Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 29 (2026), No. 5-B, pp. 1531–1535
DOI : 10.47974/JIM-2544

Approximate solutions of random ordinary differential models using modified Sumudu transform

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Abstract

An efficient combination of ADM and the Sumudu transformation for solving random ordinary differential equations (RODEs) is introduced as a novel tool to obtain an accurate analytical solution. It is an elegant combination of ST-ADM. The suggested method will convert differential equations into iterative algebraic equations, thereby reducing computational and analytical effort. The technique solves the problem of calculating the Adomian polynomials. The method's efficiency was investigated using several numerical examples, and the findings demonstrate that it is easier to use than many other numerical methods. It has also been established that (STADM) is a trustworthy technique for solving differential equations. Using the Mathematica 13.3 program. Applications are presented as examples of how the proposed technique can be used to obtain analytical or numerical solutions to certain kinds of Random Differential Equations (RODEs), thereby demonstrating its efficacy and potential.

Subject Classification: *35R11, 35R60.*

Keywords: *Sumudu transform, Analytical technique, Random differential equation, Ordinary differential equation.*

1. Introduction

Numerous Nonlinear Random Integral Equations (NRIEs) play a major role in modern modeling phenomena. Several biological, engineering, and physical challenges involve these phenomena. Also, the requirement to get accurate solutions is often seen as a challenge for these problems. Several analytical techniques for finding solutions are used, either by idealizing or simplifying the original problem. These techniques have shown to be an

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effective and efficient tool for supplying accurate results for nonlinear problems. Several approaches to solving RIDEs have been found recently. Various theorems that aid in the handling and resolution of integral equations, PDEs, ODEs, and differential equations of all types. The major goal of this study is to compute approximate solutions using a mixture Sumudu transform and the Adomian decomposition method to obtain effective solutions of RODEs [1-11].

2. Application and results

Example 1: Consider the nonlinear Random Ordinary Differential Equation as follows:

$$\mathfrak{D}'(\pi;\rho) = -\mathfrak{D}^2(\pi\,;\rho) + \mathrm{W}_\pi(\rho)$$

with initial condition $\mathfrak{D}_0(0;\rho) = 1, \pi \in [0,1]$

Solution: Using the Sumudu transform Decomposition Method (STADM) for Eq (2.1) we get: $\mathrm{S}[\mathrm{f}'(\mathrm{t})](\mathrm{u}) = \frac{1}{\mathrm{u}}\big(\mathrm{F}(\mathrm{u}) - \mathrm{f}(0)\big), \mathrm{S}^{-1}[\mathrm{uF}(\mathrm{u})](\mathrm{t}) = \int_0^{\mathrm{t}} \mathrm{f}(\mathrm{s})\,\mathrm{ds}$

$$\mathfrak{D}'(\pi;\rho) = \mathrm{W}_\pi(\rho) - \mathrm{N}(\mathfrak{D})(\pi\,;\rho)\,, \mathrm{N}(\mathfrak{D}) = \mathfrak{D}^2$$

$$\frac{1}{\mathrm{u}}(\mathrm{S}[\mathfrak{D}](\mathrm{u}) - 1) = \mathrm{S}[\mathrm{W}_\pi](\mathrm{u}) - \mathrm{S}[\mathfrak{D}^2](\mathrm{u}).$$

Hence $$\mathrm{S}[\mathfrak{D}](\mathrm{u}) = 1 + \mathrm{uS}[\mathrm{W}_\pi](\mathrm{u}) - \mathrm{uS}[\mathfrak{D}^2](\mathrm{u}).$$

Apply $\mathrm{S}^{-1}$and use $\mathrm{S}^{-1}[\mathrm{uS}[\mathrm{f}]](\mathrm{t}) = \int_0^{\mathrm{t}} \mathrm{f}(\mathrm{s})\,\mathrm{ds}$

$$\mathfrak{D}(\pi\,;\rho) = 1 + \int_0^{\mathrm{t}} \mathrm{W}_{\mathrm{s}}(\rho)\mathrm{ds} - \int_0^{\mathrm{t}} \mathfrak{D}^2(\mathrm{s},\rho)\mathrm{ds}$$

Now, by using ADM $\mathfrak{D}(\pi\,;\rho) = \sum_{\mathrm{n}=0}^{\infty} \mathfrak{D}_{\mathrm{n}}(\pi\,;\rho), \mathfrak{D}^2 = \sum_{\mathrm{n}=0}^{\infty} \mathrm{A}_{\mathrm{n}}$

Where A_{n} are the Adomian polynomials for $\mathrm{g}(\mathfrak{D}) = \mathfrak{D}^2$,

$\mathrm{A}_0 = \mathfrak{D}_0^2, \mathrm{A}_1 = 2\mathfrak{D}_0\mathfrak{D}_1, \mathrm{A}_2 = 2\mathfrak{D}_0\mathfrak{D}_2 + \mathfrak{D}_1^2, \mathrm{A}_3 = 2\mathfrak{D}_0\mathfrak{D}_3 + 2\mathfrak{D}_1\mathfrak{D}_2,$ $\mathrm{A}_4 = 2\mathfrak{D}_0\mathfrak{D}_4 + 2\mathfrak{D}_1\mathfrak{D}_3 + \mathfrak{D}_2^2$, etc. Matching like orders gives the (ST-ADM) recursion

$$\mathfrak{D}^{(0)}(\pi\,;\rho) = 1 + \int_0^{\mathrm{t}} \mathrm{W}_{\mathrm{s}}(\rho)\mathrm{ds},$$

$$\mathfrak{D}^{(1)}(\pi\,;\rho) = 1 + \int_0^{\mathrm{t}} \mathrm{W}_{\mathrm{s}}\mathrm{ds} - \int_0^{\mathrm{t}}(\,1 + \int_0^{\mathrm{s}} \mathrm{W}_{\rho}\mathrm{d}\rho\,)^2\mathrm{ds},$$

$$\mathfrak{D}^{(2)}(\pi\,;\rho) = \mathfrak{D}^{(1)}(\pi\,;\rho) - \int_0^{\mathrm{t}} 2\,(\,1 + \int_0^{\mathrm{s}} \mathrm{W}_{\rho}\mathrm{d}\rho)\,\left[\,\int_0^{\mathrm{s}}\big(1 + \int_0^{\rho} \mathrm{W}_{\eta}\mathrm{d}\eta\,\big)^2\mathrm{d}\rho\right]\mathrm{ds}$$

$$\mathfrak{D}^{(3)}(\pi\,;\rho) = \mathfrak{D}^{(2)}(\pi\,;\rho) - \int_0^{\mathrm{t}}\left\{2\mathfrak{D}_0(\mathrm{s}\,;\rho)\mathfrak{D}_2(\mathrm{s}\,;\rho) + \big(\mathfrak{D}_1(\mathrm{s}\,;\rho)\big)^2\right\}\mathrm{ds}$$

The following Figures 1-2 express the random approximate solutions and absolute error at various values of for $\mathrm{t} = 0, 0.1, 0.2, \dots, 1$.

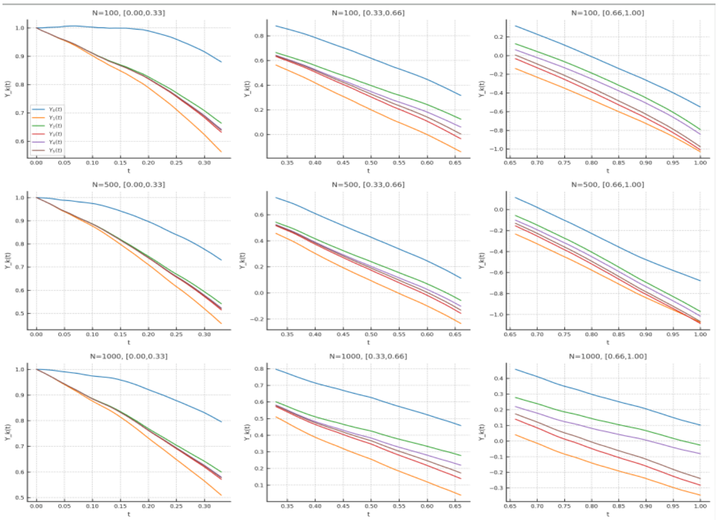

Figure 1

2Dplot graphs of random approximate solutions with N=100, 500, 1000.

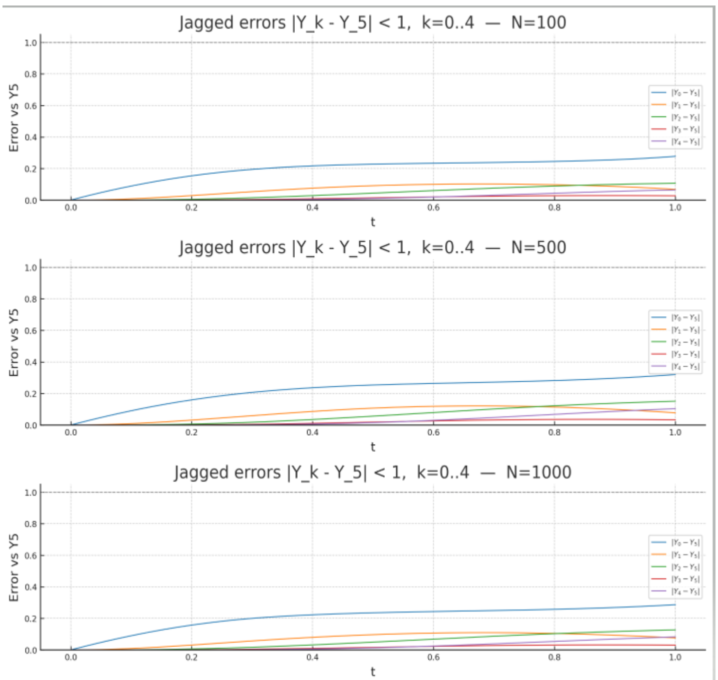

Figure 2

2D plot graphs of the absolute error

with $\mathbf{N = 100, N = 500}$ and $\mathbf{N = 1000}$ generations of Brownian motion.

3. Conclusion

The numerical results for these NRDEs demonstrate the effectiveness of the Sumudu Transform combined with the Adomian Decomposition Method (ST-ADM) in generating accurate, rapidly convergent approximate solutions. The successive approximations Y0 through Y5 showed stable behavior, and the error between each approximation and the final solution remained consistently below 1, confirming strong convergence. The method successfully captured the natural zigzag structure of the stochastic path while preserving randomness without numerical instability. Moreover, analyzing different time intervals revealed that higher-order terms significantly reduce the approximation error. The approach proved efficient at handling nonlinear and random components simultaneously, without linearization or reliance on heavy numerical schemes.

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Received November, 2025