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Analytical solutions of random integro differential equations by new combination Sumudu transform with ADM

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Abstract

A new transform Sumudu Adomian Decomposition method (STADM) for non-linear random ordinary differential equations and the non-linear Random Integro differential equation (NRIDE) are utilized in a novel way to obtain accurate solutions. The iterative implementation of the coupled transform decomposition be use to approximated solution to ordinary differentials equation with random components. We use a normal distribution to analyze the parameters and initial conditions of ordinary differential equations with random components. Using Mathematica 13.3, the graphs of the approximate solutions and the absolute error are presented. An investigation is conducted into the effects of the normal distribution on the results of random component differential equations. Based on the numerical results, this method is highly effective. Some applications are presented as examples of how the proposed technique can be utilized to obtain analytical or numerical solutions to certain kinds of random differential equations, thereby demonstrating its efficacy and potential.

Subject Classification: *35R11, 35R60.*

Keywords: *Sumudu transform, Analytical technique, Random differential equation, Ordinary differential equation.*

1. Introduction

Random Ordinary Differential Equations (RODEs) are currently significant topics that involve ordinary differential equations (ODEs) with random variables or constants. In physics, engineering, and mathematical biology, random differential equations (RDEs) are used to analyze a wide range

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of problems. In recent times, some scientific challenges spanning domains such as control theory, fluid mechanics, climate models, and physics have been successfully resolved. In the literature, there are remarkably few studies discussing random nonlinear differential equations (RNDEs). When simulating disparate phenomena, noise is crucial. Differential equations pose essential difficulties due to uncertainty and arise in the mathematical modeling of physical systems and in a number of real-world situations. Uncertainty can be incorporated into differential systems in various ways. especially in the past few decades in a variety of fields, such as the social sciences, mathematical biology, and engineering. Ordinary Differential Equations (ODEs) with a multidimensional stochastic process $\mathcal{W}_{\tau,}$ are named RDEs. In numerous applications across numerous scientific domains, RNDEs are essential. These issues are frequently unsolvable analytically. Consequently, numerous numerical techniques for solving ODEs, PDEs, RPDEs, and RODEs have been created. Decomposition method, differential transform method, finite difference method, and numerous other methods are among the many numerical techniques available [1-6]. Many authors studied some types of differentials equation. A mains goal for the this study are to compute approximate solutions using a mixture Sumudu transforms and the Adomian decompositions methods to obtains effective solutions of the RIDEs [7-12].

2. Application and results

Consider $\text{Ӽ}'(t,\omega) = e^{W_t(\omega)} - \frac{1}{3}te^{W_t(\omega)} + \int_0^1 ts\text{Ӽ}(s,\omega)\, ds$

With $\text{Ӽ}(0,\omega) = 0$

Solution: Recall the Sumudu transform $S\{f(t)\}(u) = \int_0^\infty e^{-t} f(ut)\, dt$

With basic properties $S\{f'(t)\}(u) = \frac{1}{u}\left(S\{f(t)\}(u) - f(0)\right)$,

$$S\{t^n\}(u) = n!\, u^n$$

Let $$S\{\text{Ӽ}(t,\omega)\} = \tilde{\text{Ӽ}}(u,\omega)$$

Applying S to both sides of Eq. (2.1) and using $\text{Ӽ}(0,\omega) = 0$, then

$$\frac{1}{u}\tilde{\text{Ӽ}}(u,\omega) = S\{e^{W_t(\omega)}\}(u) - \frac{1}{3}S\{t\, e^{W_t(\omega)}\}(u) + S\left\{t\int_0^1 s\, \text{Ӽ}(s,\omega)\, ds\right\}(u)$$

Let $\int_0^1 s\, \text{Ӽ}(s,\omega)\, ds =: A(\omega)$

then $S\left\{t\int_0^1 s\, \text{Ӽ}(s,\omega)\, ds\right\}(u) = A(\omega)\, S\{t\}(u) = u\, A(\omega)$

Hence $\tilde{\text{Ӽ}}(u,\omega) = u\, S\{e^{W_t(\omega)}\}(u) - \frac{u}{3}S\{t\, e^{W_t(\omega)}\}(u) + u^2 A(\omega)$

Taking the inverse Sumudu transform we get

$$\text{Ӽ}(t,\omega) = \text{Ӽ}_0(t,\omega) + S^{-1}\{u^2 A(\omega)\}(t),$$

Where $\text{Ӽ}_0(t,\omega) = S^{-1}\left\{u\, S\{e^{W_t(\omega)}\} - \frac{u}{3}S\{t\, e^{W_t(\omega)}\}\right\}(t)$

Using the derivative properties, Ӽ_0 is the exact solution of

$$X_0'(t,\omega) = e^{W_t(\omega)} - \frac{1}{3} t\, e^{W_t(\omega)}, X_0(0,\omega) = 0$$

So $X_0(t,\omega) = \int_0^t \left(e^{W_r(\omega)} - \frac{1}{3} r\, e^{W_r(\omega)}\right) dr$

Then, we can generate the higher components by

$$X_{n+1}(t,\omega) = S^{-1}\{u^2 A_n(\omega)\}(t) = \frac{t^2}{2} A_n(\omega), n \geq 0$$

Then, we can get $X_1(t,\omega) = \frac{t^2}{2} A_0(\omega)$

$$X_2(t,\omega) = \frac{t^2}{16} A_0(\omega)$$

$X_3(t,\omega) = \frac{t^2}{128} A_0(\omega), X_4(t,\omega) = \frac{t^2}{1024} A_0(\omega), X_5(t,\omega) = \frac{t^2}{8192} A_0(\omega)$

Then general pattern $X_{n+1}(t,\omega) = \frac{t^2}{2*8^n} A_0(\omega), n = 0,1,2, \dots$

The partial sums are $Y_m(t,\omega) = \sum_{n=0}^{m} X_n(t,\omega)\, , m = 0, 1, \dots, 5$

$Y_0(t,\omega) = X_0(t,\omega) = \int_0^t \left(e^{W_r(\omega)} - \frac{1}{3} r\, e^{W_r(\omega)}\right) dr$

$Y_1(t,\omega) = X_0(t,\omega) + \frac{1}{2} t^2 A_0(\omega), Y_2(t,\omega) = X_0(t,\omega) + \frac{9}{16} t^2 A_0(\omega)$

$Y_3(t,\omega) = X_0(t,\omega) + \frac{73}{128} t^2 A_0(\omega)\, , Y_4(t,\omega) = X_0(t,\omega) + \frac{585}{1024} t^2 A_0(\omega)$

$Y_5(t,\omega) = X_0(t,\omega) + \frac{4681}{8192} t^2 A_0(\omega)$

Where $A_0(\omega) = \int_0^1 s\, X_0(s,\omega)\, ds = \int_0^1 s\, [\int_0^s e^{W_r(\omega)} - \frac{1}{3} r\, e^{W_r(\omega)} dr]\, ds$

We obtain the best Brownian path when $W_t(\omega) = 0$

Figures 1 and 2 illustrate 2D plots for random approximated solution and their absolute errors for N = 100, 500, and 1000.

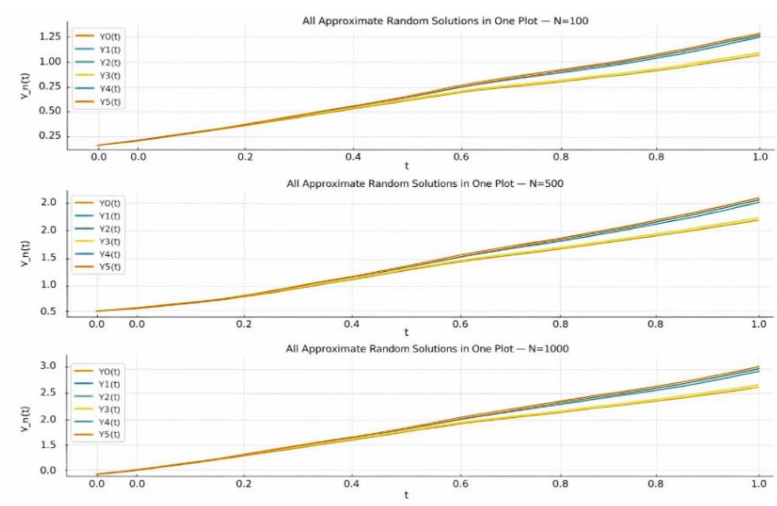

Figure 1

2D plots results of the approximate solutions when N=100, N=500 and N=1000

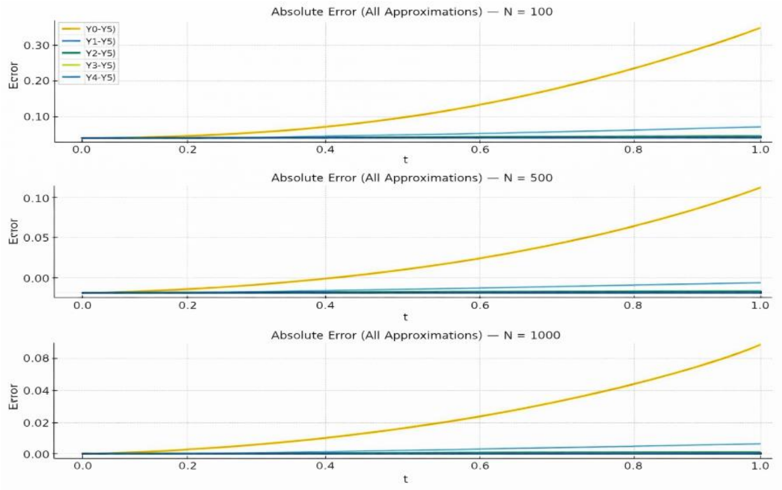

Figure 2

2D plots of results of the absolute error when N =100, N=500 and N=1000

3. Conclusion

We have successfully employed STADM to obtain approximate analytical solutions for several RODE and NRIDE applications. The proposed technique is simple to use, appropriate, and effective in achieving the desired results for these issues, and STADM provides converging series results quickly. sRIDEs' approximate analytical solution was obtained quickly and effectively. The STADM is more powerful and reliable for obtaining approximate solutions to several classes of random-component integral equations. The error used here, when the exact solution does not exist, is used to check the accuracy and convergence of the sequence of approximate results to the exact solution.

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