell}\big(\text{Ꭓ}_\xi, \text{Ꭓ}_\xi\ , \mho\text{Ꭓ}_\xi\big) \leq \ell\big[2H_{\ell}\big(\text{Ꭓ}_\xi\ , \mho\text{Ꭓ}_{\xi-1}\big) + H_{\ell}\big(\mho\text{Ꭓ}_\xi, \mho\text{Ꭓ}_\xi\ , \mho\text{Ꭓ}_{\xi-1}\big)\big]$$
$$\leq \ell\big[2H_{\ell}\big(\mho\text{Ꭓ}_{\xi-1}, \mho\text{Ꭓ}_{\xi-1}, \text{Ꭓ}_\xi\big) + \alpha\mathfrak{S}_{\ell}\big(\text{Ꭓ}_\xi, \text{Ꭓ}_\xi, \text{Ꭓ}_{\xi-1}\big)\big]$$
$$\leq \ell[2H_{\ell}\left(\mho\text{Ꭓ}_{\xi-1}, \mho\text{Ꭓ}_{\xi-1}, \emptyset\left(\text{Ꭓ}_{\xi-1}, \mho\text{Ꭓ}_{\xi-1}, \lambda_{\xi-1}, r_{\xi-1}\right)\right) + \alpha(1 - \lambda_{\xi-1}r_{\xi-1})H_{\ell}(\text{Ꭓ}_{\xi-1}, \text{Ꭓ}_{\xi-1}, \mho\Lambda_{\xi-1})]$$
$$\leq \ell\big[2\lambda_{\xi-1}r_{\xi-1}H_{\ell}\big(\text{Ꭓ}_{\xi-1}, \text{Ꭓ}_{\xi-1}, \mho\text{Ꭓ}_{-1}\big) + \alpha\big(1 - \lambda_{\xi-1}\ r_{\xi-1}\big)\mathfrak{S}_{\ell}\big(\text{Ꭓ}_{\xi-1}, \text{Ꭓ}_{\xi-1}, \mho\text{Ꭓ}_{\xi-1}\big)\big]$$
$$\leq \ell\big[2\lambda_{\xi-1}r_{\xi-1} + \alpha(1 - \lambda_{\xi-1}\ r_{\xi-1})\big]H_{\ell}\big(\text{Ꭓ}_{\xi-1}\text{Ꭓ}_{\xi-1}, \mho\text{Ꭓ}_{\xi-1}\big)$$

Put $a_\xi = \ell\big[2\lambda_{\xi-1}r_{\xi-1} + \alpha(1 - \lambda_{\xi-1}\ r_{\xi-1})\big]$. By the hypothesis, we obtain

$$H_{\ell}\big(\text{Ꭓ}_\xi, \text{Ꭓ}_\xi, \mho\text{Ꭓ}_\xi\big) \leq a_\xi H_{\ell}\big(\text{Ꭓ}_{\xi-1}, \text{Ꭓ}_{\xi-1}, \mho\text{Ꭓ}_{\xi-1}\big)$$
$$\leq \frac{1}{\ell^2}H_{\ell}\big(\text{Ꭓ}_{\xi-1}, \text{Ꭓ}_{\xi-1}, \mho\text{Ꭓ}_{\xi-1}\big)$$

So $\{H_{\ell}\big(\text{Ꭓ}_\xi, \text{Ꭓ}_\xi, \mho\text{Ꭓ}_\xi\big)\}\}$ is convergent to $\mu$. By taking limit from (1) we find $\mu = 0$, because $\mu \leq \frac{1}{\ell^2}\mu < \mu$. Also $\lim_{\xi\to\infty} \mathfrak{S}_{\ell}\big(\text{Ꭓ}_\xi, \text{Ꭓ}_\xi, \text{Ꭓ}_{\xi+1}\big) = 0$.

Therefore

$$\frac{\varepsilon_0}{2} \leq \lim_{\ell\to\infty} \sup \leq \mathfrak{S}_{\ell}\big(\Lambda_{\xi_\ell}, \Lambda_{\xi_\ell}, \Lambda_{m_\ell+1}\big) \leq \frac{2\varepsilon_0}{\ell} < \varepsilon_0,$$

Which means $\{\Lambda_\xi\}$ the Cauchy sequences in Ꟊ. Thus $\Lambda_\xi \longrightarrow \Lambda^*$ for some $\Lambda^* \in$ Ꟊ.

$\mathfrak{S}_{\ell}(\Lambda^*, \Lambda^*, \mho\Lambda^*) \leq \ell\big[2\mathfrak{S}_{\ell}\big(\Lambda^*, \Lambda^*, \Lambda_\xi\big) + \mathfrak{S}_{\ell}\big(\mho\Lambda^*, \mho\Lambda^*, \Lambda_\xi\big)\big]$Which implies that $\Lambda^* = \mho\Lambda^*$.

## 3. Some Applications

In order to shows an existences and uniqueness for the solutions integrals equations, consider:

$$o(\mathcal{L}) = v(\mathcal{L}) + \alpha \int_e^d I\big(\mathcal{L}, s, o(s)\big)ds,$$

on $\mathrm{Ꟶ} = C[e,d]$ with sup-norm, and define $\mathfrak{S}_{\mathscr{b}}$-metric as

$$\mathfrak{S}_{\mathscr{b}}(o_1,o_2,o_3) = \sup_{e\le\mathcal{L}\le d}|o_1(\mathcal{L}) - o_2(\mathcal{L})| + \sup_{e\le\mathcal{L}\le d}|o_1(\mathcal{L}) - o_3(\mathcal{L})| + \sup_{e\le w\le d}|o_2(\mathcal{L}) - o_3(\mathcal{L})|$$

Then $\mathfrak{S}_{\mathscr{b}}$ is $\mathfrak{S}_{\mathscr{b}}$-metric with $\mathscr{b} = 2$. Assume that

$$\mho o(\mathcal{L}) = v(\mathcal{L}) + \alpha\int_e^d I\big(\mathcal{L},s,o(s)\big)ds\,,\mathcal{L}\in[e,d]$$

Therefore $\mathfrak{S}_{\mathscr{b}}$ has r-convex structure if we assume $\emptyset\colon \mathrm{Ꟶ}\times\mathrm{Ꟶ}\times\{\frac{1}{4}\}\times I\to\mathrm{Ꟶ}$, $\emptyset\left(h,j,\frac{1}{4},r\right) = \frac{r}{4}h + \left(1-\frac{r}{4}\right)j$. Since

$\mathfrak{S}_{\mathscr{b}}\left(\mathfrak{H},\mathscr{g},\emptyset\left(h,j,\frac{1}{4},r\right)\right) = \|\mathfrak{H}-\mathscr{g}\| + \left\|\mathfrak{H} - (\frac{r}{4}h + \left(1-\frac{r}{4}\right)j)\right\| +$
$\left\|\mathscr{g} - (\frac{r}{4}h + \left(1-\frac{r}{4}\right)j)\right\|$
$= \left\|\frac{r}{4}(\mathfrak{H}-\mathscr{g}) + \left(1-\frac{r}{4}\right)(\mathfrak{H}-\mathscr{g})\right\| + \left\|\frac{r}{4}(\mathfrak{H}-h) + \left(1-\frac{r}{4}\right)(\mathfrak{H}-j)\right\| +$
$\left\|\frac{r}{4}(\mathscr{g}-h) + \left(1-\frac{r}{4}\right)(\mathscr{g}-j)\right\|$
$\le \frac{r}{4}(\|\mathfrak{H}-\mathscr{g}\| + \|\mathfrak{H}-h\| + \|\mathscr{g}-h\|) + \left(1-\frac{r}{4}\right)(\|\mathfrak{H}-\mathscr{g}\| + \|\mathfrak{H}-j\| +$
$\|\mathscr{g}-j\|)$
$\le \frac{r}{4}\mathfrak{S}_{\mathscr{b}}(\mathfrak{H},\mathscr{g},h) + \left(1-\frac{r}{4}\right)\mathfrak{S}_{\mathscr{b}}(\mathfrak{H},\mathscr{g},j)$

**Theorem 3.1:** *Assume that the integral equation (4) with*

1. $I\big(\mathcal{L},s,o(s)\big)$ is continuous mapping
2. $I\big(\mathcal{L},s_1,o(s_1)\big) - I\big(\mathcal{L},s_2,o(s_2)\big) \le |o(s_1) - o(s_2)|$
3. $v\in C[e,d]$
4. $|\alpha| < \frac{1}{2(d-e)}$
5. $\frac{(d-e)}{2} < 1$

So, integral eq. (4) have unique solutions on $[e,d]$.

***Proof:***

$$\mathfrak{S}_{\mathscr{b}}(\mho o_1,\mho o_2,\mho o_3) = \|\mho o_1 - \mho o_2\| + \|\mho o_1 - \mho o_3\| + \|\mho o_2 - \mho o_3\|$$
$$= \sup_{e\le\mathcal{L}\le d}|\mho o_1(\mathcal{L}) - \mho o_2(\mathcal{L})| + \sup_{e\le\mathcal{L}\le d}|\mho o_1(\mathcal{L}) - \mho o_3(\mathcal{L})| + \sup_{e\le\mathcal{L}\le d}|\mho o_2(\mathcal{L}) - \mho o_3(\mathcal{L})|$$
$$= \sup_{e\le\mathcal{L}\le d}\left|\alpha\int_e^d(I\big(\mathcal{L},s,\mathfrak{o}_1(s)\big) - \int_e^d I\big(\mathcal{L},s,\mathfrak{o}_2(s)\big)ds\,(\mathcal{L})))ds\right| +$$
$$+\sup_{e\le\mathcal{L}\le d}\left|\alpha\int_e^d(I\big(\mathcal{L},s,\mathfrak{o}_1(s)\big) - \int_e^d I\big(\mathcal{L},s,\mathfrak{o}_3(s)\big)ds\,(\mathcal{L})))ds\right| +$$
$$+\sup_{e\le\mathcal{L}\le d}\left|\alpha\int_e^d(I\big(\mathcal{L},s,\mathfrak{o}_2(s)\big) - \int_e^d I\big(\mathcal{L},s,\mathfrak{o}_3(s)\big)ds\,(\mathcal{L})))ds\right|$$

$= \sup_{e \leq \mathcal{L} \leq d} |\alpha| (\int_e^d ds) \left| \int_e^d (I(\mathcal{L}, s, \mathfrak{o}_1(s)) - \int_e^d I(\mathcal{L}, s, \mathfrak{o}_2(s)) ds\, (\mathcal{L})) \right| ds +$

$+ \sup_{e \leq \mathcal{L} \leq d} |\alpha| (\int_e^d ds) ( \left| \int_e^d (I(\mathcal{L}, s, \mathfrak{o}_1(s)) - \int_e^d I(\mathcal{L}, s, \mathfrak{o}_3(s)) ds\, (\mathcal{L})) \right| ds +$

$+ \sup_{e \leq \mathcal{L} \leq d} |\alpha| (\int_e^d ds) \left| \int_e^d (I(\mathcal{L}, s, \mathfrak{o}_2(s)) - \int_e^d I(\mathcal{L}, s, \mathfrak{o}_3(s)) ds\, (\mathcal{L})) \right| ds$

$\leq |\alpha| (d-e)^2 [\sup_{e \leq \mathcal{L} \leq d} |\mathfrak{o}_1(s) - \mathfrak{o}_2(s)| + \sup_{e \leq \mathcal{L} \leq d} |\mathfrak{o}_1(s) - \mathfrak{o}_3(s)| +$
$\sup_{e \leq \mathcal{L} \leq d} |\mathfrak{o}_2(s) - \mathfrak{o}_3(s)|] < \frac{(d-e)}{2} \mathfrak{S}_{\mathscr{b}}(\mathfrak{o}_1, \mathfrak{o}_2, \mathfrak{o}_3) < \aleph \mathfrak{S}_{\mathscr{b}}(\mathfrak{o}_1, \mathfrak{o}_2, \mathfrak{o}_3)$

Where $\aleph = \frac{(d-e)}{2} < 1$. Therefore Ʊ has unique fixed points $o \in$ Ⅎ.

## 4. Conclusion

We introduced the notions for the r-convex - metric spaces defining of r-convex structures on and prove some fixed - points theorem satisfying these r-convex structure on metric space. Our work has been supported by new theorems and an application.

## References


[1] W. Takahashi, "A convexity in metric space and nonexpansive mappings, I," *Kodai mathematical seminar reports*, vol. 22, no. 2, pp. 142–149 (1970).

[2] I. Beg, "An iteration scheme for asymptotically nonexpansive mappings on uniformly convex metric spaces," *Nonlinear Anal*, vol. 6, no. 1, pp. 27–34 (2001).

[3] I. Beg and M. Abbas, "Common fixed points and best approximation in convex metric spaces," *Soochow Journal of Mathematics*, vol. 33, no. 4, p. 729 (2007).

[4] I. Beg and M. Abbas, "Fixed-point theorem for weakly inward multivalued maps on a convex metric space," *Demonstratio Mathematica,* vol. 39, no. 1, pp. 149–160 (2006), doi: 10.1515/dema-2006-0119.

[5] S. S. Chang, J. K. Kim, and D. S. Jin, "Iterative sequences with errors for asymptotically quasi-nonexpansive type mappings in convex metric spaces," *Archives of Inequality and Applications*, vol. 2, pp. 365–374 (2004).

[6] L. Ćirić, "On some discontinuous fixed point mappings in convex metric spaces," *Czechoslovak Mathematical Journal,* vol. 43, no. 2, pp. 319–326 (1993).

[7] T. Shimizu and W. Takahashi, "Fixed point theorems in certain convex metric spaces," *Math. Japon*, vol. 37, no. 5, pp. 855–859 (1992).

[8] Y. X. Tian, "Convergence of an Ishikawa type iterative scheme for asymptotically quasi-nonexpansive mappings," *Computers & Mathematics with Applications*, vol. 49, no. 11–12 (2005).

[9] X. P. Ding, "Iteration processes for nonlinear mappings in convex metric spaces," *Journal of mathematical analysis and applications*, vol. 132, no. 1, pp. 114–122 (1988).

[10] Z. H. Abood, D. A. Kadhim, and M. K. Abd Al-Rahem, "View a new of neutrosophic in D-metric space," *Journal of Interdisciplinary Mathematics*, vol. 24, no. 7, pp. 1855–1861 (2021).

[11] Z. H. Abood, M. K. A. Al-Rahem, and D. A. Kadhim, "Fixed point theorem over orthogonal vector metric space and application for solve integral equation," *Journal of Interdisciplinary Mathematics*, vol. 28, no. 3-B, pp. 1021-1030 (2025).

[12] V. Duduka and N. R. Viswa, "Some fixed point results using (ψ, φ)-generalized almost weakly contractive maps in S-metric spaces," *Ratio Mathematica*, vol. 47, pp. 244–259 (2023), doi: 10.23755/rm.v47i0.855.

[13] T. Thaibema, Y. Rohen, T. Stephen, and O. B. Singh, "Fixed points of rational F-contractions in S-metric spaces," *Journal of Mathematical and Computational Science*, vol. 12, pp. 153–161 (2022).

[14] N. Ekiz Yazici, O. Ege, N. Mlaiki, and A. Mukheimer, "Controlled S-Metric-Type Spaces and Applications to Fractional Integrals," *Symmetry*, vol. 15, no. 5 (2023).

[15] I. Bakhtin, "The contraction mapping principle in quasimetric spaces," *Functional analysis*, vol. 30, pp. 26–37 (1989).

[16] S. Sedghi, N. Shobe, and A. Aliouche, "A generalization of fixed point theorems in S -metric spaces," *Matematički vesnik*, vol. 64, no. 249, pp. 258–266 (2012).

[17] N. Souayah and N. Mlaiki, "A fixed point theorem in Sb-metric spaces," *J. Math. Computer Sci*, vol. 16, no. 315, pp. 131–139 (2016).

*Received November, 2025*