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Novel technique for solving nonlinear fractional differential equations via ANN

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Abstract

A novel, computationally efficient technique is suggested to find exact solution of nonlinear differential equations with order is fractional, This technique is extended to fractional partial differential equations and demonstrates high accuracy of implementation. Numerical experiments confirm its efficiency and reveal key advantages over existing approaches: it obviates the need for Adomian polynomials in handling nonlinear terms, as

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required by the ADM, and eliminates the use of Lagrange multipliers characteristic of the VIM.

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1. Introduction

Fractional calculus has become an indispensable framework for modeling a wide array of real-world phenomena, including reaction–diffusion processes [1], seismic-design considerations [2], water-wave propagation [3], control-engineering applications [4], population-growth dynamics [5], decentralized wireless networks [6], medical-science problems [7], fractional stochastic systems [8], signal-processing techniques [9], diffusion mechanisms [10, 11].

Nonlinear fractional partial differential equations (NF-PDEs) extend classical differential equations by incorporating nonlocal, power-law memory effects of arbitrary positive order, making them indispensable for accurately modeling complex phenomena across disciplines including engineering, medicine, and applied science such: physics, applied mathematics, biology, chemistry, and geology. Unlike integer-order models, fractional formulations preserve hereditary characteristics and continuous functional values, enabling deep insights into systems such as business dynamics, viscoelastic surface behavior, psychological processes, human-love interactions, and well-being models. However, most NF-PDEs lack closed-form solutions, prompting the development of various analytical and numerical techniques. For example, the authors employed a Galerkin with Legendre multi-wavelet approach to solve fractional order modified KdV–ZK equation; other authors such author, utilized a homotopy perturbation (HPM) with transform of type Sumudu framework for spatial diffusion in biological populations; and other authors used an iterative with transform method of type Sumudu for Cauchy reaction–diffusion problems of time-fractional order. Transform-based variational iteration strategies have been advanced by Authors. through the Elzaki–VIM combination, while Dehestani and colleagues have proposed discrete Genocchi–Laguerre schemes and Genocchi–Petrov–Galerkin methods for fractional order to address variable order multiterm NF-PDEs and time–space independent variable Fokker–Planck equations. More recently, demonstrated the efficacy techniques of homotopy analysis (HAM) with residual power series and transform for NF-PDEs with proportional delays.

Researchers have recently developed numerous numerical techniques to solve various kinds of fractional order partial differential equations with different dimensions. In got approximate solution for nonlinear fractional PDEs by integrating the multi-Laplace transform with the Adomian decomposition method (ADM). Further advanced this area by proposing an iterative algorithm for TF-PDEs with proportional delays. Motivated by the broad applicability of NT-FPDEs and the ongoing search for efficient solvers, the authors proposed efficient Transform (YTADM). Yang's transform (YT) converts the fractional differential operators into algebraic forms [6].

2. Basic Ideas of Suggested Method

In this section, we present basic concepts of ANN for TFPDE, including struct-ure and formulations for ANN with initial conditions fractional order equations.

ANNs [6] will be used to solve different types of extremely difficult real-life.

To solve a problem using computational techniques, such as ANNs approaches, it must first be well characterized. The general presentation of a 3-lyers (FFNN) feed forward with signal input, m hidden neurons, and an output.

Consider the IVP for non-linear fractional (TFPDE) of order $\alpha, n-1 < \alpha \leq n$

$$L_t^{\alpha}\mathfrak{U}(z,\tau) + \mathcal{T}(\mathfrak{U}(z,\tau)) + \mathcal{Q}(\mathfrak{U}(z,\tau)) = \mathcal{J}(z,\tau) \tag{1}$$

Depending on initial conditions:

$$\frac{\partial^k u(z,t)}{\partial t^k}|_{t=0} = f_k(z), k \in \{0,1,\dots,n\} \tag{2}$$

where $L_t^{\alpha}(u) = \frac{\partial^{\alpha}}{\partial t^{\alpha}}, n-1 < \alpha \leq n,$ is the Caputo fractional differential operator of highest fractional order derivative depending on time variable *t*, u(z, t) is unrecognized function must be determined, *z* is the space variable, *t* is time variable, *R*(u) is remind of the linear differential operator, g(z, t) is inhomogeneous function and N(u) = f(u(z, t)) is non-linear term.

$$\mathfrak{A}(z,\tau) = \mathfrak{A}_0(z) + \tau\,\mathfrak{A}_1(z) + \tau^2\mathfrak{A}_2(z) + \cdots = \sum_{k=0}^{\infty} \tau^k\,\mathfrak{A}_k(z) \tag{3}$$

where,

$$\mathfrak{A}_k(z) = \frac{1}{k!}\frac{\partial^k\mathfrak{A}(z,t)}{\partial t^k}|_{t=0} \tag{4}$$

The next step is to determine the terms $\mathfrak{A}_n (n = 0,1,2,\dots)$.

Now, applying the Riemann-Liouville theorem for integral operator of fractional order α on both side of equation (1).

$$\int_{\tau}^{\alpha} L_t^{\alpha}\,\mathfrak{A}(z,\tau) + \int_{\tau}^{\alpha} \mathcal{T}\,(\mathfrak{A}(z,\tau)) + \int_{\tau}^{\alpha} \mathcal{Q}(\mathfrak{A}(z,\tau)) = \int_{\tau}^{\alpha} \mathcal{J}(z,\tau) \tag{5}$$

equation (5) becomes:

$$\mathfrak{A}(z,\tau) - \sum_{k=0}^{n-1}\frac{\tau^k}{k!}\frac{\partial^k\mathfrak{A}(z,\tau)}{\partial\tau^k}|_{t=0} = -J_t^{\alpha}[\mathcal{T}(\mathfrak{A}) + \mathcal{Q}(\mathfrak{A})] + J_t^{\alpha}\mathcal{J}(z,\tau) \tag{6}$$

Depending on Eq. (2) we obtain that:

$$\mathfrak{A}(z,t) = \sum_{k=0}^{n-1}\frac{t^k}{k!}f_k(x) - J_t^{\alpha}[R(u) + N(u)] + J_t^{\alpha}g(z,t) \tag{7}$$

where:

$$J_t^{\alpha}(R(\mathfrak{U})) = J_t^{\alpha}[R(\textstyle\sum_{k=0}^{\infty}\mathfrak{U}_k(z)\,t^k)] = \sum_{k=0}^{\infty}\mathcal{T}\left(u_k(z)\frac{k!}{(\alpha+k)!}t^{\alpha+k}\right) \tag{8}$$

In Eq. (7) rewrite the nonlinear term $\mathcal{Q}(\mathfrak{U})$, such the follow:

$$\mathcal{Q}(\mathfrak{U}) = \sum_{k=0}^{\infty} N_k\, t^k$$

Such that:

$$N_k(z) = \frac{1}{k!}\frac{\partial^k N(u(z,t))}{\partial t^k}|_{t=0} \tag{9}$$

Thus:

$$J_t^{\alpha}(N(u)) = J_t^{\alpha}[\sum_{k=0}^{\infty} N_k\, t^k] = \sum_{k=0}^{\infty} N_k\ J_t^{\alpha}(t^k) = \sum_{k=0}^{\infty} N_k \frac{k!}{(\alpha+k)!} t^{\alpha+k} \tag{10}$$

Again, rewrite the nonhomogeneous part such as:

$$G(z,t) = J_t^{\alpha}(g(z,t)) = \sum_{k=0}^{\infty} g_k \frac{t^k}{k!}$$

Where

$$g_k = \frac{1}{k!}\frac{\partial^k G(z,t))}{\partial t^k}|_{t=0} \tag{11}$$

Substituting Eqs. (8), (9), and (10) in Eq. (7) to have:

$$\mathfrak{U}(z,t) = \sum_{k=0}^{n-1} \frac{t^k}{k!} f_k(z) - \sum_{k=0}^{\infty} \mathcal{T}\left(\mathfrak{U}_k(z)\frac{k!}{(\alpha+k)!} t^{\alpha+k}\right) - \sum_{k=0}^{\infty} \mathcal{Q}_k \frac{k!}{(\alpha+k)!} t^{\alpha+k} + \sum_{k=0}^{\infty} \mathcal{J}_k \frac{t^k}{k!} \tag{12}$$

$$\because j \geq n\ andwith\ n-1 < \alpha \leq n \rightarrow j \geq n\ \geq \alpha > n-1$$

$$\mathfrak{U}_j(z) = \frac{1}{j!}\frac{\partial^j \mathfrak{U}(z,t)}{\partial t^j}|_{t=0} = \frac{1}{j!}\frac{\partial^j \mathfrak{U}(z,t)}{\partial t^j}[\sum_{k=0}^{n-1}\frac{t^k}{k!} f_k(z) - \sum_{k=0}^{\infty}\frac{k!}{(n+k)!}(\mathcal{T}(\mathfrak{U}_k(z)) + \mathcal{Q}_k) t^{n+k} + \sum_{k=0}^{\infty} g_k\, \tau^k]_{t=0} \forall\, j \geq n \tag{13}$$

$$\frac{\partial^j}{\partial \tau^j}\tau^k = 0\ If\ k < J \text{ and } \frac{\partial^j}{\partial t^j}\tau^k = \frac{k!}{(k-J)!}\tau^{k-j}\ if\ k \geq J$$

Thus Eq. (12) becomes:

$$u_j(z) = \frac{1}{j!}[-\sum_{k=j-n}^{\infty} \frac{k!}{(n+k)!}(Ru_j(z) + N_k)\frac{(n+k)!}{(n+k-j)!} t^{n+k-j} + \sum_{k=0}^{\infty} g_k \frac{k!}{(k-j)!} t^{k-j}]_{t=0} \tag{14}$$

$$u_j(x) = \frac{1}{j!}\left[-\frac{(j-n)!}{0!}(R(u_{j-n}) + N_{j-n}) + g_j \frac{j!}{0!} t^{k-j}\right]$$

Hence

$$u_j(x) = g_j - \frac{(j-n)!}{j!}(R(u_{j-n}) + N_{j-n}), j \geq n \tag{15}$$

Finally, substituting Eq. (15) in Eq. (3) to get $u(x,t)$.

3. Clarifying Examples

In this section, some clarifying examples are presented about solving non-linear PDEs have time independent variable with fractional order by using new suggested method.

Example 3.1: Consider the non-linear equation with fractional order in time independent variable:

$$D_t^{\alpha}\omega(z,\tau) = \omega_{zz}(z,\tau) - \omega_z(z,\tau) + \omega(z,\tau)\omega_z(z,\tau) - \omega^2(z,\tau) + \omega(z,\tau), 0 < \alpha \leq 1$$

together initial condition: $\mathfrak{A}(z,0) = e^z$, we noted that $\boldsymbol{\omega} = \mathfrak{A}$

It is clear that: $\mathcal{T}(\mathfrak{A}) = \mathfrak{A}_{zz} - \mathfrak{A}_z + \mathfrak{A}$, $\mathcal{Q}(\mathfrak{A}) = \mathfrak{A}\mathfrak{A}_z - \mathfrak{A}^2, g(z,y,t) = 0$ *then*

$$g_k = 0, \forall k = 0,1,2,\dots$$

From ICs $\mathfrak{A}_0 = e^z$. From equation (15) we get:

$$\omega_1(z) = g_1 + \frac{(1-1)!}{1!}(\mathcal{T}(\omega_0(z)) + \mathcal{Q}_0) = (\omega_{0zz} - \omega_{0z} + \omega_0) + (\omega_0\omega_{0z} - \omega_0^2)$$

$$= (2\,e^z - e^z) + ((e^z)^2 - e^{2z}) = e^x$$

$$\mathfrak{A}_2(z) = g_2 + \frac{(2-1)!}{2!}\left(\mathcal{T}\left(\mathfrak{A}_1(z)\right) + \mathcal{Q}_1\right)$$

$$= \frac{1}{2!}(\omega_{1zz} - \omega_{1z} + \omega_1) + (\omega_0\omega_{1z} + \omega_{0z}\omega_1 - 2\omega_0\omega_1)$$

$$= (2\,e^z - e^z) + (e^{2z} - e^{2z} - 2e^{2z}) = \frac{1}{2!}e^z$$

$$\mathfrak{A}_3(z) = g_3 + \frac{(3-1)!}{3!}(\mathcal{T}(\omega_2(z)) + \mathcal{Q}_2)$$

$$= \frac{2!}{3!}(\omega_{2z} - \omega_{2z} + \omega_2) + (\omega_0\omega_{2z} + \omega_{1z}\omega_1 + \omega_{0z}\omega_2 + \omega_1\omega_{1x} - 2\omega_0\omega_2 - 2\omega_1\omega_1)$$

$$= (2\,e^z - e^z) + \left(\frac{e^{2z}}{2!} + e^{2z} + \frac{e^{2z}}{2!} + e^{2z} - 2\frac{e^{2z}}{2!} - 2e^{2z}\right) = \frac{1}{3!}e^z$$

Then by equation (3), we have:

$$\mathfrak{A}(z,\tau) = e^z + e^z\tau + e^z\frac{\tau^2}{2!} + e^z\frac{\tau^3}{3!} + \cdots = e^z\left(1 + \tau + \frac{\tau^2}{2!} + \frac{\tau^3}{3!} + \cdots\right) = e^z E_1(\tau)$$

$$\mathfrak{A}(z,\tau) = e^{z+\tau}.$$

A Comparison between exact and suggested solution is illustrated in Figure 1. The stability of proposed solution for different values of α is illustrated in Figure 2.

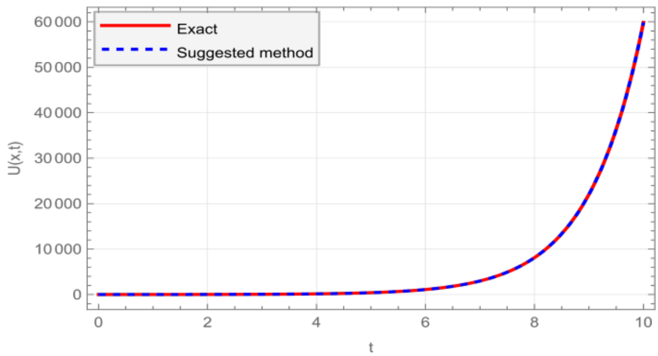

Figure 1
Comparing between exact and suggested results for Example 3.1

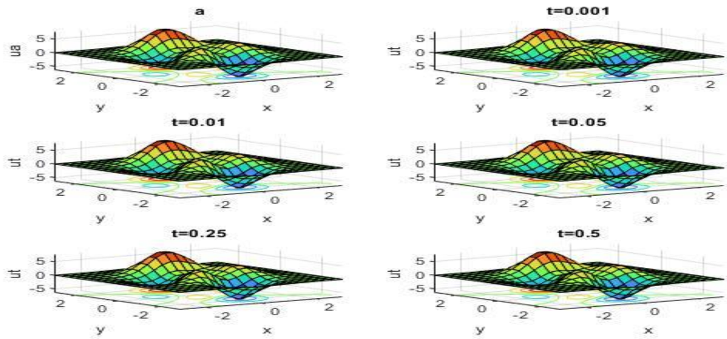

Figure 2
Stability of suggested method for Example 3.1

Example 3.2: Consider the following homogeneous nonlinear equation and fractional in time independent variable [5]:

$$D_t^{\alpha}\mathfrak{U}(z,\tau)+\mathfrak{U}(z,\tau)\mathfrak{U}_z(z,\tau)-\mathfrak{U}(z,\tau)\big(1-\mathfrak{U}(z,\tau)\big)=0, \alpha\in(0,1]$$

Depending on initial condition: $\mathfrak{U}(z,0)=e^{-z}$

It is clear that: $\mathfrak{l}(\mathfrak{U})=-\mathfrak{U}_z$, $\mathcal{Q}(\mathfrak{U})=\mathfrak{U}\,\mathfrak{U}_z+\mathfrak{U}^2$, $g(z,\tau)=0$ *then* $g_k=0, \forall k=0,1,2,.$

We denote $\mathfrak{A}$ as $\boldsymbol{\omega}$. From equation (9) we get:

$$\mathcal{Q}_0(z)=\frac{1}{0!}\frac{\partial^0\mathcal{Q}(\omega(z,\tau))}{\partial\tau^0}|_{\tau=0}=\omega_0\,\omega_{0x}+\omega_0^2$$

$$N_1(z)=\frac{1}{1!}\frac{\partial^1\mathcal{Q}(\omega(z,\tau))}{\partial\tau^1}|_{\tau=0}=\ \omega_0\,\omega_{1x}+\omega_1\,\omega_{0x}+2\omega_0\,\omega_1$$

$$N_2(x)=\frac{1}{2!}\frac{\mathcal{Q}(\mathfrak{U}(z,\tau))}{\partial\tau^2}|_{t=0}=\ \mathfrak{U}_0\,\mathfrak{U}_{2z}+\mathfrak{U}_1\,\mathfrak{U}_{1z}+\mathfrak{U}_2\,\mathfrak{U}_{0z}+\mathfrak{U}_1\,\mathfrak{U}_{1z}+2\mathfrak{U}_0\,\mathfrak{U}_2+2\mathfrak{U}_1\,\mathfrak{U}_1$$

From ICs $\mathfrak{U}_0=e^{-z}$. By equation (15) we have:

$$\mathfrak{U}_1(z)=g_1-\frac{(1-1)!}{1!}(\mathfrak{A}(\mathfrak{A}_{1-1})+N_{1-1})=0-\frac{1}{1!}\big((-\mathfrak{A}_0)+(\mathfrak{A}_0\,\mathfrak{A}_{0z}+\mathfrak{A}_0^2)\big)\ =e^{-z}$$

$$\mathfrak{A}_2(z)=g_2-\frac{(2-1)!}{2!}(\mathcal{T}(\mathfrak{A}_{2-1})+\mathcal{Q}_{2-1})$$

$$=0+\frac{1}{2!}\big((-\mathfrak{A}_1)+(\mathfrak{A}_0\,\mathfrak{A}_{1z}+\mathfrak{A}_1\,\mathfrak{A}_{0z}+2\mathfrak{A}_0\,\mathfrak{A}_1)\big)\ =\frac{1}{2!}e^{-z}$$

$$\mathfrak{A}_3(x)=g_3-\frac{(3-1)!}{3!}(\mathcal{T}(\mathfrak{A}_{3-1})+\mathcal{Q}_{3-1})$$

$$=0+\frac{2!}{3!}\big((-\omega_2)+(\omega_0\,\omega_{2\tau}+\omega_1\,\omega_{1z}+\omega_2\,\omega_{0z}+\omega_1\,\omega_{1z}+2\omega_0\omega_2+2\omega_1\,\omega_1)\big)=\frac{1}{3!}e^{-z}$$

$$\mathfrak{U}_4(x)=g_4-\frac{(4-1)!}{4!}(\mathcal{T}(\omega_{4-1})+\mathcal{Q}_{4-1})$$

$= 0 - \frac{3!}{4!}\left((-\omega_3) + (\omega_{0z}\,\omega_3 + \omega_{1z}\omega_2 + \omega_{2z}\,\omega_1 + \omega_{3z}\,\omega_0 + 2\omega_{0z}\,\omega_{3z} + 2\omega_{1z}\,\omega_{2z})\right) = \frac{1}{4!}e^{-z}$

Then by equation (3), we have:

$$\mathfrak{A}(z,\tau) = \mathfrak{A}_0(z) + \mathfrak{A}_1(z)\tau + \mathfrak{A}_2(z)\tau^2 + \mathfrak{A}_3(z)\tau^3 + \omega_4(z)\tau^4 + \cdots$$

$$= e^{-z} + \tau e^{-z} + \frac{\tau^2}{2!}e^{-z} + \frac{\tau^3}{3!}e^{-z} + \frac{\tau^4}{4!}e^{-z} + \cdots = e^{-z}\left(1 + \tau + \frac{\tau^2}{2!} + \frac{\tau^3}{3!} + \cdots\right)$$

$= e^{-z}E_1(\tau) = e^{-z+\tau}$; So, $\mathfrak{A}(z,\tau) = e^{-z+\tau}$

The difference between exact and suggested solution is clearing in Figure 3.

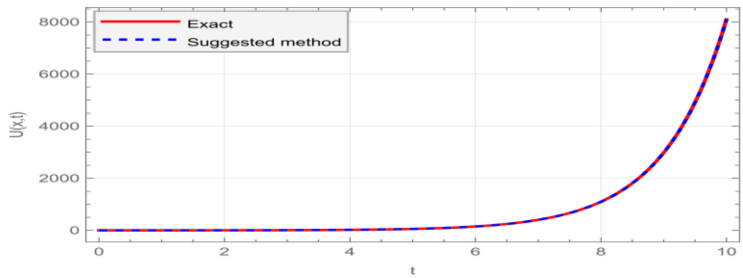

Figure 3
Difference between exact & suggested solution for Example 3.2

4. Conclusion

The present study suggested a novel technique for deriving efficient solutions of the nonlinear fractional differential equation. To validate and demonstrate the method's efficacy, two distinct examples are solved in detail. The outcomes obtained through the proposed approach coincide precisely with the established exact solutions, confirming its accuracy. Hence, this method represents a promising tool for tackling a wide range of non-linear partial differential equations have fractional order arising in diverse scientific fields and dynamical systems across applied mathematics and engineering.

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