Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 29 (2026), No. 5-B, pp. 1671–1681
DOI : 10.47974/JIM-2560

Neutrosophic crisp triple tritopological group

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Abstract

In this paper, we establish a neuromorphic crisp triple tri topological group. The neuromorphic crisp triple, a topological space and continuous function, is defined and used in constructing the new concept. We study the basic properties of this new concept, including the fundamental system of NBHDS.

Subject Classification: *54H11, 54A05.*

Keywords: *Crisp triple, Topology, Neutrosophic.*

1. Introduction

The fuzzy set was introduced in 1965 by Zadeh [1]. Atanassov [2] introduced the concept of tuition as a fuzzy set in 1986, using the notation of fuzzy sets. Neutrosophic set theory was proposed in 1998 by Smarandache [3]. In 2023, introduced a new class of Neutrosophic sets, called a Neutrosophic crisp triple set Habeeb and Hamzah [4]. In 2024, Abdulsada, Al-Swidi, and Hadi [5], introduced a new class of Neutrosophic crisp sets, and then they presented some operations via these sets, like NCT −intersection, NCT −union, and algebraic NCT−difference to arrive at the algebraic ring construction. In 2025, Habeeb, Abbas, and Kadhim [6] introduced a center bi- topological space. In this paper, we introduced the concept of a Neutrosophic crisp triple tri topological group by first defining a Neutrosophic crisp triple tri topological space and a continuous function, and then using these concepts to construct the new concept.

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2. Basic Concepts

Definition 2.1: [5]: $X \neq \emptyset$ a neutrosophic crisp triple set NCT_A is an object having the form:

$NCT_A = \langle A_1, A_2, A_3 \rangle$. Where $A_1, A_2, A_3 \subseteq X$ satisfying $A_1 \subseteq A_2$ and $A_2 \cap A_3 = \phi$ and $NCT(X) = \{NCT_A = \langle A_1, A_2, A_3 \rangle : A_1 \subseteq A_2$ and $A_2 \cap A_3 = \phi\}$ is the collection of all NCT - sets on X.

From this definition, we see that if $A_3 = X$ then $A_1 = A_2 = \phi$ and if $A_2 = X$ then $A_3 = \phi$ finally if $A_1 = X$ then $A_2 = X$ and $A_3 = \varphi$.

Definition 2.2: [5] Let $NCT_A = \langle A_1, A_2, A_3 \rangle$ $NCT_B = \langle B_1, B_2, B_3 \rangle$ and are NCT −a non-empty set X over sets. Therefore:

1. NCT_A is a NCT − subset of NCT_B if $A_3 \supseteq B_3$ and $A_1 \subseteq B_1$, $A_2 \subseteq B_2$ We write $NCT_A \subseteq NCT_B$
2. $NCT_A = NCT_B$ iff $NCT_A \subseteq NCT_B$ and $NCT_B \subseteq NCT_A$.
3. The NCT −complement of NCT − set NCT_A is $CNCT_A = \langle A_3, A_2^c, A_1 \rangle$.
4. $NCT_A \cup NCT_B = \langle A_1 \cup B_1, A_2 \cup B_2, A_3 \cap B_3 \rangle$ is a crisp triple set that is neutrosophic in union (NCT −union set).
5. $NCT_A \cap NCT_B = \langle A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3 \rangle$ is the intersection a crisp triple set that is neutrosophic (NCT −intersection sets)
6. $NCT_A - NCT_B = NCT_A \cap NCT_B$.
7. $NCT_A \Delta NCT_B = (NCT_A \cap CNCT_B) \cup (CNCT_A \cap NCT_B)$.

Now we will explain the concepts NCT −universal and NCT −null set which are among the basic concepts in our work.

Definition 2.3: [5] Let $X \neq \phi$ Then:

1. $NCT_X = \langle X, X, \phi \rangle$ is NCT −universal set.
2. $NCT_\phi = \langle \phi, \phi, X \rangle$ is NCT −null set. Clearly $CNCT_X = NCT_\varphi$ and $CNCT_\phi = NCT_X$.

The three most significant relationships NCT −union, NCT −intersection and NCT −complement listed in the following properties.

Definition 2.4: [7] Let $X \neq \phi$ and $p \in X$. So NCT −points ($NCTP$) are structure:

$$NCT_p = \langle \{p\}, \{p\}, \{p\}^c \rangle.$$

Definition 2.5: [7] Let $X \neq \phi$ and $p \in X$. $NCT_A = \langle A_1, A_2, A_3 \rangle$. Then the NCT −belong as follows:

$$NCT_{\tilde{p}} \in NCT_A \text{ if } p \in A_1$$
$$NCT_{\tilde{p}} \tilde{\in} NCT \text{ if } p \notin A_1.$$

Definition 2.6: [6] Let $f : X \to Y$ be a function. Define NCT –function $f_{NCT} : NCT(X) \to NCT(Y)$ by:

1. If $NCT_A\langle A_1, A_2, A_3\rangle \in NCT_A(X)$ then
   $f_{NCT}(NCT_A) = \langle f(A_1), f(A_2), f-(A_3)\rangle$ where $f-(A_3) = Y - (f(X - A_3))$.
2. If $NCT_B\langle R_1, R_2, R_3\rangle \in NCT_B(Y)$ then
   $f_{NCT}{}^{-1}(NCT_B) = \langle f^{-1}(R_1), f^{-1}(R_2), f^{-1}(R_3)\rangle$.

Definition 2.7: [5] The pair (NCT_X, τ^X_{NCT}) is called neutrosophic crisp triple topological space $(NCTT)$ over NCT (X), if you achieve the following:

1. $NCT_X, NCT_\phi \in \tau^X_{NCT}$ ($\in$ is the classical belonging).
2. τ^X_{NCT} is closed under the finite NCT −intersection.
3. τ^X_{NCT} is closed under the NCT −union of every subfamily of τ_{NCT}.

Any member of τNCT is called NCT −open and the complement is called NCT −closed.

Definition 2.8: [4] Let NCT_A and NCT_B be two NCT-sets we define the NCT Cartesian product as follows:

$$NCT_A \times_{NCT} NCT_B = \langle A_1 \times B_1, A_2 \times B_2, A_3 \times B_3\rangle$$

Definition 2.9: Let $(NCT_{X_1}, \tau^{X_1}_{1NCT})$ and $(NCT_{X_2}, \tau^{X_2}_{2NCT})$ be two NCT-topological space we define the NCT- product topology the family which has the base:

$\{NCT_A \times_{NCT} NCT_B / NCT_A \in \tau^{X_1}_{1NCT}$ and $NCT_B \in \tau^{X_2}_{2NCT}\}$ and denoted by:

$$(NCTX_1 \times_{NCT} NCTX_2, \tau^{X_1}_{1NCT} \times_{NCT} \tau^{X_2}_{2NCT}).$$

Definition 2.10: Let $(NCT_{X_1}, \tau^{X_1}_{1NCT}, \tau^{X_1}_{2NCT}, \tau^{X_1}_{3NCT})$ and $(NCT_{X_2}, \tau^{X_2}_{1NCT}, \tau^{X_2}_{2NCT}, \tau^{X_2}_{3NCT})$ be two NCT- tri topological space.

Then $\left(NCTX_1 \times_{NCT} NCTX_2, P^{X_1\times X_2}_{1NCT}, P^{X_1\times X_2}_{2NCT}, P^{X_1\times X_2}_{3NCT}\right)$ is called the NCT-product tri topological space. Where $P^{X_1\times X_2}_{1NCT}$ NCT_{X_1} -product tri topology with respect $\tau^{X_1}_{1NCT}$ and $\tau^{X_2}_{1NCT}$, $P^{X_1\times X_2}_{2NCT}$ NCT-product tri topology write $\tau^{X_1}_{2NCT}$ and $\tau^{X_2}_{2NCT}$ and $P^{X_1\times X_2}_{3NCT}$ NCT -product tri topology of $\tau^{X_1}_{3NCT}$ and $\tau^{X_2}_{3NCT}$.

3. Neutrosophic Crisp Triple Tri topological Space

Definition 3.1: Let NCT_X be a neutrosophic crisp triple set non- empty and τ_{1NCT}^X, τ_{2NCT}^X and τ_{3NCT}^X be three NCTTS on NCT_X. NCT_X ogether with this three NCTTS is called a NCT. Tvi topological space and is denoted to it by $(NCT_X, \tau_{1NCT}^X, \tau_{2NCT}^X, \tau_{3NCT}^X)$.

Example 3.2: Let $X = \{1,2,3\}, \tau_{1NCT} = \{NCT_\phi, NCT_X\}, \tau_{2NCT} = \{NCT_X, NCT_\phi,$ $< \{1\}, \{1,2\}, \{3\} >\}$

$$\tau_{3NCT} = \{NCT_X, NCT_\phi, < \{2\}, \{2,3\}, \{1\} >\}$$

then $(NCT_X, \tau_{1NCT}^X, \tau_{2NCT}^X, \tau_{3NCT}^X)$ is NCT- tri topological space.

Note 3.3. Any NCT- topological space is a NCT- tri topological space. Let (NCT_X, τ_{NCT}^X) be a NCT- topological space then $(NCT_X, \tau_{NCT}^X, \tau_{NCT}^X, \tau_{NCT}^X)$ is a NCT- tri topological space.

Definition 3.4: Let $(NCT_X, \tau_{1NCT}^X, \tau_{2NCT}^X, \tau_{3NCT}^X)$ be NCT- tri topological space. NCT_A is called NCT- tri open set if $NCT_A \in, \tau_{1NCT}^X \cap \tau_{2NCT}^X \cap \tau_{3NCT}^X$.

Definition 3.5: Let $(NCT_X, \tau_{1NCT}^X, \tau_{2NCT}^X, \tau_{3NCT}^X)$ be NCT- tri topological space. NCT_B is called NCT- tri closed set if NCT_B is NCT- tri open.

Definition 3.6: Let $(NCT_X, \tau_{1NCT}^X, \tau_{2NCT}^X, \tau_{3NCT}^X)$ be NCT- tri topological space. A NCT_B is called NCT-trinnd of $NCT_{\tilde{p}}$ if $\exists NCT_u$ be NCT- tri open s.t $NCT_{\tilde{p}} \in NCT_u \subseteq NCT_B$.

Definition 3.7: Let

$$f_{NCT} : (NCT_X, \tau_{1NCT}^X, \tau_{2NCT}^X, \tau_{3NCT}^X) \to (NCT_Y, \tau_{1NCT}^Y, \tau_{2NCT}^Y, \tau_{3NCT}^Y)$$

is called NCT tri- continuous if $f_{NCT}^{-1}(NCTA)$ is NCT-tri open in NCT_X for every $NCTA : NCT -$ tri open in NCT_Y.

Remark 3.8: $f_{NCT} : (NCT_X, \tau_{1NCT}^X, \tau_{2NCT}^X, \tau_{3NCT}^X) \to (NCT_Y, \tau_{1NCT}^Y, \tau_{2NCT}^Y, \tau_{3NCT}^Y)$ is NCT-tri continuous if $NCT_{\tilde{p}} \in NCT_X$ and NCT or NCT-tri open containing $f_{NCT}(NCT_{\tilde{p}}) \exists NCT_u$ $NCTtri$ open in NCT_X s.t $NCT_{\tilde{p}} \in NCT_u$ and $f_{NCT}(NCT_u) \subseteq NCT_v$.

Definition 3.9: Let

$$f_{NCT} : (NCT_X, \tau_{1NCT}^X, \tau_{2NCT}^X, \tau_{3NCT}^X) \to (NCT_Y, \tau_{1NCT}^Y, \tau_{2NCT}^Y, \tau_{3NCT}^Y)$$

is called NCT- tri open if $f_{NCT}(NCT_u)$ is NCT- tri open for every NCT_u NCT-tri open in NCT_X.

Definition 3.10: A f_{NCT} is called NCT- tri closed if $f_{NCT}(NCT_u)$ is NCT- tri closed for every NCT_u NCT-tri closed in NCT_X.

Definition 3.11: A f_{NCT} is called NCT- tri homomorphism if f_{NCT} is bijective f_{NCT} is NCT-tri continues and f_{NCT}^{-1} is NCT-tri continues.

Remark 3.12: A f_{NCT} is called NCT- tri homomorphism if f_{NCT} is bijective f_{NCT} is NCT-tri continues, NCT-tri open (closed).

4. Neutrosophic Crisp Triple Tritopological Group

Definition 4.1: Let (G,M) be a group we define:

i) $M_{NCT}: NCT(G)\times NCT(G)\to NCT(G)$ as follows:

$M_{NCT}(NCT_{\tilde{p}}, NCT_{\tilde{q}}) = M_{NCT}(\langle\{p\},\{p\},\{p\}^c\rangle)$

$\langle\{q\},\{q\},\{q\}^c\rangle = \{<\{p*q\},\{p*q\}^c>\}$

and denoted to it by

$M_{NCT}(NCT_{\tilde{p}}, NCT_{\tilde{q}}) = NCT_{\tilde{p}} *_{NCT} NCT_{\tilde{q}}$.

ii) $V_{NCT}: NCT(G)\to NCT(G)$

$V_{NCT}(NCT_{\tilde{p}}) = (NCT_{\tilde{p}})^{-1} = \{<\{p^{-1}\},\{p^{-1}\},\{p^{-1}\}^c>\} = NCT_{(\tilde{p})^{-1}}$

Definition 4.2: Let (G,M) be a group and $(NCT_G, \tau_{1NCT}^G, \tau_{2NCT}^G, \tau_{3NCT}^G)$ be NCT-tri topological space and M_{NCT} and V_{NCT} are NCT- tri continuous then the ordered pentagon $(NCT_G, \tau_{1NCT}^G, \tau_{2NCT}^G, \tau_{3NCT}^G, M_{NCT})$ is called NCT-tri topological group.

Note 4.3: For simply we write NCT_G is stead of the ordered pentagon $(NCT_G, \tau_{1NCT}^G, \tau_{2NCT}^G, \tau_{3NCT}^G)$.

Remark 4.4: Let NCT_G be a NCT- tri topological group:

1) Since M_{NCT} is NCT- tri continuous then for each NCT-nhhd of NCT_N of $NCT_{\tilde{p}} *_{NCT} NCT_{\tilde{h}}$ there exists NCT-tri open nhds NCT_u and NCT_v of $NCT_{\tilde{p}}$ and $NCT_{\tilde{h}}$ respectively such that

$$M_{NCT}(NCT_u \times_{NCT} NCT_v) = NCT_u \times_{NCT} NCT_v \subseteq NCT_N$$

2) Since V_{NCT} is NCT- tri continuous then for each NCT-nhhd of NCT_N of $NCT_{(\tilde{p})^{-1}}$ there exists NCT-tri open nhds NCT_u of $NCT_{\tilde{p}}$ such that $V_{NCT}(NCT_u) = NCT_{u^{-1}} \subseteq NCT_N$.

Proposition 4.5: NCT_G be a NCT- tri topological group if the NCT- function $(NCT_{\tilde{p}}, NCT_{\tilde{h}}) \to NCT_p *_{NCT} NCT_{(\tilde{h})^{-1}}$ is NCT- tri continuous.

Proof: NCT_N NCT-tri open nhd of $NCT_{\tilde{p}} *_{NCT} NCT_{(\tilde{h})^{-1}}$ then, since M_{NCT} is NCT- tri continuous there $NCT_{(\tilde{h})^{-1}}$ exists NCT-tri open nhds NCT_u and NCT_v of $NCT_{\tilde{p}}$ and respectively such that $NCT_u \times_{NCT} NCT_v \subseteq NCT_N$ since NCT_v is NCT-tri open nhd of $NCT_{(\tilde{h})^{-1}}$ and NCT_v NCT- tri continuous there exists NCT-tri open nhds NCT_w of $NCT_{\tilde{h}}$ s.t $NCT_{w^{-1}} \subseteq NCT_V$.

Hence $NCT_u \times_{NCT} NCT_{w^{-1}} \subseteq NCT_u \times_{NCT} NCT_v \subseteq NCT_N$. There for $(NCT_{\tilde{p}}, NCT_{\tilde{h}}) \to NCT_{\tilde{p}} *_{NCT} NCT_{(\tilde{h})^{-1}}$ is NCT- tri continuous.

Conversely 4.6: The NCT- function

$(NCT_{\tilde{p}}, NCT_{\tilde{h}}) \to NCT_p *_{NCT} NCT_{(\tilde{h})^{-1}}$ is NCT- tri continuous.

i) To prove V_{NCT} is NCT- tri continuous. Let NCT_N be NCT-tri open nhd of $NCT_{(\tilde{p})^{-1}}$. Then NCT_N is NCT-tri open nhd of

$NCT_{\tilde{e}} *_{NCT} NCT_{(\tilde{p})^{-1}} = NCT_{(\tilde{p})^{-1}} *_{NCT} NCT_{\tilde{e}} = NCT_{(\tilde{p})^{-1}}$

there exists NCT-tri open nhds NCT_u and NCT_v of $NCT_{\tilde{e}}$ and $NCT_{(\tilde{p})^{-1}}$ respectively s.t $NCT_u *_{NCT} NCT_{v^{-1}} \subseteq NCT_N$.

$$NCT_{V^{-1}} \subseteq NCT_u *_{NCT} NCT_{v^{-1}} \subseteq NCT_N$$

Thus $NCT_{V^{-1}} \subseteq NCT_N$ hence

V_{NCT} is NCT- tri continuous.

ii) To prove M_{NCT} is NCT- tri continuous. Let NCT_N NCT-tri open of $NCT_{\tilde{p}} *_{NCT} NCT_{\tilde{h}}$ then, there exists NCT-tri open nhds NCT_u and NCT_v of $NCT_{\tilde{p}}$ and $NCT_{\tilde{h}}$ respectively such that $NCT_u \times_{NCT} NCT_v \subseteq NCT_N$ since $NCT_{\tilde{p}} *_{NCT} NCT_{\tilde{h}} = NCT_{\tilde{p}} *_{NCT} NCT_{(\tilde{h}^{-1})^{-1}}$, then, There exists NCT-tri open nhds NCT_u, NCT_v of $NCT_{\tilde{p}}$ and $NCT_{(\tilde{h})^{-1}}$ respectively s.t $NCT_u *_{NCT} NCT_{v^{-1}} \subseteq NCT_N$. Since U_{NCT} is NCT- tri continuous and

$NCT_{v^{-1}}$ NCT-tri open of $NCT_{(\tilde{h})^{-1}}$ there exists NCT_W is NCT-tri open nhds of $NCT_{\tilde{h}}$ s.t $V_{NCT}(NCT_w) \subseteq NCT_V$.

Thus $NCT_W \subseteq V_{NCT}^{-1}(NCT_v) = NCT_{V^{-1}}$

So $NCT_u *_{NCT} NCT_w \subseteq NCT_u *_{NCT} NCT_{v^{-1}} \subseteq NCT_N$. Hence M_{NCT} is NCT-tri continuous.

Remark 4.7: Let NCT_G be a NCT- tri topological group and let $NCT_{\tilde{k}}$ be affixed NCT point the NCT- constant function:

$$C_{NCT} : NCT_G \to NCT_G,\ \ C_{NCT}(NCT_{\tilde{p}}) = NCT_{\tilde{k}}, \forall NCT_{\tilde{p}} \in NCT_G$$

and the NCT-identity function

$$I_{NCT} : NCT_G \to NCT_G,\ \ I_{NCT}(NCT_{\tilde{p}}) = NCT_{\tilde{p}}, \forall NCT_{\tilde{p}} \in NCT_G$$

are NCT-tri continuous function, so

$$h_{NCT} : NCT_G \to NCT_G \times_{NCT} NCT_G,\ \ h_{NCT}(NCT_{\tilde{p}}) = (NCT_{\tilde{k}}, NCT_{\tilde{p}})$$

$$M_{NCT} \circ h_{NCT} = L_{NCT_{\tilde{k}}}, I_{NCT_{\tilde{k}}}(NCT_{\tilde{p}}) = NCT_{\tilde{k}} *_{NCT} NCT_{\tilde{p}}$$

is call NCT def translation.

Similarly, we define the NCT- right translation

$$r_{NCT_{\tilde{k}}}(NCT_{\tilde{p}}) = NCT_p *_{NCT} NCT_{\tilde{k}}.$$

Proposition 4.8: Let NCT_G be a NCT- tri topological group, fix $NCT_{\tilde{k}} \in NCT_G$ then the NCT--function:

$l_{NCT_{\tilde{k}}} : NCT_G \to NCT_G$ and $r_{NCT_{\tilde{k}}} : NCT_G \to NCT_G$ are NCT-tri homomorphism.

Proof: Clear that each of $r_{NCT_{\tilde{k}}}$ and $l_{NCT_{\tilde{k}}}$ is bijective function since by definition M_{NCT} and h_{NCT} are NCT-tri continuous. So $l_{NCT_{\tilde{k}}} = M_{NCT} \circ M_{NCT}$ is NCT-tri continuous.

But $l_{NCT_{\tilde{k}}}(NCT_{\tilde{p}}) = l_{NCT_{(\tilde{k})^{-1}}}(NCT_{\tilde{p}}) = NCT_{\tilde{k}^{-1}} *_{NCT} NCT_{\tilde{p}}$. So $l^{-1}{}_{NCT_{\tilde{k}}}$ Is NCT-tri continuous?

Therefore, $l_{NCT_{\tilde{k}}}$ is an NCT-tri homomorphism, the result for NCT-right translation follows similarly.

Notation 4.9: If $NCT_A, NCT_B \subset NCT_G$ and $NCT_{\tilde{k}} \in NCT_G$, where NCT_G is a NCT- tri topological group. then

(i) $NCT_A *_{NCT} NCT_{\tilde{k}} = r_{NCT_{\tilde{k}}}(NCT_A) = NCT_{\tilde{a}} *_{NCT} NCT_{\tilde{k}} : NCT_{\tilde{a}} \in NCT_A$.

(ii) $NCT_{\tilde{k}} *_{NCT} NCT_A = l_{NCT_{\tilde{k}}}(NCT_A) = NCT_{\tilde{k}} *_{NCT} NCT_{\tilde{a}} : NCT_{\tilde{a}} \in NCT_A$.

(iii) $NCT_A *_{NCT} NCT_B = \cup_{NCT_{\tilde{b}\in NCT_B}} NCT_A *_{NCT} NCT_{\tilde{b}} = \cup_{NCT_{\tilde{a}}} NCT_{\tilde{a}} *_{NCT} NCT_B$.

(iv) $NCT_{A^{-1}} = \{NCT_{a^{-1}} : NCT_a \in NCT_A\}$.

Proposition 4.10: Let NCT_G be a NCT- tri topological group, $NCT_A, NCT_B \subset NCT_G$ and $NCT_{\tilde{k}} \in NCT_G$ then

i. NCT_A is NCT-tri open implies $NCT_A *_{NCT} NCT_{\tilde{k}}$ and $NCT_{\tilde{k}} *_{NCT} NCT_A$ are NCT-tri open.

ii. NCT_A is NCT-tri open implies $NCT_A *_{NCT} NCT_{\tilde{k}}$ and $NCT_{\tilde{k}} *_{NCT} NCT_A$ are NCT-tri open.

iii. is NCT-tri closed implies $NCT_A *_{NCT} NCT_{\tilde{k}}$ and $NCT_{\tilde{k}} *_{NCT} NCT_A$ are NCT-tri closed.

iv. NCT_A is NCT-tri closed and NCT_B is finite implies $NCT_A *_{NCT} NCT_B$ and $NCT_B *_{NCT} NCT_A$ are NCT-tri closed.

Proof: (i), (ii) $l_{NCT_{\tilde{k}}}, r_{NCT_{\tilde{k}}}$ being NCT-tri homomorphism, are both NCT-tri open and NCT-tri closed,

$NCT_A *_{NCT} NCT_{\tilde{k}} = r_{NCT_{\tilde{k}}}(NCT_A), NCT_{\tilde{k}} *_{NCT} NCT_A = l_{NCT_{\tilde{k}}}(NCT_A)$ are NCT-tri open and tri closed.

iii) $NCT_A *_{NCT} NCT_B = \cup_{NCT_{\tilde{b}}} r_{NCT_{\tilde{b}}}(NCT_A) = \cup_{NCT_{NCT_{\tilde{b}\in NCT_B}}} NCT_A *_{NCT} NCT_{\tilde{b}}$

and

$NCT_B *_{NCT} NCT_A = \cup_{NCT_{NCT_{\tilde{b}\in NCT_B}}} l_{NCT_{\tilde{b}}}(NCT_A) = \cup_{NCT_{NCT_{\tilde{b}\in NCT_B}}} NCT_{\tilde{b}} *_{NCT} NCT_A$

are aunion of NCT- tri open and hence NCT-tri open

iv) $NCT_A *_{NCT} NCT_B = \bigcup_{i=1}^{n} {}_{NCT_{\tilde{bi}}\in NCT_B} NCT_A *_{NCT} NCT_{bi}$ and

$NCT_B *_{NCT} NCT_A = \bigcup_{i=1}^{n} {}_{NCT_{\tilde{bi}}\in NCT_B} NCT_{bi} *_{NCT} NCT_A$ the union of a finite number of NCT-tri closed set is NCT-tri closed.

Proposition 4.11: Any NCT - fundamental system F_{NCT} of NCT- tri open nhds of $NCT_{\tilde{e}}$ in NCT_G has the following properties:

i. If $NCT_u, NCT_v \in F_{NCT}$ then $\exists NCT_w \in F_{NCT}$ s.t $NCT_w \subseteq NCT_u \cap NCT_v$.

ii. If $NCT_{\tilde{p}} \in NCT_u \in F_{NCT}$ then $\exists NCT_v \in F_{NCT}$ s.t $NCT_v *_{NCT} NCT_{\tilde{p}} \subseteq NCT_u$.

iii. If $NCT_u \in F_{NCT}$ then $\exists NCT_v \in F_{NCT}$ s.t $NCT_{v^{-1}} *_{NCT} NCT_v \subseteq NCT_u$.

iv. If $NCT_u \in F_{NCT}, NCT_{\tilde{k}} \in NCT_G$ then s.t
$NCT_{\tilde{k}^{-1}} *_{NCT} NCT_v, \quad NCT_{\tilde{k}} \subseteq NCT_u$.

Proof:

(i) since $NCT_u, NCT_v \in F_{NCT}$ then $NCT_u \cap NCT_v \in F_{NCT}$ so $\exists NCT_w \in F_{NCT}$ s.t $NCT_w \subseteq NCT_u \cap NCT_v$.

(ii) since $NCT_u \in F_{NCT}$ and $NCT_{\tilde{p}} \in NCT_u$ implies $NCT_u *_{NCT} NCT_{\tilde{p}^{-1}} \in F_{NCT}$ then $\exists NCT_v \in F_{NCT}$ s.t $NCT_v \subseteq NCT_u *_{NCT} NCT_{\tilde{p}^{-1}} \in F_{NCT}$ thus $NCT_v *_{NCT} NCT_{\tilde{p}} \subseteq NCT_u$.

(iii) the NCT function $f_{NCT} : NCT(G) \times NCT(G) \to NCT(G)$ given by $f_{NCT}(NCT_{\tilde{p}}, NCT_{\tilde{h}}) = NCT_{\tilde{p}^{-1}} *_{NCT} NCT_{\tilde{h}}$ is NCT-tri continuous. Thus $f^{-1}{}_{NCT}(NCT_u)$ is NCT-tri open set in $NCT(G) \times NCT(G)$ contains $(NCT_{\tilde{e}}, NCT_{\tilde{e}})$ and hence contains a NCT- set of the form $NCT_A \times NCT_B$ where NCT_A, NCT_B are NCT-tri open and contain $NCT_{\tilde{e}}$ since $NCT_A \cap NCT_B$ is NCT-tri open of $NCT_{\tilde{e}}, \exists NCT_v \in f_{NCT}$ s.t $NCT_v \subseteq NCT_A \cap NCT_B$ so that $NCT_v \subseteq NCT_A$ and $NCT_v \subseteq NCT_B$. thus $NCT_v \times_{NCT} NCT_v \subseteq NCT_A \times_{NCT} \quad NCT_B \subseteq f_{NCT}{}^{-1}(NCT_u)$ then $f_{NCT}(NCT_v \times_{NCT} NCT_v) = NCT_{v^{-1}} \times_{NCT} NCT_v \subseteq NCT_u$.

(iv) the NCT function $f_{NCT} : NCT(G) \to NCT(G)$, given by $f_{NCT}(NCT_{\tilde{p}}) = NCT_{\tilde{k}^{-1}} *_{NCT} NCT_{\tilde{p}} *_{NCT} NCT_k$ is NCT- tri continuous. Since r_{NCT_k} and $l_{NCT_{\tilde{k}^{-1}}}$ is NCT- tri continuous. So $r_{NCT_{\tilde{k}}}$ or $l_{NCT_{\tilde{k}^{-1}}}$ is NCT- tri continuous form $NCT(G)$ to $NCT(G)$,

$$f_{NCT}(NCT_{\tilde{p}}) = l_{NCT_{\tilde{k}^{-1}}} \text{ or } r_{NCT\tilde{k}}(NCT_{\tilde{p}}) = l_{NCT_{\tilde{k}^{-1}}}(r_{NCT_k}(NCT_{\tilde{p}}))$$

$$= l_{NCT_{\tilde{k}^{-1}}}(NCT_{\tilde{p}} *_{NCT} NCT_{\tilde{k}}) = NCT_{\tilde{k}^{-1}} *_{NCT} NCT_{\tilde{p}} *_{NCT} NCT_{\tilde{k}}.$$

$f_{NCT}(NCT_{\tilde{p}}) = l_{NCT_{\tilde{k}^{-1}}}$ or $r_{NCT_{\tilde{k}}}$ is NCT- tri continuous. So $f^{-1}{}_{NCT}(NCT_u)$ NCT-tri open and contain $NCT_{\tilde{e}}$ hence $\exists NCT_v \in f_{NCT}$ s.t $f_{NCT}(NCT_v) = NCT_{\tilde{k}^{-1}} *_{NCT} NCT_v *_{NCT} NCT_{\tilde{k}} \subseteq NCT_u$.

Proposition 4.12: Any NCT - fundamental system F_{NCT} of NCT- tri open nhds of $NCT_{\tilde{e}}$ also satisfies:

1. $\forall NCT_u \in f_{NCT}, \exists NCT_v \in f_{NCT} \ni NCT_{v^{-1}} \subseteq NCT_u$.
2. $\forall NCT_u \in f_{NCT}, \exists NCT_w \in f_{NCT} \ni NCT_w *_{NCT} NCT_w \subseteq NCT_u$.

Proof: By the conditions M_{NCT} and V_{NCT} are NCT- tri continuous

1. $V^{-1}{}_{NCT}(NCT_u)$ is NCT- tri open and contain $NCT_{\tilde{e}}$, hence $\exists NCT_v \in f_{NCT} \ni NCT_v \subseteq NCT_{v^{-1}}(NCT_u)$. Thus $N_{NCT}(NCT_v) = NCT_{v^{-1}} \subseteq NCT_u$.
2. Thus $M^{-1}{}_{NCT}(NCT_u)$ is NCT- tri open contain $(NCT_{\tilde{e}}, NCT_{\tilde{e}})$ and hence contains a set of the from $NCT_A \times NCT_B$ where NCT_A, NCT_B are NCT- tri open and contain $NCT_{\tilde{e}}$.

   Since $NCT_A \cap NCT_B$ is NCT- tri open nhd of $NCT_{\tilde{e}}$ $\exists NCT_w \in f_{NCT} \ni NCT_w \subseteq NCT_A \cap NCT_B$

   Then $NCT_W \times NCT_W \subseteq NCT_A \times NCT_B \subseteq M^{-1}{}_{NCT}(NCT_u)$ thus, $M_{NCT}(NCT_w\ NCT_w) = NCT_w \times_{NCT} NCT_w \subseteq NCT_u$.
3. The family f_{NCT} of all is NCT- tri open nhd of $NCT_{\tilde{e}}$ and has the extra properties that, if $NCT_u, NCT_v \in f_{NCT}$ then $NCT_u \cap NCT_v$ and $NCT_{u^{-1}} \in f_{NCT}$.

These imply that every $NCT_u \in f_{NCT}$ contains a NCT- symmetric $NCT_w \in f_{NCT}$ namely $NCT_w = NCT_u \cap NCT_{u^{-1}}$ a NCT- tri nhd NCT_w of $NCT_{\tilde{e}}$ is called symmetric if $NCT_{w^{-1}} = NCT_w$.

Conclusion

In this paper, a neuromorphic crisp triple tri topological group was established using the concepts of neuromorphic crisp triples, topological spaces, and continuous functions. The basic properties of this new structure were investigated, including the fundamental system of NBHDS, providing a foundation for further studies in neuromorphic topological group theory.

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Received November, 2025