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Intuitionistic fuzzy soft boundary and semi boundary sets in intuitionistic fuzzy soft closure spaces

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Abstract

This work introduces concept for an intuitionistic fuzzy soft(*ifs*) boundary in intuitionistic fuzzy soft(*ifs*) closure space. Additionally, the intuitionistic fuzzy soft semi-boundary in intuitionistic fuzzy soft closure spaces is presented, its characteristics are explored. Furthermore, main aims are to discuss necessary condition of function to considered (*ifs*) continuous with respect to an (*ifs*) closure spaces by employing the notions of (*ifs*) boundary and (*ifs*) semi-boundary in an intuitionistic fuzzy soft closure space.

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1. Introduction

Zadeh [1] studied a notion for the fuzzy sets, marking beginning for extensive research in this field. Following this, Chang [2] constructed concept for the fuzzy topological spaces.

Later, Coker [3] introduced the definition for intuitionistic fuzzy set, which led Atanassov [4] defined an intuitionistic fuzzy topological spaces. Molodtsov [5] then introduce soft set, and Maji, Biswas, and Roy [6] gave definition for fuzzy soft sets. A intuitionistic fuzzy soft sets was given by Maji, Biswas, and Roy [7]. Subsequently, authors in [8] contributed to the field by introducing the definition of intuitionistic fuzzy soft closure spaces.

A present study aim to describe a notion for the intuitionistic fuzzy soft boundary.

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Moreover, the necessary condition for a function to be a newly introduced (ifs) continuous and (ifs) semi-continuous function with helps of intuitionistic fuzzy soft boundary sets and (ifs) semi-boundary sets with respect to an (ifs) closure space is discussed deeply, and we explore important properties and characterizations of these functions.

Note: We write ifs to mean intuitionistic fuzzy soft.

2. Preliminaries

Definition 1: [7], [9], [10] if $\mathcal{M}$ the initials universal sets & $\mathcal{S}$ the set for every parameter and $\digamma \subseteq \mathcal{S}$. assume $\mathrm{IF}^{\mathcal{M}}$ is sets for intuitionistic fuzzy set over $\mathcal{M}$. An intuitionistic fuzzy soft (ifs) sets over $\mathcal{M}$ be pair $(\mathrm{R}, \digamma_{ifs})$ where R is a function $\mathrm{R}: \digamma \longrightarrow \mathrm{IF}^{\mathcal{M}}$ s.t $\mathrm{R}(\mathfrak{s}) = \left\{\left(m, \mu_{\digamma}(m), \lambda_{\digamma}(m)\right): m \in \mathcal{M}, \mathfrak{s} \in \mathcal{S}\right\}$. Let $ifs\ (\mathcal{M}, \mathcal{S})$ is the family of ifs soft set. We will denote to $\mu_{\digamma}(m), \lambda_{\digamma}(m)$ by $\mathrm{R}_{\mathfrak{s}}$ and $\acute{\mathrm{R}}_{\mathfrak{s}}$, respectively. The family of every fuzzy set on $\mathcal{M}$ is $\mathrm{I}^{\mathcal{M}}$.

Example 1: Assume that $\mathcal{M} = \{m_1, m_2, m_3\}$ be an initial universe set and $\mathcal{S} = \{\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3\}$ is a set of parameters. If $(\mathrm{R}, \digamma_{ifs}) = \{\mathrm{R}(\mathfrak{s}_1) = \{(m, 0.8, 0.001), (m_2, 0.07, 0.2), (m_3, 0.06, 0.3)\},$

$\mathrm{R}(\mathfrak{s}_2) = \{(m_1, 0.005, 0.4), (m_2, 0.2, 0.06), (m_3, 0.1, 0.008)\},$

$\mathrm{R}(\mathfrak{s}_3) = \{(m_1, 0, 1), (m_2, 0.09, 0.1), (m_3, 0.3, 0.005)\}\}.$

Then, $(\mathrm{R}, \digamma_{ifs})$ be an ifs set.

Definition 2: [7], [9] For two ifs sets $(\mathrm{R}, \digamma_{ifs})$ and $(\mathrm{X}, \mathrm{T}_{ifs})$ over a common universe $\mathcal{M}$, we write $(\mathrm{R}, \digamma_{ifs}) \tilde{\tilde{\sqsubseteq}} (\mathrm{X}, \mathrm{T}_{ifs})$ and state that $(\mathrm{R}, \digamma_{ifs})$ is an ifs subset of $(\mathrm{X}, \mathrm{T}_{ifs})$,if $\digamma \subseteq \mathrm{T}$ and for all $\mathfrak{s} \in \digamma$, $\mathrm{R}(\mathfrak{s}) \subseteq \mathrm{X}(\mathfrak{s})$.

The complement of an ifs set $(\mathrm{R}, \digamma_{ifs})$ is denoted by $(\mathrm{R}, \digamma_{ifs})^{c}$, defined as $(\mathrm{R}, \digamma_{ifs})^{c} = (\mathrm{R}^{c}, \digamma_{ifs})$. Where $\mathrm{R}^{c}_{\digamma} : \digamma \longrightarrow \mathrm{IF}^{\mathcal{M}}$ is a function given by $\mathrm{R}^{c}(\mathfrak{s}) = [\mathrm{R}(\mathfrak{s})]^{c}$, $\forall\ \mathfrak{s} \in \digamma$, thus if $\mathrm{R}(\mathfrak{s}) = \{\left(m, \mu_{\mathrm{R}(\mathfrak{s})}(m), \lambda_{\mathrm{R}(\mathfrak{s})}(m)\right): m \in \mathcal{M}, \mathfrak{s} \in \mathcal{S}\}$. Then, $\mathrm{R}^{c}(\mathfrak{s}) = [\mathrm{R}(\mathfrak{s})]^{c} = \left\{\left(m, \lambda_{\mathrm{R}(\mathfrak{s})}(m), \mu_{\mathrm{R}(\mathfrak{s})}(m)\right): m \in \mathcal{M}, \mathfrak{s} \in \mathcal{S}\right\}$.

Definition 3: [7], [10] A null ifs set, which $\tilde{\tilde{\varphi}}$ represents, is an ifs set, $\mathrm{R}(\mathfrak{s}) = \left\{\left(m, \mu_{\digamma}(m), \lambda_{\digamma}(m)\right): \forall m \in \mathcal{M}, \forall\ \mathfrak{s} \in \mathcal{S}\right\}$, so $\mu_{\digamma}(m) = 0, \lambda_{\digamma}(m) = 1$. An absolute ifs set, which $\tilde{\tilde{1}}$ represents, is an ifs, $\mathrm{R}(\mathfrak{s}) = \left\{\left(m, \mu_{\digamma}(m), \lambda_{\digamma}(m)\right): \forall m \in \mathcal{M}, \forall\ \mathfrak{s} \in \mathcal{S}\right\}$, so $\mu_{\digamma}(m) = 1, \lambda_{\digamma}(m) = 0$.

Definition 4: [7], [11, 12] Intersections for the two ifs set $(\mathrm{R}, \digamma_{ifs})$ & $(\mathrm{X}, \mathrm{T}_{ifs})$ over common universal $\mathcal{M}$ be an ifs sets $(\mathcal{J}, \mathrm{C}_{ifs})$, when $\mathrm{C} = \digamma \cap \mathrm{T}, \forall\ \mathfrak{s} \in \mathrm{C}$ and $\mathcal{J}(\mathfrak{s}) = \mathrm{R}(\mathfrak{s}) \cap \mathrm{X}(\mathfrak{s})$. We write $(\mathrm{R}, \digamma_{ifs})\ \tilde{\tilde{\sqcap}}\ (\mathrm{X}, \mathrm{T}_{ifs})$. And unions for two ifs set $(\mathrm{R}, \digamma_{ifs})$ & $(\mathrm{X}, \mathrm{T}_{ifs})$ over a common universal $\mathcal{M}$ an ifs sets $(\mathcal{J}, \mathrm{C}_{ifs})$, when $\mathrm{C} = \digamma \cup \mathrm{T}, \forall\ \mathfrak{s} \in \mathrm{C}$ and $\mathcal{J}(\mathfrak{s}) = \mathrm{R}(\mathfrak{s}) \cup \mathrm{X}(\mathfrak{s})$. We write $(\mathrm{R}, \digamma_{ifs})\ \tilde{\tilde{\sqcup}}\ (\mathrm{X}, \mathrm{T}_{ifs})$ as follows, $\mathcal{J}(\mathfrak{s}) = \text{Ʀ}(\mathfrak{s})$ if $\mathfrak{s} \in \text{Ƒ} - \text{Ƭ}$, $\mathcal{J}(\mathfrak{s}) = \text{Ҳ}(\mathfrak{s})$ if $\mathfrak{s} \in \text{Ƭ} - \text{Ƒ}$, $\mathcal{J}(\mathfrak{s}) = \text{Ʀ}(\mathfrak{s}) \cup \text{Ҳ}(\mathfrak{s})$ if $\mathfrak{s} \in \text{Ƒ} \cap \text{Ƭ}, \forall\, \mathfrak{s} \in \text{C}$.

Definition 5: [8] An operator $\acute{\text{Ç}}_\ell: ifs(\mathcal{M},\mathcal{S}) \longrightarrow ifs(\mathcal{M},\mathcal{S})$ is called an intuitionistic fuzzy soft closure operator ($ifs - \acute{\text{Ç}}_\ell$, for short) on $\mathcal{M}$, if for all (Ʀ, Ƒ_{ifs}) and (Ҳ, Ƭ_{ifs}) $\tilde{\tilde{\in}}$ $ifs(\mathcal{M},\mathcal{S})$ the following axioms are satisfied:

1. $\tilde{\tilde{\varphi}} = \acute{\text{Ç}}_\ell(\tilde{\tilde{\varphi}})$,
2. $(\text{Ʀ}, \text{Ƒ}_{ifs}) \tilde{\tilde{\subseteq}} \acute{\text{Ç}}_\ell(\text{Ʀ}, \text{Ƒ}_{ifs})$,
3. $(\text{Ʀ}, \text{Ƒ}_{ifs}) \tilde{\tilde{\subseteq}} (\text{Ҳ}, \text{Ƭ}_{ifs}) \Rightarrow \acute{\text{Ç}}_\ell(\text{Ʀ}, \text{Ƒ}_{ifs}) \tilde{\tilde{\subseteq}} \acute{\text{Ç}}_\ell(\text{Ҳ}, \text{Ƭ}_{ifs})$.

The triple $(\mathcal{M}, \acute{\text{Ç}}_\ell, \mathcal{S})$ named intuitionistic fuzzy soft closure spaces ($ifs - \acute{\text{Ç}}_\ell$ space). The $ifs - \acute{\text{Ç}}_\ell$ is denoted by $\tilde{\tilde{\mathcal{M}}}_{ifs_{\mathcal{S}}}$.

Example 2: By example 1, we have $\acute{\text{Ç}}_\ell(\text{Ʀ}, \text{Ƒ}_{ifs}) = (\text{Ʀ}, \text{Ƒ}_{ifs})$, $\acute{\text{Ç}}_\ell(\tilde{\tilde{\varphi}}) = \tilde{\tilde{\varphi}}$ and $\acute{\text{Ç}}_\ell\left(\tilde{\tilde{1}}\right) = \tilde{\tilde{1}}$, then $(\mathcal{M}, \acute{\text{Ç}}_\ell, \mathcal{S})$ is referred to as an $ifs - \acute{\text{Ç}}_\ell$ spaces.

Definition 6: [8] If $\tilde{\tilde{\mathcal{M}}}_{ifs_{\mathcal{S}}}$ is the $ifs - \acute{\text{Ç}}_\ell$ spaces. A subset $(\text{Ʀ}, \text{Ƒ}_{ifs})$ of an $\tilde{\tilde{\mathcal{M}}}_{ifs_{\mathcal{S}}}$ space, then interiors and closures for $(\text{Ʀ}, \text{Ƒ}_{ifs})$ are denote respectively by $\text{Int}_{\text{Ç}_\ell}(\text{Ʀ}, \text{Ƒ}_{ifs})$, $\overline{(\text{Ʀ}, \text{Ƒ}_{ifs})}$ define as following:

1. Intuitionistic fuzzy soft closed ($ifs\text{C}'\text{s}$ in short), if $(\text{Ʀ}, \text{Ƒ}_{ifs}) = \acute{\text{Ç}}_\ell\,(\text{Ʀ}, \text{Ƒ}_{ifs})$,
2. Intuitionistic fuzzy soft open ($ifs\text{O}'\text{s}$ in short), if $(\text{Ʀ}, \text{Ƒ}_{ifs}) = \text{Int}_{\text{Ç}_\ell}(\text{Ʀ}, \text{Ƒ}_{ifs})$.
3. A complements for $ifs\text{C}$ sets named $ifs\text{O}$ set.
4. If $\acute{\text{Ç}}_\ell(\text{Ʀ}, \text{Ƒ}_{ifs}) \tilde{\tilde{\subseteq}} \left(\text{Ʀ}, \text{Ƒ}_{ifs}\right)$. Then, $(\text{Ʀ}, \text{Ƒ}_{ifs})$ is an $ifs\text{C}$ set of $\tilde{\tilde{\mathcal{M}}}_{ifs_{\mathcal{S}}}$.
5. $\text{Int}_{\text{Ç}_\ell}(\text{Ʀ}, \text{Ƒ}_{ifs}) = \tilde{\tilde{\cup}}\{(\mathcal{J}, \mathcal{Q}_{ifs}): (\mathcal{J}, \mathcal{Q}_{ifs})$ is $fs\text{O}'\text{s}$: $(\mathcal{J}, \mathcal{Q}_{ifs}) \tilde{\tilde{\subseteq}} (\text{Ʀ}, \text{Ƒ}_{ifs})\}$.
6. $\overline{(\text{Ʀ}, \text{Ƒ}_{ifs})} = \tilde{\tilde{\cap}}\{(\mathcal{E}, \mathcal{F}_{ifs}): (\mathcal{E}, \mathcal{F}_{ifs})$ is $fs\text{C}'\text{s}$: $\left(\text{Ʀ}, \text{Ƒ}_{ifs}\right) \tilde{\tilde{\subseteq}} (\mathcal{E}, \mathcal{F}_{ifs})\}$.

Definition 7: Let $\tilde{\tilde{\mathcal{M}}}_{ifs_{\mathcal{S}}}$ be an $ifs - \acute{\text{Ç}}_\ell$ space and $\left(\text{Ʀ}, \text{Ƒ}_{ifs}\right) \tilde{\tilde{\subseteq}} \tilde{\tilde{\mathcal{M}}}_{ifs_{\mathcal{S}}}$. Then, semi-(interior, closure) for $(\text{Ʀ}, \text{Ƒ}_{ifs})$ be denoted respectively as $\mathcal{S}\text{Int}_{\text{Ç}_\ell}(\text{Ʀ}, \text{Ƒ}_{ifs})$ & $\mathcal{S}\overline{(\text{Ʀ}, \text{Ƒ}_{ifs})}$ define as followed:

1. Intuitionistic fuzzy soft semi-closed ($ifs - \text{SC}'\text{s}$ in short), if $\text{Int}_{\text{Ç}_\ell}(\overline{(\text{Ʀ}, \text{Ƒ}_{ifs})}) \tilde{\tilde{\subseteq}} (\text{Ʀ}, \text{Ƒ})$.
2. Intuitionistic fuzzy soft semi-open ($ifs - \text{SO}'\text{s}$ in short), if $\left(\text{Ʀ}, \text{Ƒ}_{ifs}\right) \tilde{\tilde{\subseteq}} \overline{(\text{Int}_{\text{Ç}_\ell}\left(\text{Ʀ}, \text{Ƒ}_{ifs}\right))}$. The complement of $ifs - \text{SC}'\text{s}$ set is called $ifs - \text{SO}'\text{s}$ set.
3. $\mathcal{S}\text{Int}_{\text{Ç}_\ell}(\text{Ʀ}, \text{Ƒ}_{ifs}) = \tilde{\tilde{\cup}}\{(\mathcal{J}, \mathcal{Q}_{ifs}): (\mathcal{J}, \mathcal{Q}_{ifs})$ is $fs{-}\mathcal{S}\text{O}'\text{s}$: $(\mathcal{J}, \mathcal{Q}_{ifs}) \tilde{\tilde{\subseteq}} (\text{Ʀ}, \text{Ƒ}_{ifs})\}$.

4. $\mathcal{S}\overline{(\text{Ʀ}, \text{Ƒ}_{ifs})} = \tilde{\tilde{\bigcap}}\{(\mathcal{E}, \mathcal{F}_{ifs}) : (\mathcal{E}, \mathcal{F}_{ifs})$ is $ifs - \mathcal{S}$C′s: $(\text{Ʀ}, \text{Ƒ}_{ifs}) \tilde{\tilde{\sqsubseteq}} (\mathcal{E}, \mathcal{F}_{ifs})\}$.

Proposition 1: [8] Let $\tilde{\tilde{\mathcal{M}}}_{ifs_{\mathcal{S}}}$ be an $ifs - \acute{\text{C}}_{\ell}$ space, $(\text{Ʀ}, \text{Ƒ}_{ifs})$ and $(\text{Ҳ}, \text{Ƃ}_{ifs})$ are ifs subsets over $\mathcal{M}$, then

1. $(\text{Ʀ}, \text{Ƒ}_{ifs})\tilde{\tilde{\bigcap}}(\text{Ʀ}, \text{Ƒ}_{ifs}) = (\text{Ʀ}, \text{Ƒ}_{ifs})$, $(\text{Ʀ}, \text{Ƒ}_{ifs})\tilde{\tilde{\bigcup}}(\text{Ʀ}, \text{Ƒ}_{ifs}) = (\text{Ʀ}, \text{Ƒ}_{ifs})$.
2. $(\text{Ʀ}, \text{Ƒ}_{ifs})\tilde{\tilde{\bigcup}}\,\tilde{\tilde{\varphi}} = (\text{Ʀ}, \text{Ƒ}_{ifs})$, $(\text{Ʀ}, \text{Ƒ}_{ifs})\tilde{\tilde{\bigcap}}\,\tilde{\tilde{\varphi}} = \tilde{\tilde{\varphi}}$, $(\text{Ʀ}, \text{Ƒ}_{ifs})\tilde{\tilde{\bigcup}}\,\tilde{\tilde{1}} = \tilde{\tilde{1}}$.
3. $(\text{Ʀ}, \text{Ƒ}_{ifs})\tilde{\tilde{\bigcap}}\,\tilde{\tilde{1}} = (\text{Ʀ}, \text{Ƒ}_{ifs})$, $\text{Int}_{\text{C}_{\ell}}(\tilde{\tilde{\varphi}}) = \tilde{\tilde{\varphi}}$, $\text{Int}_{\text{C}_{\ell}}(\tilde{\tilde{1}}) = \tilde{\tilde{1}}$.
4. $((\text{Ʀ}, \text{Ƒ}_{ifs})\tilde{\tilde{\bigcap}}(\text{Ҳ}, \text{Ƃ}_{ifs}))^{c} = (\text{Ʀ}, \text{Ƒ}_{ifs})^{c}\tilde{\tilde{\bigcup}}(\text{Ҳ}, \text{Ƃ}_{ifs})^{c}$.
5. $((\text{Ʀ}, \text{Ƒ}_{ifs})\tilde{\tilde{\bigcup}}(\text{Ҳ}, \text{Ƃ}_{ifs}))^{c} = (\text{Ʀ}, \text{Ƒ}_{ifs})^{c}\tilde{\tilde{\bigcap}}(\text{Ҳ}, \text{Ƃ}_{ifs})^{c}$.
6. $(\text{Int}_{\text{C}_{\ell}}(\text{Ʀ}, \text{Ƒ}_{ifs}))^{c} = \overline{((\text{Ʀ}, \text{Ƒ}_{ifs})^{c})}$, $(\overline{(\text{Ʀ}, \text{Ƒ}_{ifs})})^{c} = (\text{Int}_{\text{C}_{\ell}}(\text{Ʀ}, \text{Ƒ}_{ifs})^{c})$.
7. $\text{Int}_{\text{C}_{\ell}}(\text{Ʀ}, \text{Ƒ}_{ifs}) \tilde{\tilde{\sqsubseteq}} (\text{Ʀ}, \text{Ƒ}_{ifs})$, $\text{Int}_{\text{C}_{\ell}}(\text{Int}_{\text{C}_{\ell}}(\text{Ʀ}, \text{Ƒ}_{ifs})) = \text{Int}_{\text{C}_{\ell}}(\text{Ʀ}, \text{Ƒ}_{ifs})$.
8. If $(\text{Ʀ}, \text{Ƒ}_{ifs}) \tilde{\tilde{\sqsubseteq}} (\text{Ҳ}, \text{Ƃ}_{ifs})$, then $\text{Int}_{\text{C}_{\ell}}(\text{Ʀ}, \text{Ƒ}_{ifs}) \tilde{\tilde{\sqsubseteq}} \text{Int}_{\text{C}_{\ell}}(\text{Ҳ}, \text{Ƃ}_{ifs})$.
9. $\text{Int}_{\text{C}_{\ell}}((\text{Ʀ}, \text{Ƒ}_{ifs})\tilde{\tilde{\bigcap}}(\text{Ҳ}, \text{Ƃ}_{ifs})) = \text{Int}_{\text{C}_{\ell}}(\text{Ʀ}, \text{Ƒ}_{ifs})\tilde{\tilde{\bigcap}}\,\text{Int}_{\text{C}_{\ell}}(\text{Ҳ}, \text{Ƃ}_{ifs})$.
10. $\text{Int}_{\text{C}_{\ell}}(\text{Ʀ}, \text{Ƒ}_{ifs})\tilde{\tilde{\bigcup}}\,\text{Int}_{\text{C}_{\ell}}(\text{Ҳ}, \text{Ƃ}_{ifs}) \tilde{\tilde{\sqsubseteq}} \text{Int}_{\text{C}_{\ell}}((\text{Ʀ}, \text{Ƒ}_{ifs})\tilde{\tilde{\bigcup}}(\text{Ҳ}, \text{Ƃ}_{ifs}))$.

3. Intuitionistic Fuzzy Soft Boundary

Definition 8: Let $(\mathcal{M}, \acute{\text{C}}_{\ell}, \mathcal{S})$ and $(\mathcal{W}, \acute{\text{C}}_{\ell 1}, \mathcal{S})$ be two $ifs - \acute{\text{C}}_{\ell}$ spaces.

A function δ: $\tilde{\tilde{\mathcal{M}}}_{ifs_{\mathcal{S}}} \to \tilde{\tilde{\mathcal{W}}}_{ifs_{\mathcal{S}}}$ is called intuitionistic fuzzy soft closed (open) function, if $\delta(\mathcal{E}, \mathcal{F}_{ifs})$ is an ifsC (ifsO) subset of $(\mathcal{W}, \acute{\text{C}}_{\ell 1}, \mathcal{S})$, for every $(\mathcal{E}, \mathcal{F}_{ifs})$ is an ifsC (ifsO) subset of $(\mathcal{M}, \acute{\text{C}}_{\ell}, \mathcal{S})$.

Definition 9: Let $(\mathcal{M}, \acute{\text{C}}_{\ell}, \mathcal{S})$ and $(\mathcal{W}, \acute{\text{C}}_{\ell 1}, \mathcal{S})$ be two $ifs - \acute{\text{C}}_{\ell}$ spaces. A function δ: $\tilde{\tilde{\mathcal{M}}}_{ifs_{\mathcal{S}}} \to \tilde{\tilde{\mathcal{W}}}_{ifs_{\mathcal{S}}}$ is said to be an ifs continuous function, if $\delta^{-1}(\mathcal{E}, \mathcal{F}_{ifs})$ is an ifsC set of $\tilde{\tilde{\mathcal{M}}}_{ifs_{\mathcal{S}}}$ for each ifsC set of $\tilde{\tilde{\mathcal{W}}}_{ifs_{\mathcal{S}}}$.

Definition 10: Let $(\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs})$ be an ifs set of $ifs - \acute{\text{C}}_{\ell}$ space $\tilde{\tilde{\mathcal{M}}}_{ifs_{\mathcal{S}}}$, the intuitionistic fuzzy soft boundary of $(\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs})$ (ifsB′s in short) is defined as $\text{Ƕ}_{\acute{\text{C}}_{\ell}}(\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs}) = \overline{(\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs})}\tilde{\tilde{\bigcap}}\,\overline{(\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs})^{c}}$. The ifsB′s is denoted by $\text{Ƕ}_{\acute{\text{C}}_{\ell}}$.

Proposition 2: If $(\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs})$ be an ifs set in $ifs - \acute{\text{C}}_{\ell}$ space $\tilde{\tilde{\mathcal{M}}}_{ifs_{\mathcal{S}}}$. Then, $\text{Ƕ}_{\acute{\text{C}}_{\ell}}(\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs}) = \text{Ƕ}_{\acute{\text{C}}_{\ell}}(\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs})^{c}$.

Proof: Let $\text{Ƕ}_{\acute{\text{C}}_{\ell}}(\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs}) = \overline{(\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs})}\tilde{\tilde{\bigcap}}\,\overline{(\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs})^{c}} = \overline{(\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs})^{c}}\tilde{\tilde{\bigcap}}\,\overline{(\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs})} = \overline{(\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs})^{c}}\tilde{\tilde{\bigcap}}\,\overline{((\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs})^{c})^{c}} = \text{Ƕ}_{\acute{\text{C}}_{\ell}}(\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs})^{c}$.

Proposition 3: If $(Ɱ_{ifs}, Ꜧ_{ifs})$ be an ifsC set in $ifs - \acute{C}_\ell$ space $\tilde{\tilde{\tilde{\mathcal{M}}}}_{ifs_S}$.

Then, $Ꜧ_{\acute{C}_\ell}(Ɱ_{ifs}, Ꜧ_{ifs}) \tilde{\tilde{\subseteq}} (Ɱ_{ifs}, Ꜧ_{ifs})$.

Proof: Let $(Ɱ_{ifs}, Ꜧ_{ifs})$ be an ifsC set in $\tilde{\tilde{\tilde{\mathcal{M}}}}_{ifs_S}$, $Ꜧ_{\acute{C}_\ell}(Ɱ_{ifs}, Ꜧ_{ifs}) = \overline{(Ɱ_{\iota fs}, Ꜧ_{\iota fs})} \tilde{\tilde{\cap}} \overline{(Ɱ_{\iota fs}, Ꜧ_{\iota fs})^c} \tilde{\tilde{\subseteq}} \overline{(Ɱ_{\iota fs}, Ꜧ_{\iota fs})} = (Ɱ_{ifs}, Ꜧ_{ifs})$, hence $Ꜧ_{\acute{C}_\ell}(Ɱ_{ifs}, Ꜧ_{ifs}) \tilde{\tilde{\subseteq}} (Ɱ, Ꜧ_{ifs})$.

Proposition 4: If $(Ɱ_{ifs}, Ꜧ_{ifs})$ be an ifs set in $ifs - \acute{C}_\ell$ space $\tilde{\tilde{\tilde{\mathcal{M}}}}_{ifs_S}$. Then, $Ꜧ_{\acute{C}_\ell}(Ꜧ_{\acute{C}_\ell}(Ɱ_{ifs}, Ꜧ_{ifs})) \tilde{\tilde{\subseteq}} Ꜧ_{\acute{C}_\ell}(Ɱ_{ifs}, Ꜧ_{ifs})$.

Proof: Let $(Ɱ_{ifs}, Ꜧ_{ifs})$ be an ifs set in $\tilde{\tilde{\tilde{\mathcal{M}}}}_{ifs_S}$, $Ꜧ_{\acute{C}_\ell}(Ꜧ_{\acute{C}_\ell}(Ɱ_{ifs}, Ꜧ_{ifs})) = \overline{(Ꜧ_{\acute{C}_\ell}(Ɱ_{\iota fs}, Ꜧ_{\iota fs}))} \tilde{\tilde{\cap}} \overline{(Ꜧ_{\acute{C}_\ell}(Ɱ_{\iota fs}, Ꜧ_{\iota fs}))^c} \tilde{\tilde{\subseteq}} \overline{(Ꜧ_{\acute{C}_\ell}(Ɱ_{\iota fs}, Ꜧ_{\iota fs}))} = Ꜧ_{\acute{C}_\ell}(Ɱ_{ifs}, Ꜧ_{ifs})$.

Proposition 5: If $(Ɱ_{ifs}, Ꜧ_{ifs})$ be an ifs set in $ifs - \acute{C}_\ell$ space $\tilde{\tilde{\tilde{\mathcal{M}}}}_{ifs_S}$. Then, $\left(Ꜧ_{\acute{C}_\ell}(Ɱ_{ifs}, Ꜧ_{ifs})\right)^c = \mathrm{Int}_{C_\ell}(Ɱ_{ifs}, Ꜧ_{ifs}) \tilde{\tilde{\cup}} \mathrm{Int}_{C_\ell}\left(Ɱ_{ifs}, Ꜧ_{ifs}\right)^c$.

Proof: Let $(Ɱ_{ifs}, Ꜧ_{ifs})$ is an ifs set in $\tilde{\tilde{\tilde{\mathcal{M}}}}_{ifs_S}$, $\left(Ꜧ_{\acute{C}_\ell}(Ɱ_{ifs}, Ꜧ_{ifs})\right)^c = \left(\overline{(Ɱ_{\iota fs}, Ꜧ_{\iota fs})} \tilde{\tilde{\cap}} \overline{(Ɱ_{\iota fs}, Ꜧ_{\iota fs})^c}\right)^c = \left(\overline{(Ɱ_{\iota fs}, Ꜧ_{\iota fs})}\right)^c \tilde{\tilde{\cup}} \left(\overline{(Ɱ_{\iota fs}, Ꜧ_{\iota fs})^c}\right)^c$. By 1, we get $\mathrm{Int}_{C_\ell}\left(Ɱ_{ifs}, Ꜧ_{ifs}\right)^c \tilde{\tilde{\cup}} \mathrm{Int}_{C_\ell}\left(Ɱ_{ifs}, Ꜧ_{ifs}\right)$.

Proposition 6: If $(Ɱ_{ifs}, Ꜧ_{ifs})$ be an ifs set in $ifs - \acute{C}_\ell$ space $\tilde{\tilde{\tilde{\mathcal{M}}}}_{ifs_S}$. Then, $Ꜧ_{\acute{C}_\ell}(Ɱ_{ifs}, Ꜧ_{ifs}) = \overline{(Ɱ_{\iota fs}, Ꜧ_{\iota fs})} - \mathrm{Int}_{C_\ell}(Ɱ_{ifs}, Ꜧ_{ifs})$.

Proof: Let $(Ɱ_{ifs}, Ꜧ_{ifs})$ is an ifs set in $\tilde{\tilde{\tilde{\mathcal{M}}}}_{ifs_S}$. Since $\left(\overline{(Ɱ_{\iota fs}, Ꜧ_{\iota fs})^c}\right)^c = \mathrm{Int}_{C_\ell}\left(Ɱ_{ifs}, Ꜧ_{ifs}\right)$, therefore we have $Ꜧ_{\acute{C}_\ell}(Ɱ_{ifs}, Ꜧ_{ifs}) = \overline{(Ɱ_{\iota fs}, Ꜧ_{\iota fs})} \tilde{\tilde{\cap}} \overline{(Ɱ_{\iota fs}, Ꜧ_{\iota fs})^c} = \overline{(Ɱ_{\iota fs}, Ꜧ_{\iota fs})} - (\overline{(Ɱ_{\iota fs}, Ꜧ_{\iota fs})^c})^c = \overline{(Ɱ_{\iota fs}, Ꜧ_{\iota fs})} - \mathrm{Int}_{C_\ell}\left(Ɱ, Ꜧ_{ifs}\right)$. Thus $Ꜧ_{\acute{C}_\ell}(Ɱ_{ifs}, Ꜧ_{ifs}) = \overline{(Ɱ_{\iota fs}, Ꜧ_{\iota fs})} - \mathrm{Int}_{C_\ell}(Ɱ_{ifs}, Ꜧ_{ifs})$.

Proposition 7: If $(Ɱ_{ifs}, Ꜧ_{ifs})$ be an ifs set in $ifs - \acute{C}_\ell$ space $\tilde{\tilde{\tilde{\mathcal{M}}}}_{ifs_S}$. Then, $Ꜧ_{\acute{C}_\ell}(\mathrm{Int}_{C_\ell}(Ɱ_{ifs}, Ꜧ_{ifs})) \tilde{\tilde{\subseteq}} Ꜧ_{\acute{C}_\ell}(Ɱ_{ifs}, Ꜧ_{ifs})$.

Proof: Let $(Ɱ_{ifs}, Ꜧ_{ifs})$ is an ifs set in $\tilde{\tilde{\tilde{\mathcal{M}}}}_{ifs_S}$. $Ꜧ_{\acute{C}_\ell}(\mathrm{Int}_{C_\ell}(Ɱ_{ifs}, Ꜧ_{ifs})) = \overline{(\mathrm{Int}_{C_\ell}\left(Ɱ_{\iota fs}, Ꜧ_{\iota fs}\right))} \tilde{\tilde{\cap}} \overline{(\mathrm{Int}_{C_\ell}(Ɱ_{\iota fs}, Ꜧ_{\iota fs}))^c} = \overline{(\mathrm{Int}_{C_\ell}\left(Ɱ_{\iota fs}, Ꜧ_{\iota fs}\right))} - \mathrm{Int}_{C_\ell}(\mathrm{Int}_{C_\ell}\left(Ɱ_{ifs}, Ꜧ_{ifs}\right)) = \overline{(\mathrm{Int}_{C_\ell}\left(Ɱ_{\iota fs}, Ꜧ_{\iota fs}\right))} - \mathrm{Int}_{C_\ell}\left(Ɱ_{ifs}, Ꜧ_{ifs}\right) \tilde{\tilde{\subseteq}} \overline{\left(Ɱ_{\iota fs}, Ꜧ_{\iota fs}\right)} - \mathrm{Int}_{C_\ell}\left(Ɱ_{ifs}, Ꜧ_{ifs}\right) = Ꜧ_{\acute{C}_\ell}(Ɱ_{ifs}, Ꜧ_{ifs})$.

Proposition 8: If $(Ɱ_{ifs}, Ꜧ_{ifs})$ be an ifs set in $ifs - \acute{C}_{\ell}$ space $\tilde{\tilde{\mathcal{M}}}_{ifs_S}$. Then, $\mathrm{Int}_{C_\ell}(Ɱ_{ifs}, Ꜧ_{ifs}) \,\tilde{\tilde{\subseteq}}\, (Ɱ_{ifs}, Ꜧ_{ifs}) - Ꜧ_{\acute{C}_\ell}(Ɱ_{ifs}, Ꜧ_{ifs})$.

Proof: Let $(Ɱ_{ifs}, Ꜧ_{ifs})$ is an ifs set in $\tilde{\tilde{\mathcal{M}}}_{ifs_S}$. $(Ɱ_{ifs}, Ꜧ_{ifs}) - Ꜧ_{\acute{C}_\ell}(Ɱ_{ifs}, Ꜧ_{ifs}) = (Ɱ_{ifs}, Ꜧ_{ifs}) \,\tilde{\tilde{\sqcap}}\, (Ꜧ_{\acute{C}_\ell}(Ɱ_{ifs}, Ꜧ_{ifs}))^{c} = (Ɱ_{ifs}, Ꜧ_{ifs}) \,\tilde{\tilde{\sqcap}}\, (\overline{(Ɱ_{\iota fs}, Ꜧ_{\iota fs})} \,\tilde{\tilde{\sqcap}}\, \overline{(Ɱ_{\iota fs}, Ꜧ_{\iota fs})^{c}})^{c} = (Ɱ_{ifs}, Ꜧ_{ifs}) \,\tilde{\tilde{\sqcap}}\, (\mathrm{Int}_{C_\ell}(Ɱ_{ifs}, Ꜧ_{ifs})^{c} \,\tilde{\tilde{\sqcup}}\, \mathrm{Int}_{C_\ell}(Ɱ_{ifs}, Ꜧ_{ifs})) = ((Ɱ_{ifs}, Ꜧ_{ifs}) \,\tilde{\tilde{\sqcap}}\, \mathrm{Int}_{C_\ell}(Ɱ_{ifs}, Ꜧ_{ifs})^{c}) \,\tilde{\tilde{\sqcup}}((Ɱ_{ifs}, Ꜧ_{ifs}) \,\tilde{\tilde{\sqcap}}\, \mathrm{Int}_{C_\ell}(Ɱ_{ifs}, Ꜧ_{ifs})) = ((Ɱ_{ifs}, Ꜧ_{ifs}) \tilde{\tilde{\sqcap}} \mathrm{Int}_{C_\ell}(Ɱ_{ifs}, Ꜧ_{ifs})^{c}) \tilde{\tilde{\sqcup}} \mathrm{Int}_{C_\ell}(Ɱ_{ifs}, Ꜧ_{ifs}) \,\tilde{\tilde{\supseteq}}\, (Ɱ_{ifs}, Ꜧ_{ifs})$.

Proposition 9: Let $\tilde{\tilde{\mathcal{M}}}_{ifs_S}$ be an $ifs - \acute{C}_\ell$ space, $(Ɽ, Ƒ_{ifs})$ and $(Ӽ, Ƭ_{ifs})$ are ifs sets over $\mathcal{M}$, then $Ꜧ_{\acute{C}_\ell}((Ɽ, Ƒ_{ifs}) \tilde{\tilde{\sqcup}} (Ӽ, Ƭ_{ifs})) \,\tilde{\tilde{\subseteq}}\, Ꜧ_{\acute{C}_\ell}(Ɽ, Ƒ_{ifs}) \tilde{\tilde{\sqcup}}\, Ꜧ_{\acute{C}_\ell}(Ӽ, Ƭ_{ifs})$.

Proof: $Ꜧ_{\acute{C}_\ell}((Ɽ, Ƒ_{ifs}) \tilde{\tilde{\sqcup}} (Ӽ, Ƭ_{ifs})) = \overline{((Ɽ, Ƒ_{\iota fs}) \tilde{\tilde{\sqcup}} (Ӽ, Ƭ_{\iota fs}))} \,\tilde{\tilde{\sqcap}}\, \overline{\left((Ɽ, Ƒ_{\iota fs}) \tilde{\tilde{\sqcup}} (Ӽ, Ƭ_{\iota fs})\right)^{c}}$ $\tilde{\tilde{\subseteq}}\, (\overline{(Ɽ, Ƒ_{\iota fs})} \,\tilde{\tilde{\sqcup}}\, \overline{(Ӽ, Ƭ_{\iota fs})}\,) \,\tilde{\tilde{\sqcap}}(\overline{(Ɽ, Ƒ_{\iota fs})^{c}} \,\tilde{\tilde{\sqcap}} \overline{(Ӽ, Ƭ_{\iota fs})^{c}}\,) = (\overline{(Ɽ, Ƒ_{\iota fs})} \,\tilde{\tilde{\sqcap}}(\overline{(Ɽ, Ƒ_{\iota fs})^{c}} \,\tilde{\tilde{\sqcap}} \overline{(Ӽ, Ƭ_{\iota fs})^{c}}\,)) \,\tilde{\tilde{\sqcup}}\, \overline{(Ӽ, Ƭ_{\iota fs})} \,\tilde{\tilde{\sqcap}}\, (\overline{(Ɽ, Ƒ_{\iota fs})^{c}} \,\tilde{\tilde{\sqcap}} \overline{(Ӽ, Ƭ_{\iota fs})^{c}}\,)) = (Ꜧ_{\acute{C}_\ell}(Ɽ, Ƒ_{ifs}) \,\tilde{\tilde{\sqcap}} \overline{(Ӽ, Ƭ_{\iota fs})^{c}}) \tilde{\tilde{\sqcup}}\, (Ꜧ_{\acute{C}_\ell}(Ӽ, Ƭ_{ifs}) \,\tilde{\tilde{\sqcap}}\, \overline{(Ɽ, Ƒ_{\iota fs})^{c}}\,) \,\tilde{\tilde{\subseteq}}\, Ꜧ_{C_\ell}(Ɽ, Ƒ_{ifs}) \tilde{\tilde{\sqcup}} Ꜧ_{\acute{C}_\ell}(Ӽ, Ƭ_{ifs})$.

Proposition 10: Let $\tilde{\tilde{\mathcal{M}}}_{ifs_S}$ be an $ifs - \acute{C}_\ell$ space, $(Ɽ, Ƒ_{ifs})$ and $(Ӽ, Ƭ_{ifs})$ are ifs sets over $\mathcal{M}$. Then, $Ꜧ_{\acute{C}_\ell}((Ɽ, Ƒ_{ifs}) \tilde{\tilde{\sqcap}} (Ӽ, Ƭ_{ifs})) \,\tilde{\tilde{\subseteq}}\, Ꜧ_{\acute{C}_\ell}(Ɽ, Ƒ_{ifs}) \,\tilde{\tilde{\sqcup}}\, Ꜧ_{\acute{C}_\ell}(Ӽ, Ƭ_{ifs})$.

Proof: $Ꜧ_{\acute{C}_\ell}((Ɽ, Ƒ_{ifs}) \tilde{\tilde{\sqcap}} (Ӽ, Ƭ_{ifs})) = \overline{((Ɽ, Ƒ_{\iota fs}) \tilde{\tilde{\sqcap}} (Ӽ, Ƭ_{\iota fs}))} \,\tilde{\tilde{\sqcap}}\, \overline{\left((Ɽ, Ƒ_{\iota fs}) \tilde{\tilde{\sqcap}} (Ӽ, Ƭ_{\iota fs})\right)^{c}}$ $\tilde{\tilde{\subseteq}}\, (\overline{(Ɽ, Ƒ_{\iota fs})} \,\tilde{\tilde{\sqcap}}\, \overline{(Ӽ, Ƭ_{\iota fs})}) \,\tilde{\tilde{\sqcap}}(\overline{(Ɽ, Ƒ_{\iota fs})^{c}}$ $Ɱ(s_2) = \{(m_1, 0.2, 0.01), (m_2, 0.8, 0.1)\}\} = (Ɱ, Ꜧ_{ifs})^{c}$.

Proposition 11: Let δ: $\tilde{\tilde{\mathcal{M}}}_{ifs_S} \to \tilde{\tilde{\mathcal{W}}}_{ifs_S}$ be an ifs continuous function, then $Ꜧ_{\acute{C}_\ell}(\delta^{-1}(Ɽ, Ƒ_{ifs})) \,\tilde{\tilde{\subseteq}}\, \delta^{-1}(Ꜧ_{\acute{C}_\ell}(Ɽ, Ƒ_{ifs}))$, for each ifs set $(Ɽ, Ƒ_{ifs})$ of $\tilde{\tilde{\mathcal{W}}}_{ifs_S}$.

Proof: Let δ: $\tilde{\tilde{\mathcal{M}}}_{ifs_S} \to \tilde{\tilde{\mathcal{W}}}_{ifs_S}$ be an ifs continuous function and $(Ɽ, Ƒ_{ifs})$ be an ifs set in $\tilde{\tilde{\mathcal{W}}}_{ifs_S}$, then $Ꜧ_{\acute{C}_\ell}((\delta^{-1}(Ɽ, Ƒ_{ifs})) = \overline{((\delta^{-1}(Ɽ, Ƒ_{\iota fs}))} \,\tilde{\tilde{\sqcap}}\, \overline{((\delta^{-1}(Ɽ, Ƒ_{\iota fs}))^{c}}$ $\tilde{\tilde{\subseteq}}\, \overline{((\delta^{-1}\overline{(Ɽ, Ƒ_{\iota fs})})} \,\tilde{\tilde{\sqcap}}\, \overline{((\delta^{-1}\overline{(Ɽ, Ƒ_{\iota fs})})^{c}} = \delta^{-1}\overline{(Ɽ, Ƒ_{\iota fs})} \,\tilde{\tilde{\sqcap}}\, \delta^{-1}\overline{(Ɽ, Ƒ_{\iota fs})^{c}}$ $= \delta^{-1}(\,\overline{(Ɽ, Ƒ_{\iota fs})} \,\tilde{\tilde{\sqcap}}\, \overline{(Ɽ, Ƒ_{\iota fs})^{c}}) = \delta^{-1}(Ꜧ_{\acute{C}_\ell}(Ɽ, Ƒ_{ifs}))$.

4. Intuitionistic Fuzzy Soft Semi Boundary

Definition 11: Let $(\mathcal{M}, \acute{C}_\ell, \mathcal{S})$ and $(\mathcal{W}, \acute{C}_{\ell 1}, \mathcal{S})$ be two $ifs - \acute{C}_\ell$ spaces. A function δ: $\tilde{\tilde{\mathcal{M}}}_{ifs_S} \to \tilde{\tilde{\mathcal{W}}}_{ifs_S}$ is said to be an ifs semi-continuous function, if $\delta^{-1}(\mathcal{E}, \mathcal{F}_{ifs})$ is an $ifs - \mathrm{SC}$ set of $\tilde{\tilde{\mathcal{M}}}_{ifs_S}$ for each $ifs - \mathrm{SC}$ set of $\tilde{\tilde{\mathcal{W}}}_{ifs_S}$.

Definition 12: Let $(\mathrm{Ɱ}, \mathrm{Ᵹ}_{ifs})$ be an ifs set of $ifs-\acute{C}_\ell$ space $\tilde{\tilde{\mathcal{M}}}_{ifs_S}$, the intuitionistic fuzzy soft semi-boundary of $(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs})$ ($ifs-$SB′s in short) is defined as $\mathrm{Sᵸ}_{\acute{C}_\ell}(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs}) = \mathcal{S}\overline{(\mathrm{Ɱ}_{\iota fs}, \mathrm{Ᵹ}_{\iota fs})} \tilde{\tilde{\sqcap}}\ \mathcal{S}\overline{(\mathrm{Ɱ}_{\iota fs}, \mathrm{Ᵹ}_{\iota fs})^c}$. The $ifs{-}\mathcal{S}$B′s is denoted by $\mathcal{S}\mathrm{ᵸ}_{\acute{C}_\ell}$.

Proposition 12: If $(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs})$ be an ifs set in $ifs-\acute{C}_\ell$ space $\tilde{\tilde{\mathcal{M}}}_{ifs_S}$, then $\mathcal{S}\mathrm{ᵸ}_{\acute{C}_\ell}(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs}) = \mathcal{S}\mathrm{ᵸ}_{\acute{C}_\ell}(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs})^c$.

Proof: Since $\mathcal{S}\mathrm{ᵸ}_{\acute{C}_\ell}(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs}) = \mathcal{S}\overline{(\mathrm{Ɱ}_{\iota fs}, \mathrm{Ᵹ}_{\iota fs})} \tilde{\tilde{\sqcap}}\, \mathcal{S}\overline{(\mathrm{Ɱ}_{\iota fs}, \mathrm{Ᵹ}_{\iota fs})^c} = \mathcal{S}\overline{(\mathrm{Ɱ}_{\iota fs}, \mathrm{Ᵹ}_{\iota fs})^c} \tilde{\tilde{\sqcap}}\, \mathcal{S}\overline{(\mathrm{Ɱ}_{\iota fs}, \mathrm{Ᵹ}_{\iota fs})} = \mathcal{S}\overline{(\mathrm{Ɱ}_{\iota fs}, \mathrm{Ᵹ}_{\iota fs})^c} \tilde{\tilde{\sqcap}}\, \mathcal{S}\overline{((\mathrm{Ɱ}_{\iota fs}, \mathrm{Ᵹ}_{\iota fs})^c)^c} = \mathcal{S}\mathrm{ᵸ}_{\acute{C}_\ell}\,(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs})^c$.

Proposition 13: If $(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs})$ be an $ifs-$SC set in $ifs-\acute{C}_\ell$ space $\tilde{\tilde{\mathcal{M}}}_{ifs_S}$. Then, $\mathcal{S}\mathrm{ᵸ}_{\acute{C}_\ell}(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs}) \tilde{\tilde{\sqsubseteq}} (\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs})$.

Proof: Let $(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs})$ be an $ifs-$SC set in $\tilde{\tilde{\mathcal{M}}}_{ifs_S}$, $\mathcal{S}\mathrm{ᵸ}_{\acute{C}_\ell}(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs}) = \mathcal{S}\overline{(\mathrm{Ɱ}_{\iota fs}, \mathrm{Ᵹ}_{\iota fs})} \tilde{\tilde{\sqcap}}\, \mathcal{S}\overline{(\mathrm{Ɱ}_{\iota fs}, \mathrm{Ᵹ}_{\iota fs})^c} \ \tilde{\tilde{\sqsubseteq}}\ \mathcal{S}\overline{(\mathrm{Ɱ}_{\iota fs}, \mathrm{Ᵹ}_{\iota fs})} = (\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs})$. Hence, $\mathcal{S}\mathrm{ᵸ}_{\acute{C}_\ell}(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs}) \tilde{\tilde{\sqsubseteq}} (\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs})$.

Proposition 14: If $(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs})$ be an ifs set in $ifs-\acute{C}_\ell$ space $\tilde{\tilde{\mathcal{M}}}_{ifs_S}$. Then, $\left(\mathcal{S}\mathrm{ᵸ}_{\acute{C}_\ell}(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs})\right)^c = \mathcal{S}\mathrm{Int}_{C_\ell}(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs})\ \tilde{\tilde{\sqcup}}\ \mathcal{S}\mathrm{Int}_{C_\ell}\left(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs}\right)^c$.

Proposition 15: If $(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs})$ be an ifs set in $ifs-\acute{C}_\ell$ space $\tilde{\tilde{\mathcal{M}}}_{ifs_S}$. Then, $\mathcal{S}\mathrm{ᵸ}_{\acute{C}_\ell}(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs}) = \mathcal{S}\overline{(\mathrm{Ɱ}_{\iota fs}, \mathrm{Ᵹ}_{\iota fs})} -\ \mathcal{S}\mathrm{Int}_{C_\ell}(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs})$.

Proof: Let $(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs})$ is an ifs set in $\tilde{\tilde{\mathcal{M}}}_{ifs_S}$. $\mathcal{S}\overline{(\mathrm{Ɱ}_{\iota fs}, \mathrm{Ᵹ}_{\iota fs})} - \mathcal{S}\mathrm{Int}_{C_\ell}(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs}) = \mathcal{S}\overline{(\mathrm{Ɱ}_{\iota fs}, \mathrm{Ᵹ}_{\iota fs})} \tilde{\tilde{\sqcap}} \left(\mathcal{S}\mathrm{Int}_{C_\ell}(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs})\right)^c = \mathcal{S}\overline{(\mathrm{Ɱ}_{\iota fs}, \mathrm{Ᵹ}_{\iota fs})} \tilde{\tilde{\sqcap}}\, \mathcal{S}\overline{(\mathrm{Ɱ}_{\iota fs}, \mathrm{Ᵹ}_{\iota fs})^c} = \mathcal{S}\mathrm{ᵸ}_{\acute{C}_\ell}(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs})$.

Proposition 16: If $(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs})$ be an ifs set in $ifs-\acute{C}_\ell$ space $\tilde{\tilde{\mathcal{M}}}_{ifs_S}$. Then, $\mathcal{S}\mathrm{ᵸ}_{\acute{C}_\ell}(\mathcal{S}\mathrm{Int}_{C_\ell}(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs})) \tilde{\tilde{\sqsubseteq}}\ \mathcal{S}\mathrm{ᵸ}_{\acute{C}_\ell}(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs})$.

Proof: Let $(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs})$ is an ifs set in $\tilde{\tilde{\mathcal{M}}}_{ifs_S}$. $\mathcal{S}\mathrm{ᵸ}_{\acute{C}_\ell}(\mathcal{S}\mathrm{Int}_{C_\ell}(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs})) = \mathcal{S}\overline{(\mathcal{S}\mathrm{Int}_{C_\ell}\left(\mathrm{Ɱ}_{\iota fs}, \mathrm{Ᵹ}_{\iota fs}\right))} -\ \mathcal{S}\mathrm{Int}_{C_\ell}(\mathcal{S}\mathrm{Int}_{C_\ell}\left(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs}\right)) = \mathcal{S}\overline{(\mathcal{S}\mathrm{Int}_{C_\ell}\left(\mathrm{Ɱ}_{\iota fs}, \mathrm{Ᵹ}_{\iota fs}\right))} - \mathcal{S}\mathrm{Int}_{C_\ell}\left(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs}\right) = \mathcal{S}\overline{\left(\mathrm{Ɱ}_{\iota fs}, \mathrm{Ᵹ}_{\iota fs}\right)} - \mathcal{S}\mathrm{Int}_{C_\ell}\left(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs}\right) \tilde{\tilde{\sqsubseteq}}\ \mathcal{S}\mathrm{ᵸ}_{\acute{C}_\ell}(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs})$. Therefor, $\mathcal{S}\mathrm{ᵸ}_{\acute{C}_\ell}(\mathcal{S}\mathrm{Int}_{C_\ell}(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs})) \tilde{\tilde{\sqsubseteq}}\ \mathcal{S}\mathrm{ᵸ}_{\acute{C}_\ell}(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs})$.

Proposition 17: If $(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs})$ be an ifs set in $ifs-\acute{C}_\ell$ space $\tilde{\tilde{\mathcal{M}}}_{ifs_S}$. Then, $\mathcal{S}\mathrm{Int}_{C_\ell}\left(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs}\right)\ \tilde{\tilde{\sqsubseteq}}\ \left(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs}\right) - \mathcal{S}\mathrm{ᵸ}_{\acute{C}_\ell}(\mathrm{Ɱ}_{ifs}, \mathrm{Ᵹ}_{ifs})$.

Proof: Let $(\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs})$ is an ifs set in $\tilde{\tilde{\mathcal{M}}}_{ifs_S}$. $(\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs}) - S\text{Ƕ}_{Ć_\ell}(\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs}) = (\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs}) \tilde{\tilde{\sqcap}} (S\text{Ƕ}_{Ć_\ell}(\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs}))^c = (\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs}) \tilde{\tilde{\sqcap}} (S\overline{(\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs})} \tilde{\tilde{\sqcap}} S\overline{(\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs})^c})^c = (\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs}) \tilde{\tilde{\sqcap}} (S\text{Int}_{C_\ell}(\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs})^c \tilde{\tilde{\sqcup}} S\text{Int}_{C_\ell}(\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs})) = ((\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs}) \tilde{\tilde{\sqcap}} S\,\text{Int}_{C_\ell}(\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs})^c) \tilde{\tilde{\sqcup}} ((\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs}) \tilde{\tilde{\sqcap}} S\text{Int}_{C_\ell}(\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs})) =$

$$((\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs}) \tilde{\tilde{\sqcap}} S\text{Int}_{C_\ell}(\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs})^c) \tilde{\tilde{\sqcup}} S\text{Int}_{C_\ell}(\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs}) \tilde{\tilde{\sqsupseteq}} (\text{Ɱ}_{ifs}, \text{Ƕ}_{ifs}).$$

Proposition 18: Let $\tilde{\tilde{\mathcal{M}}}_{ifs_S}$ be an $ifs - Ć_\ell$ space, $(\text{Ʀ}, \text{Ƒ}_{ifs})$ and $(\text{Ҳ}, \text{Ƭ}_{ifs})$ are ifs sets over $\mathcal{M}$. Then, $S\text{Ƕ}_{Ć_\ell}((\text{Ʀ}, \text{Ƒ}_{ifs}) \tilde{\tilde{\sqcup}} (\text{Ҳ}, \text{Ƭ}_{ifs})) \tilde{\tilde{\sqsubseteq}} S\text{Ƕ}_{Ć_\ell}(\text{Ʀ}, \text{Ƒ}_{ifs}) \tilde{\tilde{\sqcup}} S\text{Ƕ}_{Ć_\ell}(\text{Ҳ}, \text{Ƭ}_{ifs})$.

Proof: $S\text{Ƕ}_{Ć_\ell}((\text{Ʀ}, \text{Ƒ}_{ifs}) \tilde{\tilde{\sqcup}} (\text{Ҳ}, \text{Ƭ}_{ifs})) = S\overline{((\text{Ʀ}, \text{Ƒ}_{ifs}) \tilde{\tilde{\sqcup}} (\text{Ҳ}, \text{Ƭ}_{ifs}))} \tilde{\tilde{\sqcap}} S\overline{((\text{Ʀ}, \text{Ƒ}_{ifs}) \tilde{\tilde{\sqcup}} (\text{Ҳ}, \text{Ƭ}_{ifs}))^c} \tilde{\tilde{\sqsubseteq}} (S\overline{(\text{Ʀ}, \text{Ƒ}_{ifs})} \tilde{\tilde{\sqcup}} S\overline{(\text{Ҳ}, \text{Ƭ}_{ifs})}) \tilde{\tilde{\sqcap}} (S\overline{(\text{Ʀ}, \text{Ƒ}_{ifs})^c} \tilde{\tilde{\sqcap}} S\overline{(\text{Ҳ}, \text{Ƭ}_{ifs})^c}) = (S\overline{(\text{Ʀ}, \text{Ƒ}_{ifs})} \tilde{\tilde{\sqcap}} (S\overline{(\text{Ʀ}, \text{Ƒ}_{ifs})^c} \tilde{\tilde{\sqcap}} S\overline{(\text{Ҳ}, \text{Ƭ}_{ifs})^c})) \tilde{\tilde{\sqcup}} S\overline{(\text{Ҳ}, \text{Ƭ}_{ifs})} \tilde{\tilde{\sqcap}} (S\overline{(\text{Ʀ}, \text{Ƒ}_{ifs})^c} \tilde{\tilde{\sqcap}} S\overline{(\text{Ҳ}, \text{Ƭ}_{ifs})^c})) = (S\text{Ƕ}_{Ć_\ell}(\text{Ʀ}, \text{Ƒ}_{ifs}) \tilde{\tilde{\sqcap}} S\overline{(\text{Ҳ}, \text{Ƭ}_{ifs})^c}) \tilde{\tilde{\sqcup}} (S\text{Ƕ}_{Ć_\ell}(\text{Ҳ}, \text{Ƭ}_{ifs}) \tilde{\tilde{\sqcap}} S\overline{(\text{Ʀ}, \text{Ƒ}_{ifs})^c}) \tilde{\tilde{\sqsubseteq}} S\text{Ƕ}_{Ć_\ell}(\text{Ʀ}, \text{Ƒ}_{ifs}) \tilde{\tilde{\sqcup}} S\text{Ƕ}_{Ć_\ell}(\text{Ҳ}, \text{Ƭ}_{ifs})$.

Proposition 19: Let $\tilde{\tilde{\mathcal{M}}}_{ifs_S}$ be an $ifs - Ć_\ell$ space, $(\text{Ʀ}, \text{Ƒ}_{ifs})$ and $(\text{Ҳ}, \text{Ƭ}_{ifs})$ are ifs sets over $\mathcal{M}$. Then, $S\text{Ƕ}_{Ć_\ell}((\text{Ʀ}, \text{Ƒ}_{ifs}) \tilde{\tilde{\sqcap}} (\text{Ҳ}, \text{Ƭ}_{ifs})) \tilde{\tilde{\sqsubseteq}} S\text{Ƕ}_{Ć_\ell}(\text{Ʀ}, \text{Ƒ}_{ifs}) \tilde{\tilde{\sqcup}} S\text{Ƕ}_{Ć_\ell}(\text{Ҳ}, \text{Ƭ}_{ifs})$.

Proof: $S\text{Ƕ}_{Ć_\ell}((\text{Ʀ}, \text{Ƒ}_{ifs}) \tilde{\tilde{\sqcap}} (\text{Ҳ}, \text{Ƭ}_{ifs})) = S\overline{((\text{Ʀ}, \text{Ƒ}_{ifs}) \tilde{\tilde{\sqcap}} (\text{Ҳ}, \text{Ƭ}_{ifs}))} \tilde{\tilde{\sqcap}} S\overline{((\text{Ʀ}, \text{Ƒ}_{ifs}) \tilde{\tilde{\sqcap}} (\text{Ҳ}, \text{Ƭ}_{ifs}))^c} \tilde{\tilde{\sqsubseteq}} (S\overline{(\text{Ʀ}, \text{Ƒ}_{ifs})} \tilde{\tilde{\sqcap}} S\overline{(\text{Ҳ}, \text{Ƭ}_{ifs})}) \tilde{\tilde{\sqcap}} (S\overline{(\text{Ʀ}, \text{Ƒ}_{ifs})^c} \tilde{\tilde{\sqcup}} S\overline{(\text{Ҳ}, \text{Ƭ}_{ifs})^c}) = (S\overline{(\text{Ʀ}, \text{Ƒ}_{ifs})} \tilde{\tilde{\sqcap}} (S\overline{(\text{Ʀ}, \text{Ƒ}_{ifs})^c} \tilde{\tilde{\sqcap}} S\overline{(\text{Ҳ}, \text{Ƭ}_{ifs})^c})) \tilde{\tilde{\sqcup}} S\overline{(\text{Ҳ}, \text{Ƭ}_{ifs})} \tilde{\tilde{\sqcap}} (S\overline{(\text{Ʀ}, \text{Ƒ}_{ifs})^c} \tilde{\tilde{\sqcap}} S\overline{(\text{Ҳ}, \text{Ƭ}_{ifs})^c})) = (S\text{Ƕ}_{Ć_\ell}(\text{Ʀ}, \text{Ƒ}_{ifs}) \tilde{\tilde{\sqcap}} S\overline{(\text{Ҳ}, \text{Ƭ}_{ifs})^c}) \tilde{\tilde{\sqcup}} (S\text{Ƕ}_{Ć_\ell}(\text{Ҳ}, \text{Ƭ}_{ifs}) \tilde{\tilde{\sqcap}} S\overline{(\text{Ʀ}, \text{Ƒ}_{ifs})^c}) \tilde{\tilde{\sqsubseteq}} S\text{Ƕ}_{Ć_\ell}(\text{Ʀ}, \text{Ƒ}_{ifs}) \tilde{\tilde{\sqcup}} S\text{Ƕ}_{Ć_\ell}(\text{Ҳ}, \text{Ƭ}_{ifs})$.

Proposition 20: Let $\delta: \tilde{\tilde{\mathcal{M}}}_{ifs_S} \to \tilde{\tilde{\mathcal{W}}}_{ifs_S}$ be an ifs semi-continuous function, then $(S\text{Ƕ}_{Ć_\ell}(\delta^{-1}(\text{Ʀ}, \text{Ƒ}_{ifs})) \tilde{\tilde{\sqsubseteq}} \delta^{-1}(\text{Ƕ}_{C_\ell} S(\text{Ʀ}, \text{Ƒ}_{ifs}))$, for each ifs set $(\text{Ʀ}, \text{Ƒ}_{ifs})$ of $\tilde{\tilde{\mathcal{W}}}_{ifs_S}$.

5. Discussion and Conclusion

Finally, a definition for intuitionistic fuzzy soft closure for the intuitionistic fuzzy soft sets is presented, along with a discussion of several key characteristics for the intuitionistic fuzzy soft closure spaces. After providing an intuitionistic fuzzy soft semi-interior and illustrating intuitionistic fuzzy soft semi-open sets, several of its fundamental properties are examined.

In conclusion, this study uses the notions for the intuitionistic fuzzy boundary to derive the characterization theorems for various categories for the intuitionistic fuzzy soft open functions that have been covered in earlier research. Several basic claims and theorems will also be discussed. As far as future research, we hope our results will provide a foundation for more studies on functions and to develop a clear view of special limits.

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