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Solution of special cases of PDEs with homogeneous terms

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Abstract

This study seeks to derive complete solutions for certain classes of third-order linear partial differential equations with constant coefficients involving three independent variables, expressed in the general form:

$B_1 H_{ppp} + B_2 H_{ppn} + B_3 H_{pnn} + B_4 H_{nnn} + B_5 H_{ppm} + B_6 H_{pmm} + B_7 H_{pnm} + B_8 H_{nnm} + B_9 H_{nmm} + B_{10} H_{mmm} + B_{11} H_{pp} + B_{12} H_{pn} + B_{13} H_{pm} + B_{14} H_{nn} + B_{15} H_{\mathrm{nm}} + B_{16} H_{mm} + B_{17} H_p + B_{18} H_n + B_{19} H_m + B_{20} H = 0.$

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Based on this supposition, the above equation can be simplified to a third-degree nonlinear second-order ordinary differential equation with three free functions.

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1. Introduction

Various methods of solving partial differential equations (PDEs) have been suggested by many studies [1]. Other related contributions to differential equations are also available in [2-7]. In the current work, we explore functions that can be used to address the considered problem. $v(p), k(n)$ and $u(m)$, $H(p,n,m) = e^{\int v(p)dp+\int k(n)dn+\int u(m)dm}$, Consequently, the general solution of the linear third-order partial differential equation with three independent variables is in the form as follows:

$$B_1H_{ppp} + B_2H_{ppn} + B_3H_{pnn}+B_4H_{nnn} + B_5H_{ppm} + B_6H_{pmm} + B_7H_{pnm} +B_8H_{nnm} + B_9H_{nmm} + B_{10}H_{mmm} + B_{11}H_{pp} + B_{12}H_{pn} + B_{13}H_{pm} + B_{14}H_{nn} +B_{15}H_{nm} + B_{16}H_{mm} + B_{17}H_p + B_{18}H_n + B_{19}H_m + B_{20}H = 0$$

Depending on the values, the original partial differential equations (PDEs) are reduced to a nonlinear second-order ordinary differential equation (ODE) in three independent variables.

2. On the Solution of Certain Classes of Linear Third-Order Partial Differential Equations with Constant Coefficients.

The general form of these equations is given by:

$$B_1H_{ppp} + B_2H_{ppn} + B_3H_{pnn}+B_4H_{nnn} + B_5H_{ppm} + B_6H_{pmm} + B_7H_{pnm} +B_8H_{nnm} + B_9H_{nmm} + B_{10}H_{mmm} + B_{11}H_{pp} + B_{12}H_{pn} + B_{13}H_{pm} + B_{14}H_{nn} +B_{15}H_{nm} + B_{16}H_{mm} + B_{17}H_p + B_{18}H_n + B_{19}H_m + B_{20}H = 0$$

To obtain the full answer, the equation will be divided into a few different cases as follows:

Case 1: $B_1H_{ppp} + B_3H_{pnn} + B_9H_{nmm} = 0.$ ($i.e. B_2 = B_4 = B_6 = \cdots = B_8 = B_{10} = \cdots = B_{20} = 0$) s.t. $B_1, B_3,$ and B_9 are not identically zero.

Case 2: $B_2H_{ppn} + B_3H_{pnn} + B_4H_{nnn} + B_6H_{pmm} = 0.$ ($i.e. B_1 = B_5 = B_7 = \cdots = B_{20} = 0$) s.t. B_2, B_3, B_4 and B_6 are not identically zero.

3. Proposals on Methodological Framework of the proposed Technique.

We consider a type of linear third-order partial equations of the form:

$$B_1H_{ppp} + B_2H_{ppn} + B_3H_{pnn}+B_4H_{nnn} + B_5H_{ppm} + B_6H_{pmm} + B_7H_{pnm} + B_8H_{nnm} + B_9H_{nmm} + B_{10}H_{mmm} + B_{11}H_{pp} + B_{12}H_{pn} + B_{13}H_{pm} + B_{14}H_{nn} +B_{15}H_{nm} + B_{16}H_{mm} + B_{17}H_p + B_{18}H_n + B_{19}H_m + B_{20} = 0 \quad (1)$$

To get the complete solution we compute three independent functions under the following assumption.

$$H(p,n,m) = e^{\int v(p)dp + \int k(n)dn + \int u(m)dm} \tag{2}$$

by substituting

$$H, H_p, H_{pp}, H_{ppp}, H_{pn}, H_{ppn}, H_{pnn}, H_n, H_{nn}, H_m, H_{pm}, H_{nm}, H_{mm}, H_{ppm}, \\ H_{pmm}, H_{nnm}, H_{nmm}, H_{pnm}$$

and H_{mmm} into the equation (1), we get

$$B_1(v'' + 3vv' + v^3) + B_2k(v' + v^2) + B_3v(k' + k^2) + B_4(k'' + 3kk' + k^3) \\ +B_5u(v' + v^2) + B_6v(u' + u^2) + B_7vku + B_8u(k' + k^2) + B_9k(u' + u^2) \\ +B_{10}(u'' + 3uu' + u^3) + B_{11}(v' + v^2) + B_{12}vk + B_{13}vu + B_{14}(k' + k^2) \\ +B_{15}ku + B_{16}(u' + u^2) + B_{17}v + B_{18}k + B_{19}u + B_{20} = 0 \tag{3}$$

It is a nonlinear two-order ordinary differential equation with three independent functions, as presented in Equation (3).

4. On the Full Solution of the Linear Third-Order Partial Differential Equations.

The entire solution of the linear partial third-order partial equation with constant coefficients is established by re-examining the situations described in the beginning of this paper.

Case 1: If $B_2 = B_4 = B_6 = \cdots = B_8 = B_{10} = \cdots = B_{20} = 0, B_1H_{ppp} + B_3H_{pnn} + B_9H_{nmm} = 0$, then the equation (3) becomes

$$B_1(v'' + 3vv' + v^3) + B_3v(k' + k^2) + B_9k(u' + u^2) = 0.$$

i) $(p,n,m) = e^{\gamma p + \frac{\delta^2 n}{2\gamma B_3}}[d_1 cos\sqrt{\frac{B_1\gamma^2}{B_3} - \left(\frac{\delta^2}{2\gamma B_3}\right)^2}\ n +$

$$Hd_2\, sin\sqrt{\frac{B_1\gamma^2}{B_3} - \left(\frac{\delta^2}{2\gamma B_3}\right)^2}\ n][d_3\, cos\ \frac{\delta}{\sqrt{B_9}}m + \ d_4\, sin\frac{\delta}{\sqrt{B_9}}m\,]$$

$d_1 = A\cos(b_1g_2)\,, d_2 = A\sin(b_1g_2)\,, d_3 = \cos(\frac{\delta}{\sqrt{B_9}}g_1)\,, d_4 = \sin(\frac{\delta}{\sqrt{B_9}}g_1).$

Where $d_1, d_2, d_3, d_4, \delta$ and γ are arbitrary constants, if $\frac{B_1\gamma^2}{B_3} \neq (\frac{\delta^2}{2\gamma B_3})^2$.

ii) $H(p,n,m) = (k - g)e^{\gamma p + \frac{\delta^2 n}{2\gamma B_3}}[d_3\, cos\ \frac{\delta}{\sqrt{B_9}}m + \ d_4\, sin\ \frac{\delta}{\sqrt{B_9}}m\,]$

Where d_3, d_4, δ and γ are arbitrary constants, if $\frac{B_1\gamma^2}{B_3} = (\frac{\delta^2}{2\gamma B_3})^2$.

Proof: Since $B_1(v'' + 3vv' + v^3) + B_3v(k' + k^2) + B_9k(u' + u^2) = 0.$

Since we cannot use the separation of variables method we take representing to an arbitrary constant. This results in the simplified version of the equation:

$$B_1\gamma^3 + B_3\gamma(k' + k^2) + B_9 k(u' + u^2) = 0$$
$$\Rightarrow \frac{B_1\gamma^3}{k} + B_3\gamma(\frac{k' + k^2}{k}) + B_9(u' + u^2) = 0$$

This equation is variable separable equation [1], to solve it
Let

$$\frac{B_1\gamma^3}{k} + B_3\gamma(\frac{k' + k^2}{k}) = -B_9(u' + u^2) = -\delta^2$$

So:

$$\frac{B_1\gamma^3}{k} + B_3\gamma(\frac{k'+k^2}{k}) + \delta^2 = 0 \Rightarrow k' + k^2 + \frac{\delta^2}{\gamma B_3}k + \frac{B_1\gamma^2}{B_3} = 0 \quad (4)$$

also:

$$B_9(u' + u^2) - \delta^2 = 0 \Rightarrow u' + u^2 - \frac{\delta^2}{B_9} = 0 \quad (5)$$

From the equation (4): i) If $\frac{B_1\gamma^2}{B_3} \neq (\frac{\delta^2}{2\gamma B_3})^2$, we get:

$$\frac{dk}{(k+\frac{\delta^2}{2\gamma B_3})^2+b_1^2} + dn = 0;\ b_1^2 = \frac{B_1\gamma^2}{B_3} - (\frac{\delta^2}{2\gamma B_3})^2$$

$$\Rightarrow \frac{1}{b_1}tan^{-1}[\frac{k+\frac{\delta^2}{2\gamma B_3}}{b_1}] = g_1 - n \Rightarrow\ k = b_1\tan(b_1 g_1 - b_1 n) - \frac{\delta^2}{2\gamma B_3} \quad (6)$$

And from equation (5), we get $\frac{du}{u^2+\frac{\delta^2}{B_9}} + dm = 0 \Rightarrow \frac{B_9}{\delta^2}\frac{du}{(\frac{\sqrt{B_9}}{\delta}u)^2+1} + dm = 0$

$$\Rightarrow \frac{\sqrt{B_9}}{\delta}tan^{-1}(\frac{\sqrt{B_9}}{\delta}u) = g_2 - m \Rightarrow\ u = \frac{\delta}{\sqrt{B_9}}\tan(\frac{\delta}{\sqrt{B_9}}g_2 - \frac{\delta}{\sqrt{B_9}}m) \quad (7)$$

$$H(p,n,m) = e^{\int\gamma dp+\int[b_1\tan(b_1 g_2-b_1 n)-\frac{\delta^2}{2\gamma B_3}]dn+\int[\frac{\beta}{\sqrt{B_9}}\tan(\frac{\delta}{\sqrt{A_9}}g-\frac{\delta}{\sqrt{B_9}}m)]dm}$$

$$H(p,n,m) = A[\cos(b_1 g_2 - b_1 n)]\cos(\frac{\delta}{\sqrt{B_9}}g_1 - \frac{\delta}{\sqrt{B_9}}m)\ e^{\gamma p-\frac{\delta^2}{2\gamma B_3}n}\ ; A = e^h$$

And the complete solution is given by:

$$H(p,n,m) = e^{\gamma p-\frac{\delta^2}{2\gamma B_3}n}\ [d_1 cos\sqrt{\frac{B_1\gamma^2}{B_3} - (\frac{\beta^2}{2\gamma B_3}))^2}\ n +$$
$$d_2 sin\sqrt{\frac{B_1\gamma^2}{B_3} - \left(\frac{\beta^2}{2\gamma B_3}\right)^2}\ n][d_3\ cos\frac{\delta}{\sqrt{B_9}}m + d_4\ sin\frac{\delta}{\sqrt{B_9}}m]$$

$$d_1 = A\cos(b_1 g_2)\,, d_2 = A\sin(b_1 g_2)\,, d_3 = \cos(\frac{\delta}{\sqrt{B_9}}g_1)\,, d_4 = \mathrm{si\,n}(\frac{\delta}{\sqrt{B_9}}g_1).$$

Where $d_1, d_2, d_3, d_4, \delta$ and γ are arbitrary constants.

ii) If $\frac{B_1\gamma^2}{B_3} = (\frac{\delta^2}{2\gamma B_3})^2$ we get

$$\frac{dk}{(k+\frac{\delta^2}{2\gamma B_3})^2} + dn = 0\ \Rightarrow \frac{-1}{k+\frac{\delta^2}{2\gamma B_3}} = g_3 - n\ \Rightarrow k = \frac{1}{n-g_3} - \frac{\delta^2}{2\gamma B_3}\ (8)$$

And

$$u = \frac{\delta}{\sqrt{B_9}}\tan(\frac{\delta}{\sqrt{B_9}}g_2 - \frac{\delta}{\sqrt{B_9}}m),$$

so,

$$H(p,n,m) = e^{\int \gamma dp + \int [\frac{1}{n-g_3} - \frac{\delta^2}{2\gamma B_3}]dn + \int [\frac{\delta}{\sqrt{B_9}}\tan(\frac{\delta}{\sqrt{B_9}}g_2 - \frac{\delta}{\sqrt{B_9}}m)]dm}$$

$$H(p,n,m) = A(n-g_3)e^{\gamma p - \frac{\delta^2}{2\gamma B_3}n}\,[\cos(\frac{\delta}{\sqrt{B_9}}g_2 - \frac{\delta}{\sqrt{B_9}}m)]\,; A = e^h,$$

The complete solution is given by:

$$H(p,n,m) = A(n-g)e^{\gamma p - \frac{\delta^2}{2\gamma B_3}n}\,[d_3\, cos\frac{\delta}{\sqrt{B_9}}m + d_4\, sin\frac{\delta}{\sqrt{B_9}}m]$$

Case 2: If $B_1 = B_5 = B_7 = \cdots = B_{20} = 0$, given by: $B_2H_{ppn} + B_3H_{pnn} + B_4H_{nnn} + B_6H_{pmm} = 0$, then the equation (3) becomes
$B_2k(v' + v^2) + B_3v(k' + k^2) + B_4(k'' + 3kk' + k^3) + B_6v(u' + u^2) = 0.$

i) $H(p,n,m) = e^{-\frac{\delta^2}{2\gamma B_2}p + \gamma n}\,[d_1cos\sqrt{\frac{B_4\gamma^2}{B_2} - (\frac{\delta^2}{2\gamma B_2})})[7^2\, p +$

$d_2sin\sqrt{\frac{B_4\gamma^2}{B_2} - \left(\frac{\delta^2}{2\gamma B_2}\right)^2}\,p][d_3\cos(\sqrt{\frac{\delta^2-\gamma^2B_3}{B_6}}m) + d_4\, sin\,(\sqrt{\frac{\delta^2-\gamma^2B_3}{B_6}}m)];$

$d_1 = \cos(b_2g_4)\,, d_2 = \sin(b_2g_4)\,, d_3 = \cos(\sqrt{\frac{B_3\,\gamma^2-\delta^2}{B_6}}g_5),$

i) $d_4 = \sin(\sqrt{\frac{B_3\,\gamma^2-\delta^2}{B_6}}g_5).$

Where $d_1, d_2, d_3, d_4, \delta$ and γ are arbitrary constants, if $\frac{B_4\gamma^2}{B_2} \neq \left(\frac{\delta^2}{2\gamma B_2}\right)^2.$

ii) $H(p,n,m) = A(p-g)e^{-\frac{\delta^2}{2\gamma B_2}p + \gamma n}$

$[d_3\cos(\sqrt{\frac{\delta^2-\gamma^2B_3}{B_6}}m) + d_4\, sin(\sqrt{\frac{\delta^2-\gamma^2B_3}{B_6}}m)]\,; A = e^h$

$d_3 = \cos(\sqrt{\frac{B_3\,\gamma^2-\delta^2}{B_6}}g_6),\ d_4 = \sin(\sqrt{\frac{B_3\,\gamma^2-\delta^2}{B_6}}g_6)$

Where d_3, d_4, A, δ and γ are arbitrary constants, if $\frac{B_4\gamma^2}{B_2} = \left(\frac{\delta^2}{2\gamma B_2}\right)^2$.

Proof: Since $B_2k(v' + v^2) + B_3v(k' + k^2) + B_4(k'' + 3kk' + k^3) + B_6v(u' + u^2) = 0. \therefore B_2\gamma(v' + v^2) + B_3\gamma^2v + B_4\gamma^3 + B_6v(u' + u^2) = 0$

$$\Rightarrow B_2\gamma(\frac{v'+v^2}{v}) + B_3\gamma^2 + \frac{B_4\gamma^3}{v} + B_6(u' + u^2) = 0.$$

$$B_2\gamma(\frac{v'+v^2}{v}) + \frac{B_4\gamma^3}{v} = -B_3\gamma^2 - B_6(u' + u^2) = -\delta^2$$

So:

$$B_2\gamma(\frac{v'+v^2}{v})+\frac{B_4\gamma^3}{v}+\delta^2=0 \Rightarrow v'+v^2+\frac{\delta^2}{B_2\gamma}v+\frac{B_4\gamma^2}{B_2}=0 \tag{9}$$

also:

$$B_3\gamma^2+B_6(u'+u^2)-\delta^2=0 \Rightarrow u'+u^2+\frac{B_3\,\gamma^2-\delta^2}{B_6}=0 \tag{10}$$

From the equation (9): i) If $\frac{B_4\gamma^2}{B_2}\neq\left(\frac{\delta^2}{2\gamma B_2}\right)^2$, we get

$$\frac{dv}{(v+\frac{\delta^2}{2\gamma B_2})^2+b_2^2}+dp=0\,;\ b_2^2=\frac{B_4\gamma^2}{B_2}-\left(\frac{\delta^2}{2\gamma B_2}\right)^2,$$

$$\Rightarrow \frac{1}{b_2}tan^{-1}(\frac{v+\frac{\delta^2}{2\gamma B_2}}{b_2})=g_4-p \Rightarrow v=b_2\tan(b_2g_4-b_2p)-\frac{\delta^2}{2\gamma B_2} \tag{11}$$

From the equation (10), we get

$$\frac{du}{u^2+\frac{B_3\,\gamma^2-\delta^2}{B_6}}+dm=0\ \Rightarrow \frac{B_6}{B_3\,\gamma^2-\delta^2}\frac{du}{(\sqrt{\frac{B_6}{B_3\,\gamma^2-\delta^2}}u)^2+1}+dm=0$$

$$\Rightarrow \sqrt{\frac{B_6}{B_3\,\gamma^2-\delta^2}}\,tan^{-1}(\sqrt{\frac{B_6}{B_3\,\gamma^2-\delta^2}}u)=g_5-m$$

$$\Rightarrow u=\sqrt{\frac{B_3\,\gamma^2-\delta^2}{B_6}}\ \tan(\sqrt{\frac{B_3\,\gamma^2-\delta^2}{B_6}}g_5-\sqrt{\frac{B_3\,\gamma^2-\delta^2}{B_6}}m) \tag{12}$$

So,

$$H(p,n,m)$$
$$=e^{\int[b_2\tan(b_2g-b_2p)-\frac{\delta^2}{2\gamma B_2}]dp+\int\gamma dn+\int[\sqrt{\frac{B_3\,\gamma^2-\delta^2}{B_6}}\tan(\sqrt{\frac{B_3\,\gamma^2-\delta^2}{B_6}}g-\sqrt{\frac{B_3\,\gamma^2-\delta^2}{B_6}}m)]dm}$$

$$H(p,n,m)=A\ \cos(b_2g_4-b_2p)\cos(\sqrt{\frac{B_3\,\gamma^2-\delta^2}{B_6}}g_5-\sqrt{\frac{B_3\,\gamma^2-\delta^2}{B_6}}m)\,e^{-\frac{\delta^2}{2\gamma B_2}p+\gamma n}$$

$$A=e^h.$$

And the complete solution is given by:

$$H(p,n,m)=e^{-\frac{\delta^2}{2\gamma B_2}p+\gamma n}\ [d_1cos\sqrt{\frac{B_4\gamma^2}{B_2}-\left(\frac{\delta^2}{2\gamma B_2}\right)^2}\ p+$$
$$d_2sin\sqrt{\frac{B_4\gamma^2}{B_2}-\left(\frac{\delta^2}{2\gamma B_2}\right)^2}\ p][d_3\cos(\sqrt{\frac{B_3\,\gamma^2-\delta^2}{B_6}}m)+d_4\sin(\sqrt{\frac{B_3\,\gamma^2-\delta^2}{B_6}}m)]$$

Where $d_1, d_2, d_3, d_4, \delta$ and γ are arbitrary constants.

ii) If $\frac{B_4\gamma^2}{B_2}=\left(\frac{\delta^2}{2\gamma B_2}\right)^2$, we get: From (9)

$$\frac{dv}{(v+\frac{\delta^2}{2\gamma B_2})^2}+dp=0\ \Rightarrow \frac{-1}{v+\frac{\delta^2}{2\gamma B_2}}=g_6-p\ \Rightarrow v=\frac{1}{p-g_6}-\frac{\delta^2}{2\gamma B_2} \tag{13}$$

And

$$u=\sqrt{\frac{B_3\,\gamma^2-\delta^2}{B_6}}\ \tan(\sqrt{\frac{B_3\,\gamma^2-\delta^2}{B_6}}g_6-\sqrt{\frac{B_3\,\gamma^2-\delta^2}{B_6}}m)$$

so,

$$H(p,n,m) = e^{\int[\frac{1}{p-g_5}-\frac{\delta^2}{2\gamma B_2}]dp+\int \gamma dn+\int[\sqrt{\frac{B_3\,\gamma^2-\delta^2}{B_6}}\tan(\sqrt{\frac{B_3\,\gamma^2-\delta^2}{B_6}}g_6-\sqrt{\frac{B_3\,\gamma^2-\delta^2}{B_6}}m)]dm}$$

so, the complete solution is given by:

$$H(p,n,m) = A(p-g_5)e^{-\frac{\delta^2}{2\gamma B_2}p+\gamma n}$$

$$d_3\cos(\sqrt{\frac{B_3\,\gamma^2-\delta^2}{B_6}}m) + d_4\sin(\sqrt{\frac{B_3\,\gamma^2-\delta^2}{B_6}}m)]\ ;\ A = e^h$$

Example 4.1: To find a solution to the following partial differential equation:

$$3H_{ppp} - \frac{1}{4}H_{pnn} + 2H_{nmm} = 0, B_1 = 3, B_3 = -\frac{1}{4}, B_9 = 2.$$

Since $\frac{B_1\,\gamma^2}{B_3} \neq (\frac{\delta^2}{2\gamma\, B_3})^2$, then by using the formula (as in case-1-i)

$$H(p,n,m) = e^{\gamma p-\frac{\delta^2}{2\gamma B_3}n}\,[d_1\cos\sqrt{-12\gamma^2-\frac{4\delta^4}{\gamma^2}}\,n$$

$$+\,d_2\,sin\sqrt{-12\gamma^2-\frac{4\delta^4}{\gamma^2}}\,n][d_3\,cos\,\frac{\delta}{\sqrt{2}}m +\ d_4\,sin\frac{\delta}{\sqrt{2}}m\,]$$

Where $d_1, d_2, d_3, d_4, \delta$.

Conclusion

This study presented complete solutions for certain classes of third-order linear partial differential equations with constant coefficients involving three independent variables. The original equation was reduced to a third-degree nonlinear second-order ordinary differential equation with three free functions, providing a simplified and effective approach for solving such equations.

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