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A novel analytical approach to solve some applications of nonlinear problems

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Abstract

The Temimi and Ansari Technique (TAM), a new and powerful analytical method, has been proposed for solving nonlinear problems. As demonstrated by the findings, the method converges to the correct solution while handling nonlinear problems elegantly, accurately, and comfortably. The TAM result shows suggested methods are accurated and effective. The TAM-obtained solution is close to the actual solution. In addition, this approach is uncomplicated, derivative-free, easy to use, comfortable, and requires minimal computing power. To show how well the recommended approach applies, the solution graphs are displayed.

Subject Classification: *35R11, 35R60.*

Keywords: *Semi-analytical method, Temimi and Ansari method, Analytical method, Perturbation methods.*

1. Introduction

Modelling nonlinear processes involves using several nonlinear equations. Numerous mechanical, biological, physical, and engineering issues can involve these phenomena. For these issues, it is also difficult to find precise solutions. The original problem is idealized or simplified in many analytical techniques that aim to offer a solution. Perturbation methods include power series expansions solution with a few condition and some unknown coefficient must be assessed to solve the problem. These techniques have proven to be a useful and

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efficient tool for providing precise solutions to various nonlinear problems. Originating from the Homotopy Perturbation Method (HAM), TAM was recently introduced. Numerous phenomena across a variety of scientific and technical fields can be theoretically described by a wide range of linear and nonlinear differential equations, which reveal their behaviors and effects. Since many of these equations have no analytical solution, semi-analytical or numerical methods must be used to solve them [1-5]. The behavior of the pertinent parameters is described by these solutions in the absence of approximation solutions. under some circumstances, the resulting approximate solutions that satisfy the beginning and boundary criteria converge to the exact solution. Recent studies propose the novel iterative technique, TAM. This approach offers the benefit of requiring less computational effort and avoiding the restrictive assumptions of other iterative methods, such as VIM and DTM. [6-11].

2. Applications and Results

2.1 Example 1: Put a coupled systems for the non-linear second - order differentials equation be:

$$\begin{bmatrix} Ɣ''(\varrho) + Ɣ(\varrho)\theta'(\varrho) - \theta^2(\varrho) - \theta(\varrho) + 1 = 0 \\ \theta''(\varrho) + Ɣ''(\varrho) - \theta'(\varrho))^2 + Ɣ'(\varrho) = 0 \end{bmatrix} \tag{1}$$

the boundary condition $Ɣ(0) = 0$ and the value for $Ɣ(1) = \sin(1)$, $\theta(0) = 1$,

$$\theta(1) = \cos(1).$$

Solution: By applying TAM on both sides of Eq. (1), we defined linear & non - linear part for equation in follows:

$$\begin{bmatrix} R_1(Ɣ, \theta) = Ɣ''(\varrho) \\ N_1(Ɣ, \theta) = Ɣ(\varrho)\theta'(\varrho) - \theta^2(\varrho) - \theta(x) \\ g_1(x) = 1 \end{bmatrix} \text{ and}$$

$$\begin{bmatrix} R_2(Ɣ, \theta) = \theta''(\varrho) \\ N_2(Ɣ, \theta) = Ɣ^2(\varrho) - (\theta'(\varrho))^2 + Ɣ'(\varrho) \\ g_2(\varrho) = 0 \end{bmatrix}$$

Then, the initial problem: $Ɣ_0''(\varrho) + 1 = 0$, $\theta_0''(\varrho) = 0$, and $Ɣ_0(0) = 0$, $Ɣ_0(1) = \sin(1)$, $\theta_0(0) = 1$, $\theta_0(1) = \cos(1)$. Then, the first iteration can be given by: $Ɣ_0(\varrho) = \frac{-\varrho^2}{2} + \varrho\sin(1) + \frac{1}{2}\varrho$, $\theta_0(\varrho) = \varrho\cos(1) - \varrho + 1$. A second iterations is carried trough's a systems:

$$\begin{bmatrix} Ɣ_1''(\varrho) = -Ɣ_0(\varrho)\,\theta_0'(\varrho) + \theta_0^2(\varrho) + \theta_0'(x) - 1 \\ \theta_1''(x) = -Ɣ_0^2(x) + (\theta_0')^2(x) - Ɣ_0'(x) \end{bmatrix} \tag{2}$$

And the boundary condition: $Ɣ_1(0) = 0$, and the $Ɣ_1(1) = \sin(1)$, $\theta_1(0) = 1$, $\theta_1(1) = cos(1)$. Then, we get: $Ɣ_1(x) = -0.0015439\,x^4 - 0.05045\,x^3 - 0.2298x^2$, $\theta_1(\varrho) = 0.00833\,\varrho^6 + 0.06707\,\varrho^5 - 0.1499\,\varrho^4 + 0.1666\,\varrho^3$

$$-0.56507\varrho^2 + 0.02993\varrho + 1$$

The resulting approximation solutions $\gamma_0\,(\varrho)\;\gamma_1(\varrho)\;\gamma(\varrho)$ and $\theta_0(\varrho), \theta_1(\varrho)$ $\theta(\varrho)$ are illustrated in Figures 1 and 2, respectively:

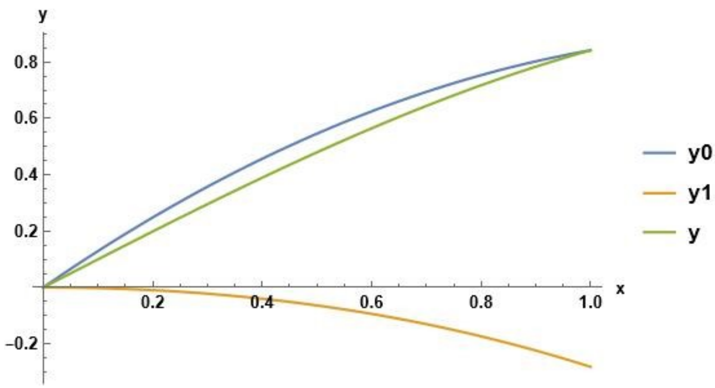

Figure 1
Approximation solutions $\boldsymbol{\gamma_0\,(\varrho)\;\gamma_1(\varrho)}$ and $\boldsymbol{\gamma(\varrho)}$

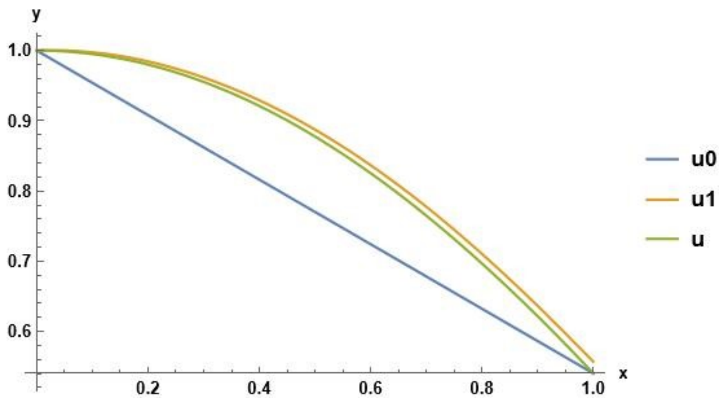

Figure 2
Approximation solutions $\boldsymbol{\theta_0(\varrho), \theta_1(\varrho)}$ and $\boldsymbol{\theta(\varrho)}$

2.2 Example 2: Put a Newell – Whitehead - Segel equations by follows:

$$\gamma_t(\varrho, t) + \gamma_{\varrho\varrho}(\varrho, t) = -3\gamma(\varrho, t) \tag{3}$$

With $\gamma(\varrho, 0) = e^{2\varrho}$

Solution: By applying TAM on both sides of Eq. (3), we get the linear and nonlinear operators as:

γ_t=$R(\gamma(\varrho, t))$, and the $(\gamma(\varrho, t)) = -\gamma_{\varrho\varrho} + 3\gamma$,

with the $f(\varrho, t)$ is equal to zero.

The approximate solution of Eq. (3) takes the form:

$$R(\text{Ɣ}_0(\varrho,t)) + f(\varrho,t) = 0 \tag{4}$$

$$\int_0^t \text{Ɣ}'_0\,(\varrho,t) = \text{Ɣ}_0(\varrho,t) - \text{Ɣ}_0(\varrho,0) = 0 \;\Rightarrow\; \text{Ɣ}_0(x,t) \;=\; e^{2x}$$

The second iterative solution of the problem gives:

$$R(\text{Ɣ}_1(\varrho,t))+f(\varrho,t)+ N(\text{Ɣ}_0(\varrho,t)) = 0. \tag{5}$$

By using $\text{Ɣ}(\varrho,t) = \lim_{n\to\infty} \text{Ɣ}_n(\varrho,t)$,
Leads to $\text{Ɣ}(\varrho,t) = e^{2\varrho+t}$
Which is the exact solution, as shown in Figure 3:

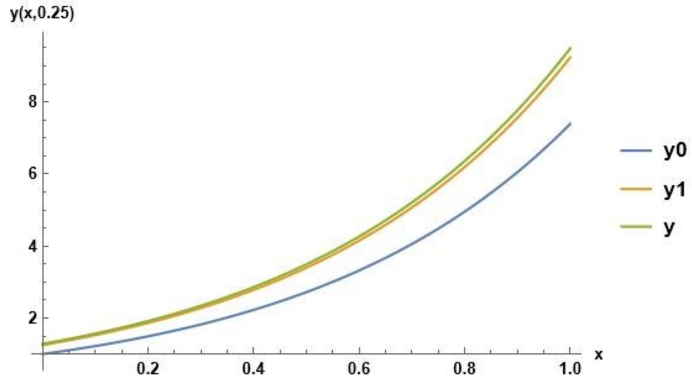

Figure 3
Solutions of $\text{Ɣ}_0(\varrho)$, $\text{Ɣ}_1(\varrho)$ and $\text{Ɣ}(\varrho)$ with $t = 0.25$

3. Conclusion

The approach that is suggested by TAM has the advantage of solving problems efficiently and rapidly while generating the analytical approximations in the converges series form with fewer terms.

To demonstrate the effectiveness and precision of the suggested approaches with tastefully calculated components, a few cases are tried.

The outcomes demonstrated that the approach manages nonlinear problems with solutions that converge to the correct solution in an elegant, accurate, and comfortable manner.

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