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Calculating triple integrals with continuous integrals numerically using the method RS (RMS)

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Abstract

An objective of this article is to calculate continuous triple integrals numerically using the RS (RMS) method by applying Romberg acceleration to complex rules (the Simpson rule for the inner dimension x and the midpoint rule for the middle dimension y). The Simpson's rules to outer dimensions z, denoted as symbol RS (RMS), where an inner dimension x is equal to the middle dimensions y and not equal to the outer dimensions z. We achieved high accuracy and a large number of sub-periods in a short time.

Subject Classification: *65D30, 65D32.*

Keywords: *Triple integral, Continuous function, Simpson rules, Midpoint rule.*

1. Introduction

Subjects for numerical analysis are distinguished by devising methods to find approximate solutions to specific mathematical problems in an effective manner. The efficiency of this method depends on both accuracy and ease of use.

Modern numerical analysis is the numerical frontier for broad fields of applied analysis. Studying for errors is central to numerical analysis, since the results we obtain from applying most numerical methods are only approximations to true solutions [4-7].

It is important to understand the errors in the results and how to estimate them using the calculation sets.

Finding the value for the integral for this type is not easy, except in a few cases, so there is an urgent need to find an approximated value of this integral.

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Triple integrals are for great importance in find volume, intermediate center, and moments of inertia for volume, and finding masses with varying densities, for example, the volume located inside B^2+R^2=4B and above z=0 and below B^2+R^2=4B, and calculating the intermediate center of the volume located inside B^2+R^2=9 & above plane z=0 & below plane B+z=4, as well as finding messes with varying densities, like as piece for thin wires or a thin sheets for the metal, the authors in [1], prompt the numbers for the researcher to works in fields for the triple integral.

The authors in [2] present six numerical method composed of Newton – Cotes formula (mid - point, trapezoidal, Simpson formulas (STM, SMT, TMS, TSM, MST and MTS with derivations for corrections limits) and the errors formulas) for evaluating three-dimensionals integral numerical, whose integral be continuous or continuous but have poor partials derivatives or is poor at one end for integrations regions.

He used the Romberg acceleration methods with them and named them (RSTM, RSMT, RTMS, RTSM, RMST, and RMTS). In 2021, Aljassas, Kadhim, and Habeeb [3] presented two numerical RO(MSuSu) & RO(SuMM) methods with equal dimensions that yielded good results, and two additional numerical methods, RO(SuMSu) & RO(MSuM), for computing the triple integral of continuous vectors.

2. Numerical Techniques RS(RMS)

Let the function $g(x, y, z)$ is continuous and differentiable at all points in regions $[\mathfrak{a}, \mathfrak{b}] \times [\mathfrak{c}, \mathfrak{d}] \times [\mathfrak{e}, \mathfrak{g}]$, so the approximate values for triple integrals $\mathcal{T} = \int_{\mathfrak{e}}^{\mathfrak{g}} \int_{\mathfrak{c}}^{\mathfrak{d}} \int_{\mathfrak{a}}^{\mathfrak{b}} f(x, y, z)\, dx\, dy\, dz$ can be calculated as follows:

$$\mathcal{T} = \int_{\mathfrak{e}}^{\mathfrak{g}} \int_{\mathfrak{c}}^{\mathfrak{d}} \int_{\mathfrak{a}}^{\mathfrak{b}} g(x, y, z)\, dx\, dy\, dz \tag{1}$$

To finding its approximated values, we written it by:

$$\int_{\mathfrak{e}}^{\mathfrak{g}} \mathcal{O}(z)\, dz \tag{2}$$

Where:

$$\mathcal{O}(z) = \int_{c}^{d} \int_{a}^{b} g(x, y, z)\, dx\, dy \tag{3}$$

An approximated values for integrals eq. (2) over an outer dimensions z use a mid - point rules are:

$$\mathcal{T} = \frac{\not{f}}{3} \sum_{k=1}^{m} \mathcal{O}(z_k) + E(\not{f}) \tag{4}$$

If $z_k = e + 2k\not{f}$,

$\not{f} = \frac{g-e}{m}$ and that:

$$E(\not{f}) = \mathcal{F}_S \not{f}^4 + \mathcal{B}_S \not{f}^6 + \cdots (5)$$

$\mathcal{F}_S, \mathcal{B}_S$ are constants [4], [5]

t be corrections limits on outer dimensions z.

To calculated a value for the $F(z_k)$ approximate, we substituted value for z_k in eq. (3), so:

$$\mathcal{O}(Z_K) = \int_c^d \int_a^b g(x, y, z_k)\, dx\, dy \tag{6}$$

When applying the (MS) rule:

$$MS = \frac{f^2}{3} \sum_{j=1}^{2n} \begin{bmatrix} g(\mathrm{a}, \mathrm{y}_j) + g(\mathrm{b}, \mathrm{y}_j) + 4\sum_{i=1}^{n} g(\mathrm{x}_{2i-1}, \mathrm{y}_j) + \\ 2\sum_{i=1}^{n-1} g(\mathrm{x}_{2i}, \mathrm{y}_j) \end{bmatrix} \tag{7}$$

To integration eq. (6), so

$$\mathcal{O}(Z_K) = \frac{f^3}{27} \sum_j^{n_1} \sum_i^{n_2} g(x_i, y_j, z_K) + \overline{E(f)} \tag{8}$$

For, $x_{2i-1} = a + (2i - 1)f$,

$$i = 1,2, \dots, n-1,$$

$$x_{2i} = a + 2if,$$

$$j = 1,2, \dots, 2n$$

$$, y_j = c + 2jf$$

A formulas of corrections limit for it, where integrals functions are continuous be the:

$$MS(f) = \mathcal{U}_{MS} f^2 + \mathcal{N}_{MS} f^4 + \mathcal{T}_{MS} f^6 + \cdots \tag{9}$$

$\mathcal{U}_{MS}, \mathcal{N}_{MS}, \mathcal{T}_{MS}$ are constant [4], [6].

3. Examples

Example 3.1: A integrant for the

$$\int_1^2 \int_1^2 \int_1^2 \log(x + y + z)\, dx\, dy\, dz \tag{10}$$

which define of $(x, y, z)\ belong\ to\ the [\mathbb{1}, \mathbb{2}] \times [\mathbb{1}, \mathbb{2}] \times [\mathbb{1}, \mathbb{2}]$ that analytic valued.

Apply $RS(RMS)$ where $m = 32, n1 = n2 = 32$.

Table 1

Convergence of the RS(RMS) method for different values of m with $n_1 = n_2 = 32$.

m	RS(RMS)	n1 = n2
2	1.498324496194	32
4	1.497805342381	32
8	1.497802296228	32
16	1.497802288585	32
32	1.497802288575	32

Example 3.2: The integrant of

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x\, e^{(y+z)}\, dx\, dy\, dz \tag{11}$$

That defined for $(x, y, z) \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right] \times \left[\frac{\pi}{4}, \frac{\pi}{2}\right] \times \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$.
Apply the $RS(RMS)$ when $m = 32, n1 = n2 = 32$.

Table 2

Effect of the parameter m on the RMS residual RS (RMS) with n1 = n2 = 32.

m	RS(RMS)	n1=n2
2	1.993930635529	32
4	2.006091127946	32
8	2.006237032468	32
16	2.006237085068	32
32	2.006237085119	32

Example 3.3: The integrant of

$$\int_1^2 \int_1^2 \int_1^2 (xyz)^{\frac{1}{yz}}\, dx\, dy\, dz \tag{12}$$

That defined for (x, y, z) $belong\ to$ $[1,2] \times [1,2] \times [1,2]$
Apply $RS(RMS)$ where

$$m = 64, n1 = n2 = 64$$

Table 3

Effect of parameter m on RS RMS values with n1 equal to n2 equal to 64.

M	RS(RMS)	n1=n2
2	1.690558087181	64
4	1.688787389456	64
8	1.688366745380	64
16	1.688356587978	64
32	1.688356521713	64
64	1.688356521694	64

4. Conclusion

A result for the article shows calculations for the approximated value of the triple integral of a continuous function by using the methods RS (RMS), which yield good accuracy, a rapid convergence to the exact integral values, and a short computation time. Tables 1, 2, and 3 illustrate the convergence behavior of the RS (RMS) method for different values of m. These results confirm the efficiency of the method in terms of accuracy, convergence speed, and low computational cost.

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