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Orbitally convergent of inverse shadowing property on G-spaces

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Abstract

The sufficient conditions were obtained under which the uniform convergence of mappings shows a dynamical behavior at a point of inverse shadowing in G-space. Since orbital convergence is stronger than uniform convergence, we observe that the relations among inverse shadowing have some properties like transitivity, a dense set of periodic points, dense small periodic sets, weak mixing, and sensitive in G- orbital convergence. And then investigate that orbital convergence deserves G-chaotic. The quantity demanded and

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investment in research and development, while the other model focuses on a more realistic relationship between the quantity demanded and the price.

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1. Introduction

One of the motivations when we study numerical dynamical systems is (InShPr), which is the importance of dynamical systems for the theory of perturbations. The concept was initially established for homeomorphisms and group actions in 1995 and was later extended through the introduction of numerical methods. These developments have provided a valuable framework for analyzing the behavior of dynamical systems under perturbations and for investigating the relationship between exact and approximate trajectories [1], [2].

The project of the system dynamically deals with the attitude of a distributed orbit, but that does not mean it can track down the genuine attitude of each orbit forever, nor does it mean it can divine the genius of an orbit by approximating it with a sequence of points. The modeling of a system in mathematics obtains a system as a parataxis of the genetic discrete or continuous system. Authors have proved that a $(ToTr)$ mappings imply $(ToTr)$ in the uniform limit of [3], [4].

It is well known that the limit mapping is continuous (Riemann integrable) if a uniformly convergent sequence of continuous (Riemann integrable) mappings, respectively, while of differentiability is not proven. The investigation of properties for dynamics which taken by the mappings in the sequence is hereditary by the limit mapping is an interesting problem in this direction. For example, the sequence of $(ToTr)$ mappings in the limit mapping of a uniformly convergent sequence need not be $(ToTr)$, and we obtained sufficient conditions to be $(ToTr)$ under uniform limit mapping. The Devaney's chaos studied the uniform limit mapping [5]. Physics and engineering are interesting applications, which some of the studies done in this direction [6], [7], [8]. Since no more studies of these properties in $\mathfrak{G} -$ spaces, so in our future works we will study these applications in more detail on $\mathfrak{G} -$ spaces. In 2012 also studied this convergence in $(SP)_{\mathfrak{G}}$ [9].

2. Preliminaries

Let a homeomorphism mapping. $\mathfrak{f}_{\mathrm{g}}$ be from $\mathfrak{M}$ to $\mathfrak{W}$, which $\mathfrak{M}$ is $(SP)_{\mathfrak{G}}$, and $dist$ is a metric. A group $\mathfrak{G}$ is finitely generated, an action Θ is defined on $\mathfrak{G}$, and a generating set $\mathfrak{S}$ is a finite symmetric.

Definition 2.1: [9] Let $\Gamma = \{\gamma_{\mathrm{g}} \in \mathrm{C}(\mathfrak{M}, \mathfrak{M}) : \mathrm{g} \in \mathfrak{G}\}$ be a family. For an action Θ, define $(\mathrm{Md})_{\mathfrak{d}}$, with respect to $\mathfrak{S}$, as : $\mathrm{dist}(\gamma_{\mathrm{sg}}(\mathrm{x}), \Theta(\mathrm{s}, \gamma_{\mathrm{g}}(\mathrm{x})))$ larger than $\mathfrak{d}$, g

belong to $\mathfrak{G}$, s belong to $\mathfrak{S}$, x belong to $\mathfrak{M}$; where the identity is $\gamma_{\mathrm{e}} = \mathrm{Id}$. An orbit was defined of the method Γ as a family $\{\mathrm{x}_{\mathfrak{g}} = \gamma_{\mathfrak{g}}(\mathrm{x}_{\mathrm{e}}) \in \mathfrak{M} : \mathfrak{g} \in \mathfrak{G}\}$. For mapping $\mathfrak{f}_{\mathfrak{g}}$, denoted for all $(\mathrm{Mds})_{\mathfrak{d}}$ as a set by $\mathcal{T}_0(\mathfrak{f}_{\mathfrak{g}}, \mathfrak{d})$.

Definition 2.2: [9] Let Γ be a $(\mathrm{Md})_{\mathfrak{d}}$. It is called a **continuous** $(\mathrm{Md})_{\mathfrak{d}}$ (simply $(\mathrm{ConMd})_{\mathfrak{d}}$) if it is continuous. For mapping $\mathfrak{f}_{\mathfrak{g}}$, all $(\mathrm{ConMds})_{\mathfrak{d}}$ was denoted as a set by $\mathcal{T}_c(\mathfrak{f}_{\mathfrak{g}}, \mathfrak{d})$.

Definition 2.3: [9] Let Θ be an action on the set $\mathfrak{S}$. A mapping $\mathfrak{f}_{\mathfrak{g}}$ is called has the $(\mathrm{InShPr})_{\mathfrak{G}}$ If there is an $\{\mathrm{x}_{\mathfrak{g}}\}$ an orbit of a method Γ fulfilling the inequalities $\mathrm{dist}(\mathfrak{f}_{\mathfrak{g}}(\mathrm{p}), \mathrm{x}_{\mathfrak{g}})$ smaller than ε, $\mathfrak{g}$ belong to $\mathfrak{G}$, where $\mathfrak{f}_{\mathfrak{g}}(\mathrm{p}) = \Theta_{\mathfrak{f}}(\mathfrak{g}, \mathrm{p})$, for each ε larger than 0 there is $\mathfrak{d}$ larger than 0 like for all p belong to $\mathfrak{M}$ and each $(\mathrm{Md})_{\mathfrak{d}}$ Γ for the set $\mathfrak{S}$.

Definition 2.4: [9] Let a continuous $\mathfrak{f}_{\mathfrak{g}}$ be a mapping from $\mathfrak{M}$ to itself. It is called a **pseudo equivariant (simply (Sdq))** if $\mathfrak{f}_{\mathfrak{g}}(\mathrm{x}) = \mathfrak{f}_{\mathfrak{g}'}(\mathrm{x})$ for all x belong to $\mathfrak{M}$ and all $\mathfrak{g}$ belong to $\mathfrak{G}$, there is a $\mathfrak{g}'$ belong to $\mathfrak{G}$.

Definition 2.5: [9] Let x be a point belong to $\mathfrak{M}$. For a mapping $\mathfrak{f}_{\mathfrak{g}}$, it is called a $\mathfrak{G}$ $-$periodic point (simply $(\mathrm{PerPo})_{\mathfrak{G}}$) where $\mathfrak{f}_{\mathfrak{g}\mathfrak{g}_{\mathrm{n}}}(\mathrm{x}) = \left(\underbrace{\mathfrak{f}_{\mathfrak{g}} \circ \mathfrak{f}_{\mathfrak{g}} \circ \ldots \circ \mathfrak{f}_{\mathfrak{g}}}_{\text{n times}}\right)$ $(\mathrm{x}) = \mathrm{x}$, if subsists an integer n larger than 0 and $\mathfrak{g}$ belong to $\mathfrak{G}$.

Definition 2.6: [9] Let $\mathfrak{f}_{\mathfrak{g}}$ be a mapping. It is called $(\mathrm{ToTr})_{\mathfrak{G}}$ if $\mathfrak{f}_{\mathfrak{g}}(\mathrm{U}) \cap \mathrm{V} \neq \emptyset$, for each U and V of $\mathfrak{M}$ which are nonempty open subsets (simply (NONeos)).

Definition 2.7: [9] Let $\mathfrak{f}_{\mathfrak{g}}$ be a mapping. If $\mathfrak{f}_{\mathfrak{g}\mathfrak{g}_{\mathrm{n}}}$ is $(\mathrm{ToTr})_{\mathfrak{G}}$ for all $\mathrm{n} > 1$, then $\mathfrak{f}_{\mathfrak{g}}$ is said to be totally $\mathfrak{G}$ $-$ttransitive (simply $(\mathrm{TotTr})_{\mathfrak{G}}$).

Definition 2.8: [9] Let $\mathfrak{f}_{\mathfrak{g}}$ be a mapping. It is called topological $\mathfrak{G}$ $-$micing simply $(\mathrm{ToMix})_{\mathfrak{G}}$ if $\mathfrak{f}_{\mathfrak{g}\mathfrak{g}_{\mathrm{n}}}(\mathrm{U}) \cap \mathrm{V} \neq \emptyset$, n larger than N, for any pair U, V (NONeos) of $\mathfrak{M}$, there exists N belong to $\mathbb{N}$, like for each $\mathfrak{g}_{\mathrm{n}}$belong to $\mathfrak{G}$.

Definition 2.9: [9] Let $\mathfrak{f}_{\mathfrak{g}}$ be a mapping. It is called weakly $\mathfrak{G}$ $-$ɱicing (simply $(\mathrm{WkMix})_{\mathfrak{G}}$) if $\mathfrak{f}_{\mathfrak{g}} \times \mathfrak{f}_{\mathrm{h}}(\mathrm{U} \times \mathrm{V}) \cap (\mathrm{E} \times \mathrm{F}) \neq \emptyset$ where the Cartesian product $\mathfrak{f}_{\mathfrak{g}} \times \mathfrak{f}_{\mathfrak{g}}$ is, $(\mathrm{ToTr})_{\mathrm{G}\times\mathrm{G}}$ that means, for every (NONeos) which are basic $\mathrm{U} \times \mathrm{V}$, $\mathrm{E} \times \mathrm{F}$ of $\mathfrak{M} \times \mathfrak{M}$, there exist $(\mathfrak{g}, \mathrm{h})$ belong to $\mathfrak{G} \times \mathfrak{G}$ and k belong to $\mathbb{N}$. That's the same with: $\mathfrak{f}_{\mathfrak{g}}(\mathrm{U}) \cap (\mathrm{E})$ not empty; and $\mathfrak{f}_{\mathrm{h}}(\mathrm{V}) \cap (\mathrm{F})$ not empty.

Definition 2.10: [9] Let $\mathfrak{f}_{\mathfrak{g}}$ be a mapping. It is called that it has densue şmall of $\mathfrak{G}$ $-$periodic sets (simply$(\mathrm{DenSmPrSets})_{\mathfrak{G}}$) if $\mathfrak{f}_{\mathfrak{g}}(\mathrm{A}) = \mathrm{A}$, for any (NONes) $\mathrm{U} \subset \mathfrak{M}$ there be a closed subset $\mathrm{A} \subset \mathrm{U}$.

3. $(\mathbf{InShPr})_{\mathfrak{G}}$ with $(\mathbf{OrCv})_{\mathfrak{G}}$

In this section, we prove the main results, including the relations between $\mathfrak{G}-$orbitally convergent $(\mathrm{OrCv})_{\mathfrak{G}}$ with $\mathfrak{G}-$inverse shadowing property $(\mathrm{InShPr})_{\mathfrak{G}}$.

Proposition 3.1: Let $(\mathfrak{M}, \mathrm{dist})$ be $(\mathrm{CpMtSp})_{\mathfrak{G}}$. It has $(\mathrm{InShPr})_{\mathfrak{G}}$ If a sequence $\{n \in \mathbb{N}_0, \mathfrak{g} \in \mathfrak{G}\}$ of mappings which have $(\mathrm{InShPr})_{\mathfrak{G}}$ for all n, is $\mathfrak{G}-$uniformly convergence to $\mathfrak{f}_{\mathfrak{g}}$.

Proof: Let $\varepsilon > 0$ and put $\varepsilon_1 = \frac{\varepsilon}{2}$. We assume that $(\mathfrak{f}_i)_{\mathfrak{g}}$ has the $(\mathrm{InShPr})_{\mathfrak{G}}$ for every $i \in \mathbb{N}_0$. Let $\Gamma_i = \{(\gamma_i)_{\mathfrak{g}}$ belong to $C(\mathfrak{M}, \mathfrak{M})$, $\mathfrak{g}$ belong to $\mathfrak{G}, 0 \leq i \leq n\}$ be $(\mathrm{Md})_{\mathfrak{d}_i}$. Thus, there exists an orbit $\{(y_i)_{\mathfrak{g}}\}$ fulfilling $\mathrm{dist}\big((\mathfrak{f}_i)_{\mathfrak{g}}(m^i), (y_i)_{\mathfrak{g}}\big) < \varepsilon$, for any $\varepsilon_1 > 0$ for any $m^i \in \mathfrak{M}$ and any $(\mathrm{Md})_{\mathfrak{d}_i}$ for $\mathfrak{S}$.

Since the sequence $\{(\mathfrak{f}_n)_{\mathfrak{g}}\}$ is $\mathfrak{G}-$uniformly convergence to $\mathfrak{f}_{\mathfrak{g}}$, then for every $\varepsilon_2 = \frac{\varepsilon}{4}$ such that there is $i_0 \in \mathbb{N}_0$ satisfying $\mathrm{dist}\Big((\mathfrak{f}_n)_{\mathfrak{g}}(m), \mathfrak{f}_q(m)\Big) < \varepsilon_2$, for all $\mathfrak{g}, q \in \mathfrak{G}, m \in \mathfrak{M}$, and $n > i_0$. So there exists an orbit $\{y_{\mathfrak{g}}\}$ satisfying

$$\mathrm{dist}\left(f_g(p), y_g\right) < \mathrm{dist}\left(f_g(p), (f_n)_g(y)\right) + \mathrm{dist}\left((f_n)_g(y), f_q(y)\right) + \mathrm{dist}\left(f_q(y), y_g\right) < \varepsilon.$$

for any point p belong to $\mathfrak{M}$ and any $(\mathrm{Md})_{\mathfrak{d}}$ for $\mathfrak{S}$. Therefore $\mathfrak{f}_{\mathfrak{g}}$ also has $(\mathrm{InShPr})_{\mathfrak{G}}$.

In 2012 [10], the authors defined $(\mathrm{OrCv})_{\mathfrak{G}}$ as follows:

Let a sequence $\{\mathfrak{f}_n\}$ be of continuous self-mappings defined on $(\mathfrak{M}, \mathfrak{d})$. It is called $(\mathrm{OrCv})_{\mathfrak{G}}$ to a mapping $\mathfrak{f}$ from $\mathfrak{M}$ to $\mathfrak{M}$ if for all ε larger than 0, there is p belong to N such that $\mathrm{dist}\Big((\mathfrak{f}_{nm})_{\mathfrak{g}}(x), (\mathfrak{f}_m)_k(x)\Big)$ smaller than ε for all x belong to $\mathfrak{M}$, for all m belong to N, for all $\mathfrak{g}, k$ belong to $\mathfrak{G}$, and for all n larger than or equal p.

And they obtained a condition which is sufficient under the limit mapping of a sequence is $(\mathrm{ToTr})_{\mathfrak{G}}$ as in Theorem 4.11, [10].

Let a sequence $\{(\mathfrak{f}_n)_{\mathfrak{g}}\}$ be of $(ToTr)_{\mathfrak{G}}$ mappings on a $(CpMtSp)_{\mathfrak{G}}$ which is $(OrCv)_{\mathfrak{G}}$ to a mapping $\mathfrak{f}_{\mathfrak{g}}$. Then $\mathfrak{f}_{\mathfrak{g}}$ *is also* $(ToTr)_{\mathfrak{G}}$.

To prove the following proposition, we need to recall the relation in between has a dense set of $(PerPo)_{\mathfrak{G}}$ and $(\mathrm{InShPr})_{\mathfrak{G}}$ from [10].

Let pseudoequivalent $\mathfrak{f}_{\mathfrak{g}}$ be $(ToTr)_{\mathfrak{G}}$ mapping on $\mathfrak{M}$. It has a dense set of $(PerPo)_{\mathfrak{G}}$ if it has the $(\mathrm{InShPr})_{\mathfrak{G}}$.

Proposition 3.2: A sequence $\mathcal{F}_{\mathfrak{g}} = \{(\mathfrak{f}_n)_{\mathfrak{g}}, n \in \mathbb{N}_0, \mathfrak{g} \in \mathfrak{G}\}$ is $(OrCv)_{\mathfrak{G}}$ to $\mathfrak{f}_{\mathfrak{g}}$. then $\mathfrak{f}_{\mathfrak{g}}$ has a dense set of $(PerPo)_{\mathfrak{G}}$ if the mappings $(\mathfrak{f}_i)_{\mathfrak{g}}$ is $(ToTr)_{\mathfrak{G}}$ of $(\mathrm{InShPr})_{\mathfrak{G}}$ for all $0 \leq i \leq n$.

Proof: Let $\{(\mathfrak{f}_n)_{\mathfrak{g}}\}$ be a sequence $(OrCv)_{\mathfrak{G}}$to $\mathfrak{f}_g$. So, by Proposition 3.1, $\mathfrak{f}_{\mathfrak{g}}$ has $(\mathrm{InShPr})_{\mathfrak{G}}$. Suppose $(\mathfrak{f}_i)_{\mathfrak{g}}$ is $(ToTr)_{\mathfrak{G}}$ and since $\{(\mathfrak{f}_n)_{\mathfrak{g}}\}$ be $(OrCv)_{\mathfrak{G}}$ to $\mathfrak{f}_{\mathfrak{g}}$, then by [10], $\mathfrak{f}_{\mathfrak{g}}$ *also is* $(ToTr)_{\mathfrak{G}}$. Hence $\mathfrak{f}_{\mathfrak{g}}$ has a dense set of $(PerPo)_{\mathfrak{G}}$.

In. They studied how to get $(DenSmPrSets)_{\mathfrak{G}}$ from $(\mathrm{InShPr})_{\mathfrak{G}}$ *as follows:*

Let $\mathfrak{f}_{\mathrm{g}}$ be (Sdq) with $(ToTr)_{\mathfrak{G}}$ mapping on $\mathfrak{M}$. It has $(DenSmPrSets)_{\mathfrak{G}}$ if it has the $(\mathrm{InShPr})_{\mathfrak{G}}$.

Proposition 3.3: Let a sequence $\mathcal{F}_{\mathrm{g}} = \{(\mathfrak{f}_n)_{\mathrm{g}}, n \in \mathbb{N}_0, \mathrm{g} \in \mathfrak{G}\}$ be $(OrCv)_{\mathfrak{G}}$to $\mathfrak{f}_{\mathrm{g}}$. It has $(DenSmPrSets)_{\mathfrak{G}}$ if the mappings $(\mathfrak{f}_i)_{\mathrm{g}}$ is $(ToTr)_{\mathfrak{G}}$ with $(\mathrm{InShPr})_{\mathfrak{G}}$ for all $0 \le i \le n$.

Proof: Let $\{(\mathfrak{f}_n)_{\mathrm{g}}\}$ be a sequence $(OrCv)_{\mathfrak{G}}$to $\mathfrak{f}_{\mathrm{g}}$. *So, b*y Proposition 3.1, $\mathfrak{f}_{\mathrm{g}}$ has $(\mathrm{InShPr})_{\mathfrak{G}}$. Suppose $(\mathfrak{f}_i)_{\mathrm{g}}$ is $(ToTr)_{\mathfrak{G}}$ and since $\{(\mathfrak{f}_n)_{\mathrm{g}}\}$ be $(OrCv)_{\mathfrak{G}}$to $\mathfrak{f}_{\mathrm{g}}$, then**,** $\mathfrak{f}_{\mathrm{g}}$ *also is* $(ToTr)_{\mathfrak{G}}$. Hence, $\mathfrak{f}_{\mathrm{g}}$ has $(DenSmPrSets)_{\mathfrak{G}}$.

Corollary 3.4: *Let* $\mathfrak{f}_{\mathrm{g}}$ *be a* (Sdq) *on* $\mathfrak{M}$ *with* $(TotTr)_{\mathfrak{G}}$ *mapping. It is* $(WkMix)_{\mathfrak{G}}$ *if it has* $(\mathrm{InShPr})_{\mathfrak{G}}$.

Proposition 3.5: Let a sequence $\mathcal{F}_{\mathrm{g}} = \{(\mathfrak{f}_n)_{\mathrm{g}}, n \in \mathbb{N}_0, \mathrm{g} \in \mathfrak{G}\}$ be $(OrCv)_{\mathfrak{G}}$ to $\mathfrak{f}_{\mathrm{g}}$. The mapping $\mathfrak{f}_{\mathrm{g}}$ is $(WkMix)_{\mathfrak{G}}$ if the mappings $(\mathfrak{f}_i)_{\mathrm{g}}$ are $(TotTr)_{\mathfrak{G}}$ with the $(\mathrm{InShPr})_{\mathfrak{G}}$ for all $0 \le i \le n$.

Proof: Let $\{(\mathfrak{f}_n)_{\mathrm{g}}\}$ be a sequence $(OrCv)_{\mathfrak{G}}$ to $\mathfrak{f}_{\mathrm{g}}$. *So, b*y Proposition 3.1, $\mathfrak{f}_{\mathrm{g}}$ has $(\mathrm{InShPr})_{\mathfrak{G}}$. Suppose $(\mathfrak{f}_i)_{\mathrm{g}}$ is $(TotTr)_{\mathfrak{G}}$. Since $\{(\mathfrak{f}_n)_{\mathrm{g}}\}$ be $(OrCv)_{\mathfrak{G}}$ to $\mathfrak{f}_{\mathrm{g}}$, then by [11]**,** $\mathfrak{f}_{\mathrm{g}}$ *also is* $(TotTr)_{\mathfrak{G}}$. Then, $\mathfrak{f}_{\mathrm{g}}$ is $(WkMix)_{\mathfrak{G}}$.

Finally, from [12]**,** $(\mathrm{WkMix})_{\mathfrak{G}}$ implies $(\mathrm{Snv})_{\mathfrak{G}}$ (simply $(\mathrm{Snv})_{\mathfrak{G}}$) *as we see* in Remark 3.11**:** $(\mathrm{WkMix})_{\mathfrak{G}}$ implies $(\mathrm{Snv})_{\mathfrak{G}}$ as one can see from the proof of Proposition 3.10.

Corollary 3.6: *Let a sequence* $\mathcal{F}_{\mathrm{g}} = \{(\mathfrak{f}_n)_{\mathrm{g}}, n \in \mathbb{N}_0, \mathrm{g} \in \mathfrak{G}\}$ *be* $(OrCv)_{\mathfrak{G}}$ *to a mapping* $\mathfrak{f}_{\mathrm{g}}$ *on* $\mathfrak{M}$. *If the mappings* $(\mathfrak{f}_i)_{\mathrm{g}}$ *are* $(Snv)_{\mathfrak{G}}$ *and have* $(\mathrm{InShPr})_{\mathfrak{G}}$ *for all* $0 \le i \le n$*, then* $\mathfrak{f}_{\mathrm{g}}$ is $(Snv)_{\mathfrak{G}}$.

Proof: Directly from Proposition 3.2.

Corollary 3.7: *Let a sequence* $\mathcal{F}_{\mathrm{g}} = \{(\mathfrak{f}_n)_{\mathrm{g}}, n \in \mathbb{N}_0, \mathrm{g} \in \mathfrak{G}\}$ *be* $(OrCv)_{\mathfrak{G}}$ *to a mapping* $\mathfrak{f}_{\mathrm{g}}$ *on* $\mathfrak{M}$. *If the mappings* $(\mathfrak{f}_i)_{\mathrm{g}}$ *are* $(Snv)_{\mathfrak{G}}$ *and have* $(\mathrm{InShPr})_{\mathfrak{G}}$ *for all* $0 \le i \le n$*, then* $\mathfrak{f}_{\mathrm{g}}$ is $G-\ chaotic$.

Proof: Directly from Corollary 3.6.

Conclusion

We conclude all results in the following sentences, in compact metric $\mathfrak{G}$ −şpace:

1. $(\mathfrak{f}_{\mathrm{n}})_{\mathrm{g}}$ is $\mathfrak{G}$ −uniformly convergence to $\mathfrak{f}_{\mathrm{g}}$ then: for all n,

$$(\mathfrak{f}_{\mathrm{n}})_{\mathrm{g}} \text{ is } (\mathrm{InShPr})_{\mathfrak{G}} \rightarrow \mathfrak{f}_{\mathrm{g}} \text{ is} (\mathrm{InShPr})_{\mathfrak{G}} .$$

2. $(\mathfrak{f}_n)_g$ is $(OrCv)_{\mathfrak{G}}$ with $(ToTr)_{\mathfrak{G}}$ and $(\mathrm{InShPr})_{\mathfrak{G}}$: for all $0 \leq i \leq n$
then $(\mathfrak{f}_i)_g$ are dense set of $(PerPo)_{\mathfrak{G}} \rightarrow \mathfrak{f}_g$ is a dense set of $(PerPo)_{\mathfrak{G}}$.
3. $(\mathfrak{f}_n)_g$ is $(OrCv)_{\mathfrak{G}}$ with $(ToTr)_{\mathfrak{G}}$ and $(\mathrm{InShPr})_{\mathfrak{G}}$: for all $0 \leq i \leq n$
then $(\mathfrak{f}_i)_g$ are $(DenSmPrSets)_{\mathfrak{G}} \rightarrow \mathfrak{f}_g\ is\ (DenSmPrSets)_{\mathfrak{G}}$.
4. $(\mathfrak{f}_n)_g$ is $(OrCv)_{\mathfrak{G}}$: for all $0 \leq i \leq n$
then $(\mathfrak{f}_i)_g$ are $(TotTr)_{\mathfrak{G}}$ with the $(\mathrm{InShPr})_{\mathfrak{G}} \rightarrow \mathfrak{f}_g$ is $(WkMix)_{\mathfrak{G}}$.
5. $(\mathfrak{f}_n)_g$ is $(OrCv)_{\mathfrak{G}}$: for all $0 \leq i \leq n$
then $(\mathfrak{f}_i)_g$ are $(Snv)_{\mathfrak{G}}$ with $(\mathrm{InShPr})_{\mathfrak{G}} \rightarrow \mathfrak{f}_g$ is $(Snv)_{\mathfrak{G}}$.
6. $(\mathfrak{f}_n)_g$ is $(OrCv)_{\mathfrak{G}}$: for all $0 \leq i \leq n$
then $(\mathfrak{f}_i)_g$ are $(Snv)_{\mathfrak{G}}$ with $(\mathrm{InShPr})_{\mathfrak{G}} \rightarrow \mathfrak{f}_g$ is $G-\ chaotic$.

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