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# A comparative study of approximate solutions to solving second-order linear complex partial differential equations

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**Abstract**

This study addresses linear complex partial differential equations (LCPDEs), which constitute vital mathematical frameworks across diverse fields in physics and applied science. We utilize the modified Variational Iteration Method (VIM) and the Homotopy Perturbation Method (HPM), techniques ideally suited for generalized LCPDEs. The efficacy and precision of both methods were rigorously evaluated on various problem sets. The findings substantiate the method's ability to generate accurate and reliable approximate solutions, achieving excellent concordance with established analytical results. This confirms the competence of the modified VIM and HPM in effectively resolving LCPDEs.

***Subject Classification:*** *Primary 93A30, Secondary 49K15.*

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## 1. Introduction

Complex partial differential equations (CPDEs) are equations that involve partial derivatives of an unknown function, where the function and its derivatives are complex-valued [1-3]. CPDEs have significant applications in various fields of mathematics and physics [4-6], including fluid dynamics [7], [8], electromagnetism, quantum mechanics, and general relativity [9]. Furthermore, CPDEs can be connected to the study of Diophantine equations through various mathematical approaches, such as analytical methods, p-adic analysis, or frameworks that connect nonlinear PDEs to rational solutions of certain equations. For more details about these equations and their applications, see e.g. [10-12]. It is important to note that CPDEs have been linked to many applications in spectral graph theory; for more details on graphs and their applications, see, e.g. [13-14]. However, solving CPDEs analytically is notoriously challenging and often requires numerical methods. As a result, the study of CPDEs remains an active and vibrant area of research, characterized by numerous unsolved problems and difficulties. This article provides an overview of CPDEs, exploring their characteristics and presenting some techniques employed in their analysis [15-16]. The crucial role of complex numbers in physics lies in their ability to simplify the process of solving physical equations. Later in this paper, we will explore specific examples of solving differential equations that employ complex numbers. In quantum mechanics, complex numbers play a central role, as they represent the state, or wave function, of quantum systems. This application of complex numbers enables us to understand the behavior and predict the properties of quantum systems. Consequently, in recent years, researchers have dedicated significant attention to studying CPDEs using various methods [17-25]. This work presented a straightforward method of finding a general solution for the CPDEs [26]. We utilize two effective techniques, namely (VIM) and (HPM). VIM is an efficient method for approximating ordinary and partial differential equations that are essential in scientific disciplines, physical phenomena, engineering, mechanics, and more. Introduced by He [27], VIM is a method for solving nonlinear (linear) differential equations and integral equations. Numerous researchers have employed VIM, as demonstrated in works such as "Variational Approach for Differential Equations" [27-29], to estimate solutions for both types of equations. Another powerful technique for approximating solutions to ordinary and partial differential equations is the HPM method. Widely applicable in fields such as physics, engineering, and mechanics, HPM were developed in 1999 to solve also nonlinear (linear) differential equations, as well as integral equations. Authors have utilized HPM to approximate solutions for integral and differential equations. To evaluate the performance and precision of both methods, we will test them by solving examples and examining their ability to reach solutions.

## 2. Exploring the Variational Iteration Method: An Analysis

In the following section, we provide a comprehensive overview of VIM as discussed in the work by He [27]. The focus of our review will be on utilizing

VIM to address Linear CPDEs. Specifically, we will consider the following equation:

$$\frac{\partial \omega}{\partial t} = (\theta_1(x,y,t) + \kappa_1 i)\frac{\partial^2 \omega}{\partial x^2} + (\theta_2(x,y,t) + \kappa_2 i)\frac{\partial^2 \omega}{\partial y^2} + (\theta_3(x,y,t) + \kappa_3 i)\frac{\partial^2 \omega}{\partial x \partial y} + (\theta_4(x,y,t) + \kappa_4 i)\frac{\partial \omega}{\partial x} + (\theta_5(x,y,t) + \kappa_5 i)\frac{\partial \omega}{\partial y} + (\theta_6(x,y,t) + \kappa_6 i)\omega, \tag{1}$$

where $\kappa_1, \kappa_2, \kappa_3, \kappa_4, \kappa_5, \kappa_6$ are real constants, $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$ are complex, unknown functions, $t$ is a nonnegative real value, $x, y$ are a real value, and $i = \sqrt{-1}$ with initial conditions:

$$\omega(x,y,0) = \xi(x,y), \tag{2}$$

An iterative approach is employed to define the corrective function for Eq. (1).

$$\omega_{\mu+1} = \omega_\mu + \int_0^t \lambda(\frac{\partial \omega_\mu}{\partial t} - (\theta_1 + \kappa_1 i)\frac{\partial^2 \omega_\mu}{\partial x^2} - (\theta_2 + \kappa_2 i)\frac{\partial^2 \omega_\mu}{\partial y^2} - (\theta_3 + \kappa_3 i)\frac{\partial^2 \omega_\mu}{\partial x \partial y} - (\theta_4 + \kappa_4 i)\frac{\partial \omega_\mu}{\partial x} - (\theta_5 + \kappa_5 i)\frac{\partial \omega_\mu}{\partial y} - (\theta_6 + \kappa_6 i)\omega_\mu)dt. \tag{3}$$

A large multiplier may be identified without difficulty, $\lambda = -1$,

A followed iterations formula is obtained as a consequence.

$$\omega_{\mu+1} = \omega_\mu - \int_0^t (\frac{\partial \omega_\mu}{\partial t} - (\theta_1 + \kappa_1 i)\frac{\partial^2 \omega_\mu}{\partial x^2} - (\theta_2 + \kappa_2 i)\frac{\partial^2 \omega_\mu}{\partial y^2} - (\theta_3 + \kappa_3 i)\frac{\partial^2 \omega_\mu}{\partial x \partial y} - (\theta_4 + \kappa_4 i)\frac{\partial \omega_\mu}{\partial x} - (\theta_5 + \kappa_5 i)\frac{\partial \omega_\mu}{\partial y} - (\theta_6 + \kappa_6 i)\omega_\mu)dt. \tag{4}$$

The iterations in the proposed modified VIM proceed by utilizing a simple initial function, denoted as $\omega_0$, until convergence is achieved at a fixed point. At this stage, updated function $\omega_{\mu+1}$ can be regarded as an approximate solution to Equation (1). The variational iteration method, known as modified VIM, has demonstrated its effectiveness, simplicity, and high accuracy in solving a wide range of linear problems. Approximations obtained through this method exhibit rapid convergence towards accurate solutions. To implement the modified VIM for solving CPDEs represented by Eq. (4), we follow the algorithm outlined below:

**Algorithm 2.1:** Let $\mu$ be the iteration index.

- Step 1: Set an appropriate value for the tolerance $\varepsilon$ for numerical purposes.
- Step 2: Choose an appropriate $\omega_0$, and set $\mu = 0$.
- Step 3: Use the calculated values of $\omega_\mu$ to compute $\omega_{\mu+1}$ from equation (4).
- Step 4: If Max $|\omega_\mu - \omega_{\mu+1}| < \varepsilon$ stop, otherwise continue.
- Step 5: Set $\mu = \mu + 1$, and define $\omega_\mu = \omega_{\mu+1}$ and return to Step 3.

**3. Exploring the Homotopy Perturbation Methods: An Analysis**

We provide a comprehensive review of the HPM, as discussed in Anjum and He [29]. The focus of our paper is on utilizing HPM to solve CPDEs. To begin, we initialize the approximation process by selecting an initial function $\omega_0(x,y,t)$ as $\xi(x,y)$. Next, we can construct HPM for solving CPDE (1) using the following procedure:

$$H(\omega(x,y,t),p) = (1-p)(\frac{\partial\omega}{\partial t} - \omega_0) + p(\frac{\partial\omega}{\partial t} - (\theta_1+\kappa_1 i)\frac{\partial^2\omega}{\partial x^2} - (\theta_2 + \kappa_2 i)\frac{\partial^2\omega}{\partial y^2} - (\theta_3+\kappa_3 i)\frac{\partial^2\omega}{\partial x\partial y} - (\theta_4+\kappa_4 i)\frac{\partial\omega}{\partial x} - (\theta_5+\kappa_5 i)\frac{\partial\omega}{\partial y} - (\theta_6+\kappa_6 i)\omega) = 0. \quad (5)$$

Thirdly, we make use of Taylor expansion by replacing the coefficient functions with their respective Taylor expansions. This allows us to express the solution $\omega(x,y,t)$ as a series expansion. However,

$$H(\omega(x,y,t),p) = (1-p)(\sum_{\mu=0}^{\infty} p^\mu \frac{\partial\omega_\mu}{\partial t} - \omega_0) + p(\sum_{\mu=0}^{\infty} p^\mu \frac{\partial\omega_\mu}{\partial t} - (\theta_1 + \kappa_1 i)\sum_{\mu=0}^{\infty} p^\mu \frac{\partial^2\omega_\mu}{\partial x^2} - (\theta_2+\kappa_2 i)\sum_{\mu=0}^{\infty} p^\mu \frac{\partial^2\omega_\mu}{\partial y^2} - (\theta_3\kappa_3 i)\sum_{\mu=0}^{\infty} p^\mu \frac{\partial^2\omega_\mu}{\partial x\partial y} - (\theta_4 + \kappa_4 i)\sum_{\mu=0}^{\infty} p^\mu \frac{\partial\omega_\mu}{\partial x} - (\theta_5+\kappa_5 i)\sum_{\mu=0}^{\infty} p^\mu \frac{\partial\omega_\mu}{\partial y} - (\theta_6+\kappa_6 i)\sum_{\mu=0}^{\infty} p^\mu \omega_\mu) = 0. \quad (6)$$

Fourthly, the form of the solution to Equation (6) is assumed to be

$$\omega(x,y,t) = \omega_0 + p\omega_1 + p^2\omega_2(z) + p^3\omega_3(z) + \cdots \quad (7)$$

Fifthly, the following equations demonstrate the process of grouping similar powers of $p$ together:

$$p^0: \frac{\partial\omega_0}{\partial t} = \omega_0, \quad (8)$$

$$p^1: \frac{\partial\omega_1}{\partial t} = (\theta_1+\kappa_1 i)\frac{\partial^2\omega_0}{\partial x^2} + (\theta_2+\kappa_2 i)\frac{\partial^2\omega_0}{\partial y^2} + (\theta_3+\kappa_3 i)\frac{\partial^2\omega_0}{\partial x\partial y} + (\theta_4+\kappa_4 i)\frac{\partial\omega_0}{\partial x} + (\theta_5+\kappa_5 i)\frac{\partial\omega_0}{\partial y} + (\theta_6+\kappa_6 i)\omega_0, \quad (9)$$

$$p^2: \frac{\partial\omega_2}{\partial t} = (\theta_1+\kappa_1 i)\frac{\partial^2\omega_1}{\partial x^2} + (\theta_2+\kappa_2 i)\frac{\partial^2\omega_1}{\partial y^2} + (\theta_3+\kappa_3 i)\frac{\partial^2\omega_1}{\partial x\partial y} + (\theta_4+\kappa_4 i)\frac{\partial\omega_1}{\partial x} + (\theta_5+\kappa_5 i)\frac{\partial\omega_1}{\partial y} + (\theta_6+\kappa_6 i)\omega_1, \quad (10)$$

$$p^3: \frac{\partial\omega_3}{\partial t} = (\theta_1+\kappa_1 i)\frac{\partial^2\omega_2}{\partial x^2} + (\theta_2+\kappa_2 i)\frac{\partial^2\omega_2}{\partial y^2} + (\theta_3+\kappa_3 i)\frac{\partial^2\omega_2}{\partial x\partial y} + (\theta_4+\kappa_4 i)\frac{\partial\omega_2}{\partial x} + (\theta_5+\kappa_5 i)\frac{\partial\omega_2}{\partial y} + (\theta_6+\kappa_6 i)\omega_2, \quad (11)$$

...

Hence, for $n = 1,2,3,\ldots$ We have,

$$p^n: \frac{\partial\omega_n}{\partial t} = (\theta_1+\kappa_1 i)\frac{\partial^2\omega_{n-1}}{\partial x^2} + (\theta_2+\kappa_2 i)\frac{\partial^2\omega_{n-1}}{\partial y^2} + (\theta_3+\kappa_3 i)\frac{\partial^2\omega_{n-1}}{\partial x\partial y} + (\theta_4+\kappa_4 i)\frac{\partial\omega_{n-1}}{\partial x} + (\theta_5+\kappa_5 i)\frac{\partial\omega_{n-1}}{\partial y} + (\theta_6+\kappa_6 i)\omega_{n-1}, \quad (12)$$

Finally, we integral the Equations (8)-(12) with some simplifications, so a solutions for the Eq.(12) be

$$\omega(x,y,t) = \omega_0(x,y,t) + \omega_1(x,y,t) + \omega_2(x,y,t) + \omega_3(x,y,t) + \cdots \tag{13}$$

## 4. Implementations

To assess the accuracy of solving linear CPDEs using VIM and HPM, we present various examples that compare the approximate solutions obtained by these methods with their corresponding exact solutions. These examples demonstrate the effectiveness and efficiency of both VIM and HPM in producing accurate approximations. To support our analysis, we use the MATLAB package, which facilitates implementing the solution methods and visualizing the results. Through the MATLAB programs and accompanying figures, we investigate the following aspects:

**Example 4.1:** Let's consider the given CPDE:

$$\frac{\partial\omega}{\partial t} = i\frac{\partial^2\omega}{\partial x^2} + \frac{\partial\omega}{\partial x} + i\omega, \tag{14}$$

subject to the initial condition $\omega(x,0) = \exp(ix)$.

By the VIM, we get that the initial approximation has the form $\omega_0 = \exp(ix)$ we obtain the following successive approximations:

$$\omega_1 = \exp(ix)(1 + it),$$

$$\omega_2 = \exp(ix)(1 + it - \frac{t^2}{2}),$$

$$\omega_3 = \exp(ix)(1 + it - \frac{t^2}{2} - \frac{it^3}{6}),$$

$$\omega_4 = \exp(ix)(1 + it - \frac{t^2}{2} - \frac{it^3}{6} + \frac{t^4}{24}),$$

Then, when $\mu \to \infty$, The solution of Equation (14) is written as follows:

$$\omega_\mu = \exp(ix)(1 + it - \frac{t^2}{2} - \frac{it^3}{6} + \frac{t^4}{24} + \cdots + \frac{i^n t^n}{n!} + \cdots) = \exp(ix + it). \tag{15}$$

That is an exact solution for this example (see Figure 1). By the HPM, using Equations (8)-(12), we get

$$\omega_1 = it\exp(ix),$$

$$\omega_2 = -\frac{t^2}{2}\exp(ix),$$

$$\omega_3 = -\frac{it^3}{6}\exp(ix),$$

$$\omega_4 = \frac{t^4}{24}\exp(ix),$$

Then, and so on, the equation (14) can be solved, and the solution is written as shown in Figure 1:

$$\omega = \omega_0 + \omega_1 + \omega_2 + \cdots = \exp(ix + it). \tag{16}$$

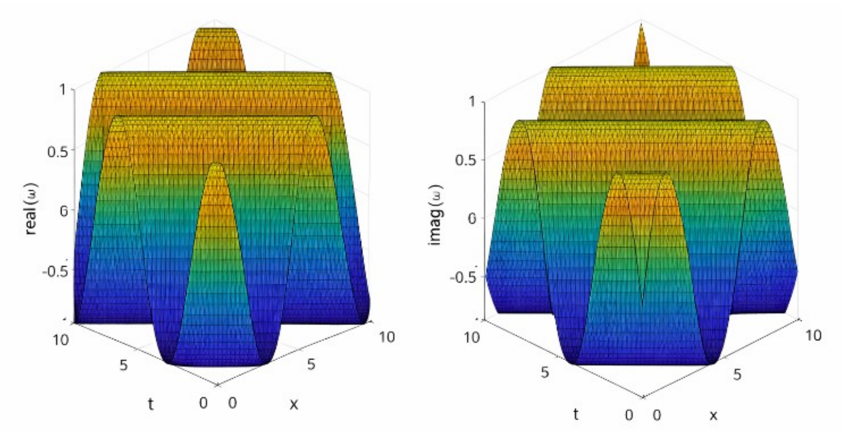


**Figure 1**
**Real and imaginary parts of the approximation solution of Example 4.1**

**Example 4.2:** Let's consider the given CPDE:

$$\frac{\partial\omega}{\partial t} = 0.5i\frac{\partial^2\omega}{\partial x^2} - 0.5i\frac{\partial\omega^2}{\partial y^2} + \frac{\partial\omega}{\partial x}, \tag{17}$$

constrained by initial conditions $\omega(x, y, 0) = \exp(ix)\cos(y)$.

By the VIM, we get the initial approximation in the form $\omega_0 = \exp(ix)\cos(y)$, applying algorithm7, we get:

$$\omega_1 = \exp(ix)\cos(y)(1 + it),$$

$$\omega_2 = \exp(ix)\cos(y)(1 + it - \frac{t^2}{2}),$$

$$\omega_3 = \exp(ix)\cos(y)(1 + it - \frac{t^2}{2} - \frac{it^3}{6}),$$

Then, when $\mu \to \infty$Equation (17) has its solution written as follows:

$$\omega_\mu = \exp(ix + it)\cos(y). \tag{18}$$

By the HPM, using Equations (8)-(12), we get

$$\omega_1 = it\exp(ix)\cos(y),$$

$$\omega_2 = -\frac{t^2}{2}\exp(ix)\cos(y),$$

$$\omega_3 = -\frac{it^3}{6}\exp(ix)\cos(y),$$

$$\dots$$

Then, the solution to Equation (17) is expressed as shown in Figure 2:

$$\omega = \omega_0 + \omega_1 + \omega_2 + \omega_3 + \cdots = \exp(ix + it)\cos(y). \tag{19}$$

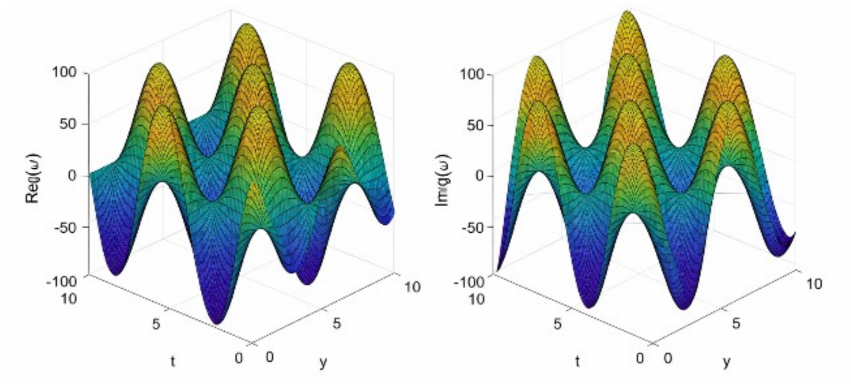


**Figure 2**

**Real and imaginary parts of the approximation solution of Example 4.2 at $x = 1$.**

**Example 4.3:** Let's consider the given CPDE:

$$\frac{\partial \omega}{\partial t} = \frac{ty^2}{y^2-x^2}\frac{\partial^2 \omega}{\partial x^2} - \frac{tx^2}{y^2-x^2}\frac{\partial \omega^2}{\partial y^2}, \tag{20}$$

accompanied by the initial condition $\omega(x, y, 0) = \exp(ix^2 + iy^2)$.

By the VIM, applying algorithm 7, we get:

$$\omega_0 = \exp(ix^2 + iy^2),$$

$$\omega_1 = \exp(ix^2 + iy^2)(1 + it^2),$$

$$\omega_2 = \exp(ix^2 + iy^2)(1 + it^2 - \frac{t^4}{2}),$$

$$\omega_3 = \exp(ix^2 + iy^2)(1 + it^2 - \frac{t^4}{2} - \frac{it^6}{6}),$$

$$\omega_4 = \exp(ix^2 + iy^2)(1 + it^2 - \frac{t^4}{2} - \frac{it^6}{6} + \frac{it^8}{24}),$$

Then, when $\mu \to \infty$, The solution to Equation (20) can be expressed as follows:

$$\omega_\mu = \exp(ix^2 + iy^2 + it^2). \tag{21}$$

These are exact solutions for these examples (see Figure 3). By the HPM, using Equation (8)-(12), we get

$$\omega_1 = it^2 \exp(ix^2 + iy^2),$$

$$\omega_2 = -\frac{t^4}{2}\exp(ix^2 + iy^2),$$

$$\omega_3 = -\frac{it^6}{6}\exp(ix^2 + iy^2),$$

$$\omega_4 = \frac{it^8}{24}\exp(ix^2 + iy^2).$$

Then, the solution to Equation (20) can be represented as shown in Figure 3:

$$\omega_\mu = \omega_0 + \omega_1 + \omega_2 + \omega_3 + \cdots = \exp(ix^2 + iy^2 + it^2). \tag{22}$$

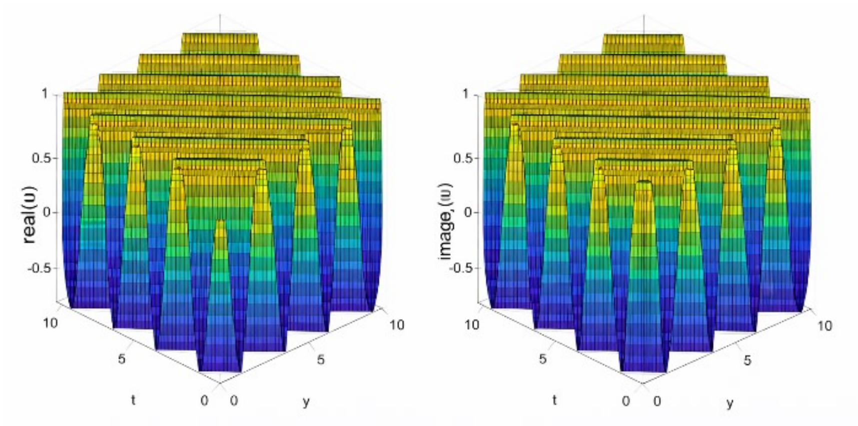


**Figure 3**
**Real and imaginary parts of the approximation solution of Example 4.3 at $x = 1$.**

**Example 4.4:** Let's consider the given CPDE:

$$\frac{\partial \omega}{\partial t} = x\frac{\partial^2 \omega}{\partial x^2} - \frac{\partial \omega}{\partial x} + x + t, \tag{23}$$

accompanied by the initial condition $\omega(x, y, 0) = yx^2 i$. By the VIM, applying algorithm 7, we get:

$$\omega_0 = yx^2 i,$$
$$\omega_1 = \frac{t^2}{2} + yx^2 i + xt,$$
$$\omega_2 = yx^2 i + xt,$$
$$\omega_3 = yx^2 i + xt,$$

Then, $\omega_{\mu+1} = \omega_\mu$ for all $\mu \geq 2$, the solution for Eq. (23) is written:

$$\omega_\mu = x(xyi + t), \tag{24}$$

that an exact solution for this example (see Figure 4). By the HPM, using Equation (8)-(12), we get

$$\omega_0 = yx^2 i,$$
$$\omega_1 = \frac{t^2}{2} + xt,$$
$$\omega_2 = -\frac{t^2}{2},$$
$$\omega_3 = 0,$$
$$\omega_4 = 0.$$
$$\dots$$

Then, Equation (23) can be solved to obtain the following solution

$$\omega_\mu = \omega_0 + \omega_1 + \omega_2 = x(xyi + t). \tag{25}$$

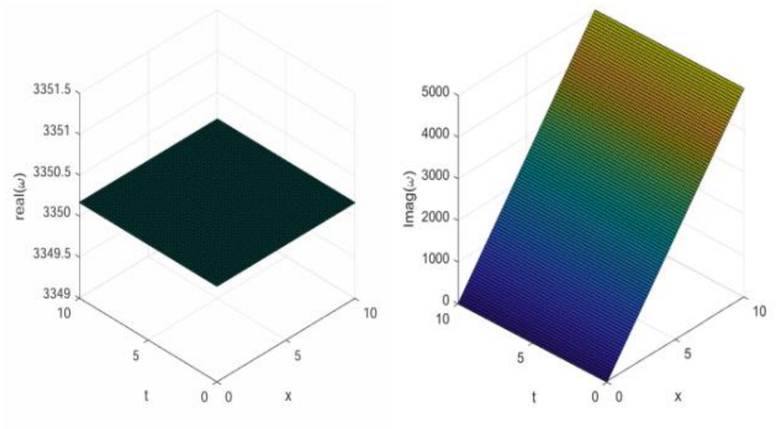


**Figure 4**
**Real and imaginary parts of the approximation solution of Example 4.4 at $y = 1$.**

## 5. Discussion and Conclusion

This study delved into the realm of second-order linear complex partial differential equations, which play a pivotal role as mathematical models in various physically significant fields and applied sciences. We employed the modified VIM and HPM to solve these Second Order Linear Complex Partial Differential Equations. Through extensive testing on diverse problem implementations, we showcased the efficiency and accuracy of both methods. The results of our analysis revealed that the proposed methods offer reliable and accurate approximations, as evidenced by the close agreement between the obtained solutions and the corresponding analytical solutions for the tested problems. This underscores the efficacy of modified VIM and HPM in tackling non-linear complex partial differential equations. The outcomes of this research contributed to a broader understanding of solving second-order linear complex partial differential equations, offering valuable insights for researchers and practitioners in the field. By leveraging VIM and HPM, scientists can address complex mathematical models in a wide range of applied sciences with enhanced efficiency and accuracy. Future studies may explore further refinements and applications of these methods, as well as their integration with other numerical techniques to tackle more complex problems in various scientific domains.

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