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# Deriving a new complex rule for calculating double integrals using Boole's Rule

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**Abstract**

The primary region. The first rule is composed of Boole's rule for the inner dimension and the midpoint rule for the outer dimension. The second rule is composed of Boole's rule on both internal dimensions. The goal of the research is to formulate two rules for determining the values of the continuous double integral inside the integration x, y, with the extraction of the correction limits that come with the two rules, and making use of them by extracting them, using the Romberg-Regardson acceleration, and comparing the accuracy and speed of approach to the exact value.

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## 1. Introduction

Many researchers in numerical analysis have devoted their efforts to developing formulas and methods for calculating continuous double integrals over the integration region. Recent studies addressed the computing continuous double integrals with Gauss's rule and the midpoint rule with Remberg acceleration [1, 2]. The Authors [3] introduced

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a technique in 2025 for computing continuous double integrals using Rumer's acceleration, based on Simpson's rule and the midpoint rule. Three approaches based on the principles (trapezoidal, midpoint, and Simpson) with Rumer's acceleration.

Furthermore, the author [1] presented a novel compound rule derived from the midpoint rule with Rumberg acceleration in 2020, where the Rumer's acceleration with Simpson's rule and the midpoint rule, and he proposed an approach akin to Akkar's method. In 2016, two publications by Authors [2] and [4] revealed six 'methods'. Al-sharify and King [2] depended on Richardson and Romberg's accelerations using the midpoint and trapezoidal rules, whereas another author relied on the same two accelerations using Simpson's rule and the trapezoidal rule. In 2018, Hilal [6] presented a method similar to Akkar's method and derived a numerical method using Romberg acceleration with the trapezoidal and midpoint rules to calculate the unknown integrals at the limits of integration.

The authors [7] also presented a 2020 study to numerically determine double integrals using a novel method based on the Ramberg acceleration rule, it has been developed by many other researchers in this field. Among those who worked in the field of numerical integration are authors [5].

Our study focuses on two (($BB$, $MB$) composite rules with associated error formulas, the Paul and mid-point rules. We also use Richardson and Romberg accelerations to improve the results of these rules. We have presented the following four composite methods: ($RBB$, $RMB$, $\overline{R}MB$, $\overline{R}BB$) We arrived at the conclusion that the rule $BB$ outperforms the other rule $MB$ in terms of accuracy and speed of approaching the exact value by comparing the outcomes of applying the derived rules.

Our conclusion was that the method $\overline{R}BB$ is more accurate and approaches the exact answer faster than methods ($RBB$, $RMB$, $\overline{R}MB$) the rule $\overline{R}MB$ is the least effective of the above rules, we also determined.

We go over a few fundamentals and the approach priorities used in deriving methods.

Presume that we have integration.

$$I = \int_a^b F(\chi)\, d\chi$$

The above integral can be calculated numerically using the midpoint rule and Boole's rule after dividing $[a,b]$ the period of integration into equal subintervals, the number of which is $\eta$.

The following two formulas determine the partial period's length $\lambda$ whereas:

$$\lambda = \frac{b-a}{\eta}$$

1. The midpoint rule (we will symbolize it *M*)

$$I = \int_{\chi_0=a}^{\chi_\eta=b} F(\chi)\, d\chi = \lambda \sum_{i=0}^{\eta-1} F_{i+\frac{\lambda}{2}} + \mathrm{E_M}(\lambda)$$

2. Boole's rule (which we shall represent symbolically B)

$$I = \int_{\chi_0=a}^{\chi_\eta=b} F(\chi)\, d\chi = \frac{2\lambda}{45}\left[7(F_0+F_\eta)+32\sum_{i=1,5,9,\ldots}^{\eta-3}(F_i+F_{i+2})+12\sum_{i=2,6,10,\ldots}^{\eta-2}(F_i)+14\sum_{i=4,8,12,\ldots}^{\eta-4}(F_i)\right]+\mathrm{E}_B(\lambda)$$

Whereas:

$$F_k = F(\chi_0 + k\lambda)$$

$$\mathrm{E_M}(\lambda) = M_1\lambda^2 + M_2\lambda^4 + M_3\lambda^6 + \cdots$$

$$\mathrm{E}_B(\lambda) = B_1\lambda^6 + B_2\lambda^8 + B_3\lambda^{10} + \cdots$$

Whereas $M_1, M_2, M_3, \cdots,\ B_1, B_2, B_3, \cdots$ The values of their constants do not affect the amount of error.

### 1.1 *Derivation of the base MB and the complex base BB for calculating double integrals:*

Let us have the following double integral

$$I = \int_{\gamma_0}^{\gamma_\eta}\int_{\chi_0}^{\chi_\eta} F(\chi,\gamma)\, d\chi d\gamma \qquad \cdots(1)$$

Where the function $F(\chi,\gamma)$ is continuous and differentiable within the region $[\chi_0,\chi_\eta]\times[\gamma_0,\gamma_\eta]$.

We will divide the period of internal integration (by distance $\chi$) to $\eta$ the equal length sub-intervals where the length of one sub-interval is $\lambda$. Additionally, we split the exterior integration period (on the distance $\gamma$) into equal-length partial periods so that the internal dimension's partial period lengths are $\bar{\lambda}$ in order for the length of the inner dimension's $\chi$

partial periods to be equivalent to the length of the outer dimension's $\gamma$ partial periods. that is $(\lambda = \overline{\lambda})$.

1.1.1 The rule's derivation *MB*

Using Boole's integral rule, we calculate the inner integral over distance $\chi$ of equation (1) taking into $\gamma$ account the constant. The result is:

$$I = \int_{\gamma_0}^{\gamma_\eta} \int_{\chi_0}^{\chi_\eta} F(\chi,\gamma)\, d\chi d\gamma =$$

$$\int_{\gamma_0}^{\gamma_\eta} \left[ \frac{2\lambda}{45} \left[ 7(F_{0,\gamma} + F_{\eta,\gamma}) + 32 \sum_{i=1,5,9,\dots}^{\eta-3} (F_{i,\gamma} + F_{i+2,\gamma}) + 12 \sum_{i=2,6,10,\dots}^{\eta-2} (F_{i,\gamma}) + 14 \sum_{i=4,8,12,\dots}^{\eta-4} (F_{i,\gamma}) \right] + B_1\lambda^6 + B_2\lambda^8 + B_3\lambda^{10} + \cdots \right] d\gamma \tag{2}$$

Whereas $F_{i,\gamma} = F(\chi_0 + i\lambda, \gamma)$

$$I = \int_{\gamma_0}^{\gamma_\eta} \int_{\chi_0}^{\chi_\eta} F(\chi,\gamma)\, d\chi d\gamma =$$

$$\int_{\gamma_0}^{\gamma_\eta} \left[ \frac{2\lambda}{45} \left[ 7(F_{0,\gamma} + F_{\eta,\gamma}) + 32 \sum_{i=1,5,9,\dots}^{\eta-3} (F_{i,\gamma} + F_{i+2,\gamma}) + 12 \sum_{i=2,6,10,\dots}^{\eta-2} (F_{i,\gamma}) + 14 \sum_{i=4,8,12,\dots}^{\eta-4} (F_{i,\gamma}) \right] \right] d\gamma$$

$$\int_{\gamma_0}^{\gamma_\eta} [+B_1\lambda^6 + B_2\lambda^8 + B_3\lambda^{10} + \cdots]\, d\gamma \tag{3}$$

Equation (3)'s first integral is computed by applying the midpoint rule to the outer dimension $\gamma$ of each integration limit. In relation to the variable $\gamma$, the second integral is analytically integrated, giving us:

$$I = \int_{\gamma_0}^{\gamma_\eta} \int_{\chi_0}^{\chi_\eta} F(\chi,\gamma)\, d\chi d\gamma =$$

$$MB(\lambda) = \frac{2\lambda^2}{45} \sum_{j=0}^{\eta-1} \left[ 7(F_{0,j+\frac{\lambda}{2}} + F_{\eta,j+\frac{\lambda}{2}}) + 32 \sum_{i=1,5,9,\dots}^{\eta-3} (F_{i,j+\frac{\lambda}{2}} + F_{i+2,j+\frac{\lambda}{2}}) + 12 \sum_{i=2,6,10,\dots}^{\eta-2} (F_{i,j+\frac{\lambda}{2}}) + 14 \sum_{i=4,8,12,\dots}^{\eta-4} (F_{i,j+\frac{\lambda}{2}}) \right]$$
$$+ \left[ \alpha_1 \lambda^2 + \alpha_2 \lambda^4 + \alpha_3 \lambda^6 + \alpha_4 \lambda^8 + \alpha_4 \lambda^{10} + \cdots \right]$$

$$I - MB(\lambda) = E_{MB}(\lambda) = \left[ \alpha_1 \lambda^2 + \alpha_2 \lambda^4 + \alpha_3 \lambda^6 + \alpha_4 \lambda^8 + \alpha_4 \lambda^{10} + \cdots \right] \tag{4}$$

Whereas $F_{i,j+\frac{\lambda}{2}} = F(\chi_0 + i\lambda, \gamma_0 + j\lambda + \frac{\lambda}{2})$ and $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \cdots$ Its values are dependent on the constant values, and have no bearing on the integral's value $B_1, B_2, M_{11}, M_{21}, M_{31}, \cdots$.

1.1.2 The rule's derivation *BB*

We will proceed similarly to the previous approach of computing the internal integral (on the dimension $\chi$), assuming we have the integral in equation (1). Using Boole's integral rule with a constant factor $\gamma$ and adding correction terms accompanying the rule

$$I=\int_{\gamma_0}^{\gamma_\eta}\int_{\chi_0}^{\chi_\eta}F(\chi,\gamma)\,d\chi d\gamma=$$

$$\int_{\gamma_0}^{\gamma_\eta}\Bigg[\frac{2\lambda}{45}\Bigg[7(F_{0,\gamma}+F_{\eta,\gamma})+32\sum_{i=1,5,9,\ldots}^{\eta-3}(F_{i,\gamma}+F_{i+2,\gamma})+12\sum_{i=2,6,10,\ldots}^{\eta-2}(F_{i,\gamma})+14\sum_{i=4,8,12,\ldots}^{\eta-4}(F_{i,\gamma})\Bigg]$$

$$+B_1\lambda^6+B_2\lambda^8+B_3\lambda^{10}+\cdots\Bigg]d\gamma \tag{5}$$

Then, we apply Boole's method once again to integrate each of the resultant terms with respect to the variable $\gamma$, and we apply analytical integration to the corrective terms.

$$I=\int_{\gamma_0}^{\gamma_\eta}\int_{\chi_0}^{\chi_\eta}F(\chi,\gamma)\,d\chi d\gamma=$$

$$\left(\frac{2\lambda}{45}\right)^2\Bigg(49(F_{0,0}+F_{\eta,0}+F_{0,\eta}+F_{\eta,\eta})\;+32\sum_{i=1,5,9,\ldots}^{\eta-3}\Bigg[7(F_{i,0}+F_{i+2,0}+F_{i,\eta}+F_{i+2,\eta}+F_{0,i}+F_{0,i+2}+F_{\eta,i}+F_{\eta,i+2})+$$

$$\sum_{j=1,5,9,\ldots}^{\eta-3}32(F_{i,j}+F_{i+2,j}+F_{i,j+2}+F_{i+2,j+2})\Bigg]+12\sum_{i=2,6,10,\ldots}^{\eta-2}\Bigg[7(F_{i,0}+F_{i,\eta}+F_{0,i}+F_{\eta,i})+\sum_{j=2,6,10,\ldots}^{\eta-2}12(F_{i,j})+$$

$$\sum_{j=1,5,9,\ldots}^{\eta-3}32(F_{i,j}+F_{i,j+2}+F_{j,i}+F_{j+2,i})\Bigg]+14\sum_{i=4,8,12,\ldots}^{\eta-4}\Bigg[7(F_{i,0}+F_{i,\eta}+F_{0,i}+F_{\eta,i})+\sum_{j=4,8,12,\ldots}^{\eta-4}14(F_{i,j})+$$

$$\sum_{j=1,5,9,\ldots}^{\eta-3}32(F_{i,j}+F_{i,j+2}+F_{j,i}+F_{j+2,i})+\sum_{j=2,6,10,\ldots}^{\eta-2}12(F_{i,j}+F_{j,i})\Bigg]\Bigg)+$$

$$+\int_{\gamma_0}^{\gamma_\eta}\left[B_1\lambda^6+B_2\lambda^8+B_3\lambda^{10}+\cdots\right]d\gamma+B_{10}\lambda^6+B_{20}\lambda^8+\cdots+B_{1\eta}\lambda^6+B_{2\eta}\lambda^8+\cdots$$

$$+\sum_{i=1,5,\ldots}^{\eta-3}B_{1i}\lambda^6+B_{2i}\lambda^8+\cdots+\sum_{i=2,6,\ldots}^{\eta-2}B_{1i}\lambda^6+B_{2i}\lambda^8+\cdots+\sum_{i=4,8,\ldots}^{\eta-4}B_{1i}\lambda^6+B_{2i}\lambda^8+\cdots \tag{6}$$

Than

$$I=\int_{\gamma_0}^{\gamma_\eta}\int_{\chi_0}^{\chi_\eta}F(\chi,\gamma)\,d\chi d\gamma=$$

$$\left(\frac{2\lambda}{45}\right)^2\Bigg(49(F_{0,0}+F_{\eta,0}+F_{0,\eta}+F_{\eta,\eta})\;+32\sum_{i=1,5,9,\ldots}^{\eta-3}\Bigg[7(F_{i,0}+F_{i+2,0}+F_{i,\eta}+F_{i+2,\eta}+F_{0,i}+F_{0,i+2}+F_{\eta,i}+F_{\eta,i+2})+$$

$$\sum_{j=1,5,9,\ldots}^{\eta-3}32(F_{i,j}+F_{i+2,j}+F_{i,j+2}+F_{i+2,j+2})\Bigg]+12\sum_{i=2,6,10,\ldots}^{\eta-2}\Bigg[7(F_{i,0}+F_{i,\eta}+F_{0,i}+F_{\eta,i})+\sum_{j=2,6,10,\ldots}^{\eta-2}12(F_{i,j})+$$

$$\sum_{j=1,5,9,\ldots}^{\eta-3} 32(F_{i,j}+F_{i,j+2}+F_{j,i}+F_{j+2,i})\Bigg]+14\sum_{i=4,8,12,\ldots}^{\eta-4}\Bigg[7(F_{i,0}+F_{i,\eta}+F_{0,i}+F_{\eta,i})+\sum_{j=4,8,12,\ldots}^{\eta-4}14(F_{i,j})+$$

$$\sum_{j=1,5,9,\ldots}^{\eta-3} 32(F_{i,j}+F_{i,j+2}+F_{j,i}+F_{j+2,i})+\sum_{j=2,6,10,\ldots}^{\eta-2}12(F_{i,j}+F_{j,i})\Bigg]\Bigg)$$

$$+\delta_1\lambda^6+\delta_2\lambda^8+\delta_3\lambda^{10}+\cdots$$

$$I-BB(\lambda)=E_{BB}(\lambda)=[\delta_1\lambda^6+\delta_2\lambda^8+\delta_3\lambda^{10}+\cdots] \tag{7}$$

Whereas $\delta_1,\delta_2,\delta_3,\cdots$ is constant Its values are dependent on the constant values and have no bearing on the integral's value $B_1,B_2,\cdots,B_{11},B_{21},B_{31},\cdots$.

## 2. Examples

Using the Romberg and Richardson accelerations to improve the integration results, we have applied the two rules (*MB, BB*) to numerous double integrals and attempted to take advantage of the correction limits that go along with them. The results of four of these integrations are documented in the tables that are attached to the research. We summarized each integral in terms of the number of integer rankings we obtained and the number of subintervals associated with them in the following tables, whereas the details of Table 1 follow:

1. The integrals and their precise values are listed in the first column.
2. In the second column, the number of right ranks that are acquired after applying the rule *BB* and its associated sub-periods are listed.
3. The number of right ranks attained with the number of corresponding sub-periods when using the method $\overline{R}BB$ and the method *RBB*, respectively, are shown in the third and fourth columns (Tables 2 and 3).
4. The number of right ranks attained after applying the rule *MB* and the number of related sub-periods are shown in the fourth column.
5. The fifth and sixth columns contain the number of correct ranks obtained with the number of sub-periods associated with them when applying the two methods $\overline{R}MB$ and *RMB* respectively.

**Table 1**

**Function and its analytical value**

| | Function and its analytical value | Number of correct positions using the rule *BB*/ number of sub-periods | Number of correct positions using the rule RBB / number of sub-periods | Number of correct positions using the rule RBB / number of sub-periods | Number of correct positions using the rule MB/ number of sub-periods | The number of correct ranks using the rule RMB / number of subintervals | The number of correct ranks using the rule RMB/ number of subintervals |
|---|---|---|---|---|---|---|---|
| 1 | $\int_1^2\int_1^2 (xy)\ln(x+y)dxdy$ 2.53437764950325 | $\frac{13}{56}$ | $\frac{14}{24}$ | $\frac{14}{32}$ | $\frac{7}{564}$ | $\frac{12}{36}$ | $\frac{14}{256}$ |
| 2 | $\int_1^2\int_0^1 xe^{(-x-y)}dxdy$ 0.06144772819733 | $\frac{14}{80}$ | $\frac{14}{24}$ | $\frac{14}{32}$ | $\frac{9}{1920}$ | $\frac{12}{36}$ | $\frac{14}{64}$ |
| 3 | $\int_1^2\int_2^3 \sqrt{(xy)/2}dxdy$ 1.36054043633161 | $\frac{14}{124}$ | $\frac{14}{24}$ | $\frac{14}{64}$ | $\frac{8}{888}$ | $\frac{13}{36}$ | $\frac{14}{64}$ |
| 4 | $\int_2^3\int_2^3 (xy)^{(1/y)}dxdy$ Unknown analytical value | | $\frac{14}{24}$ | $\frac{14}{64}$ | | $\frac{11}{32}$ | $\frac{13}{64}$ |

**Table 2**

**Function *BB* and $\bar{R}BB$**

| | $\eta$ | $BB$ | $\bar{R}BB$ | | | | |
|---|---|---|---|---|---|---|---|
| | | | $\lambda^6$ | $\lambda^8$ | $\lambda^{10}$ | $\lambda^{12}$ | $\lambda^{14}$ |
| $\int_2^3\int_2^3 (xy)^{(1/y)} dxdy$ Unknown analytical value | 4 | 2.08319627778622 | | | | | |
| | 8 | 2.08319746976841 | 2.08319748868876 | | | | |
| | 12 | 2.08319749284559 | 2.08319749506655 | 2.08319749532551 | | | |
| | 16 | 2.08319749479424 | 2.08319749521615 | 2.08319749523279 | 2.08319749522726 | | |
| | 20 | 2.08319749511331 | 2.08319749522666 | 2.08319749522878 | 2.08319749522830 | 2.08319749522838 | |
| | 24 | 2.08319749518960 | 2.08319749522802 | 2.08319749522843 | 2.08319749522837 | 2.08319749522837 | 2.08319749522837 |

**Table 3**

**Function *BB* and *RBB***

| | $\eta$ | $BB$ | $RBB$ | | | |
|---|---|---|---|---|---|---|
| | | | $\lambda^6$ | $\lambda^8$ | $\lambda^{10}$ | $\lambda^{12}$ |
| $\int_2^3\int_2^3 (xy)^{(1/y)} dxdy$ Unknown analytical value | 4 | 2.08319627778622 | | | | |
| | 8 | 2.08319746976841 | 2.08319748868876 | | | |
| | 16 | 2.08319749479424 | 2.08319749519148 | 2.08319749521698 | | |
| | 32 | 2.08319749522143 | 2.08319749522821 | 2.08319749522836 | 2.08319749522837 | |
| | 64 | 2.08319749522827 | 2.08319749522837 | 2.08319749522837 | 2.08319749522837 | 2.08319749522837 |

**Table 4**

**Function *MB* and $\bar{R}MB$**

| | | *MB* | $\bar{R}MB$ | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | $\lambda^2$ | $\lambda^4$ | $\lambda^6$ | $\lambda^8$ | $\lambda^{10}$ | $\lambda^{12}$ | $\lambda^{14}$ |
| $\int_2^3\int_2^3 (xy)^{(1/y)}dxdy$ | 4 | 2.08288346981 | | | | | | | |
| | 8 | 2.08311917820 | 2.08319774767 | | | | | | |
| | 12 | 2.08316270263 | 2.08319752217 | 2.08319746666 | | | | | |
| | 16 | 2.08317792730 | 2.08319750189 | 2.08319749250 | 2.08319749810 | | | | |
| Unknown analytical value | 20 | 2.08318497262 | 2.08319749761 | 2.08319749465 | 2.08319749541 | 2.08319749487 | | | |
| | 24 | 2.08318879929 | 2.08319749629 | 2.08319749505 | 2.08319749525 | 2.08319749520 | 2.08319749526 | | |
| | 28 | 2.08319110652 | 2.08319749577 | 2.08319749516 | 2.08319749523 | 2.08319749522 | 2.08319749523 | 2.08319749522 | |
| | 32 | 2.08319260395 | 2.08319749553 | 2.08319749520 | 2.08319749523 | 2.08319749523 | 2.08319749523 | 2.08319749523 | 2.08319749523 |

**Table 5**

**Function *MB* and *RMB***

| | $\eta$ | *MB* | *RMB* | | | | |
|---|---|---|---|---|---|---|---|
| | | | $\lambda^2$ | $\lambda^4$ | $\lambda^6$ | $\lambda^8$ | $\lambda^{10}$ |
| $\int_2^3\int_2^3 (xy)^{(1/y)}dxdy$ | 4 | 2.0828834698102 | | | | | |
| | 8 | 2.0831191782018 | 2.0831977476656 | | | | |
| Unknown analytical value | 16 | 2.0831779273031 | 2.0831975103369 | 2.0831974945150 | | | |
| | 32 | 2.0831926039451 | 2.0831974961590 | 2.0831974952138 | 2.0831974952249 | | |
| | 64 | 2.0831962724510 | 2.0831974952863 | 2.0831974952281 | 2.0831974952284 | 2.0831974952284 | |
| | 128 | 2.0831971895367 | 2.0831974952320 | 2.0831974952284 | 2.0831974952284 | 2.0831974952284 | 2.0831974952284 |

## 3. Discussion and Conclusion

We conclude from the data shown in the lookup tables (Tables 2 to 5), that when the two rules, *MB*, *BB* are applied to calculate continuous double integrals within the region of integration, they produce accurate values with a varying number of partial intervals and even several decimal places (compared to the exact values of the integrals).

Although a large number of partial periods were employed, we note that the base *MB* gave an accuracy range between (7-9) proper decimal places, and with partial periods ranging between (564-1920) partial periods. This is a relatively good accuracy. This accuracy is considered extremely good, especially given the sparsity of the partial periods. The base *BB* produced an accuracy range between (13–14) proper decimal places and partial periods ranging between (56–124) partial periods.

The following results were obtained by improving integration results and achieving a higher number of correct decimal places by utilizing correction limitations and applying Romberg and Richardson's acceleration with the rules *MB* and *BB*).

1. When using the rule *RMB* The partial intervals fell between (64-256) digits, while the right decimal places fell between (13–14) digits.
2. When applying the base $\overline{R}MB$, the correct decimal places ranged between (11-12) places, with partial periods ranging between (32-36).
3. Using the rule *RBB*, the right decimal places were (14) places with partial intervals between (32–64).
4. The correct decimal places became (14) places, and the partial periods became (24) when the rule $\overline{R}BB$ was applied.

It is noteworthy that the rules were applied to a large number of different integrals, and the outcomes were extremely similar to those reported above. This indicates that the four methods offered in this study may be used to numerically compute double integrals. It is evident that the rule is better due to its precision and its sparing use of partial periods. We should not forget the importance of the four methods in finding the values of double integrals whose analytical values are unknown, as in the fourth integral, where we notice that the results approach a certain amount, then the value remains constant horizontally, no matter how much we increase the correction limits (where the series of correction limits is a convergent Cauchy series), and the goal of the series convergence is the value of the integration.

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