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Using ZMA-transform for solving the linear delay differential equations

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Abstract

In this operation, construct ZMA-transform (ZMAT) for solving initial-value problem (IVP) of linear first and second (1st and 2nd) order delay differential equation (DDEs), to realize this method produce provided theorem used it to solve applications to acquire the exact solution (EX).

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Keywords: *ZMA-transform, Initial–value problem, Exact solution, Delay differential equation, Integral transform.*

1. Introduction

Many fields exhibit in the past the (DDEs) in science, engineering, biology, physics, and other scientific [1-5]. The (DDEs) have interest applications in [6-8]. Moreover many researchers informed approximate solutions of (DDEs) submitted in [9-15]. The integral transform plays major roles for solving (DDEs) combination with numerical methods like Adomian Decomposition iterative technique, Homotopy perturbation method and other technique to find the analytic solution [16-19], [20-21]. The main result in this survey presented (ZMAT) has been used of provide theorem to acquire the (EX) of linear (1stand2nd) order (DDEs) and to comprehend these techniques applied the (IVP) of (DDEs) through construct two applications.

2. Preliminaries

The main study in this work solves the linear 2nd order (DDE) in formed as:

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$$\mathcal{Q}''(\mathfrak{t}) + \mathcal{o}\,\mathcal{Q}'(\mathfrak{t}) + \mathcal{p}\mathcal{Q}'(\mathfrak{t}-\mathcal{r}) + \mathcal{q}\mathcal{Q}(\mathfrak{t}) + \ell\mathcal{Q}(\mathfrak{t}-\mathcal{r}) = \mathcal{g}(\mathfrak{t}) \quad \ldots(1)$$

$$\mathcal{Q}(\mathfrak{t}) = \mathcal{K}(\mathfrak{t}), -\mathcal{r} \le \mathfrak{t} \le 0;\ \mathcal{Q}'(0) = \mu \quad \ldots(2)$$

Where $\mathcal{o}, \mathcal{p}, \mathcal{q}, \ell$: are real constant.,$\mathcal{g}(\mathfrak{t}), \mathcal{K}(\mathfrak{t})$: are known the estimate in $\Re$ and a sufficiently smoothly assignment.,μ: There is in $\Re$.,$\mathcal{r}$: The a positive constant large delay.

Definition 1: [19] Let for all $\mathfrak{t}$ realized the function $\mathcal{Q}(\mathfrak{t})$ to prepare (ZMAT):

$$\mathbb{Z}_{\mathrm{MA}}(\mathcal{v}, \mathcal{s}) = \Lambda\{\mathcal{Q}(\mathfrak{t})\} = \frac{1}{\mathcal{s}} \int_0^{\infty} \mathcal{Q}(\mathcal{v}\,\mathfrak{t})\ \mathrm{e}^{-\frac{\mathfrak{t}}{\mathcal{s}}}\,\mathrm{d}\mathfrak{t} \quad \ldots(3)$$

Where the inverse (ZMAT) informed as:

$$\mathcal{Q}(\mathfrak{t}) = \Lambda^{-1}\{\mathbb{Z}_{\mathrm{MA}}(\mathcal{v}, \mathcal{s})\} \quad \ldots(4)$$

Remark 1: In [19], the function $\mathcal{Q}(\mathfrak{t})$ has the following properties as:

i- For every finite interval $0 < \mathfrak{t} < \mathfrak{t}_1, \mathfrak{t}_1 > 0$ then $\mathcal{Q}(t)$ is piecewise continuous.

ii- $\mathcal{Q}(\mathfrak{t})$ is the exponential order, then $\exists \lambda, \hbar, \mathfrak{t}_1 > 0\ , \mathrm{e}^{-\frac{\mathfrak{t}}{\lambda}} < \hbar\ , \mathfrak{t} > \mathfrak{t}_0$.

3. Basic Function of (ZMAT) [19]

In this part presented the basic function as shown in Table 1:

Table 1

Basic Function

S.No.	$\boldsymbol{\varpi(t)}$	$\boldsymbol{\Lambda\{\varpi(t)\}}$
1	1	1
2	T	sv
3	e^{at}	$\frac{1}{1-s(av)}$
4	$cos\, a\, t$	$\frac{1}{1+s^2(a^2v^2)}$
5	$sin\, a\, t$	$\frac{sav}{1+s^2(a^2v^2)}$
6	t^n	$n!\ s^n v^n$
7	t^n	$\Gamma(n+1)s^n v^n$
8	$e^{at}\, sin\, b\, t$	$\frac{b\frac{1}{sv}}{(\frac{1}{sv}-a)^2+b^2}$
9	$e^{at}\, cos\, b\, t$	$\frac{(\frac{1}{sv}-a)}{(\frac{1}{sv}-a)^2+b^2}$
10	$t\, cos \quad at$	$\frac{\frac{1}{sv}(\frac{1}{s^2v^2}-a^2)}{(\frac{1}{s^2v^2}+a^2)^2}$
11	$t\, sin\, a\, t$	$\frac{2a\frac{1}{s^2v^2}}{(\frac{1}{s^2v^2}+a^2)^2}$

4. Derivative of (ZMAT) [19]

Take $\mathcal{Q}(\mathfrak{t})$ be the function offered as:

$$\Lambda\{\mathcal{Q}'(\mathfrak{t})\} = \frac{1}{\mathcal{s}\,\mathcal{v}}\,\mathbb{Z}_{\mathrm{MA}}(\mathcal{v},\mathcal{s}) - \frac{1}{\mathcal{s}\,\mathcal{v}}\,\mathcal{Q}(0) \quad \ldots(5)$$

$$\Lambda\{\mathcal{Q}''(\mathfrak{t})\} = \frac{1}{\mathcal{s}^2\,\mathcal{v}^2}\,\mathbb{Z}_{\mathrm{MA}}(\mathcal{v},\mathcal{s}) - \frac{1}{\mathcal{s}^2\,\mathcal{v}^2}\,\mathcal{Q}(0) - \frac{1}{\mathcal{s}\,\mathcal{v}}\,\mathcal{Q}'(0) \quad \ldots(6)$$

$$\Lambda\{\mathcal{Q}^n(\mathfrak{t})\} = \frac{1}{\mathcal{s}^2\,\mathcal{v}^2}\,\mathbb{Z}_{\mathrm{MA}}(\mathcal{v},\mathcal{s}) - \sum_{\mathcal{k}=0}^{n-1}\frac{1}{\mathcal{s}^{n-\mathcal{k}}\,\mathcal{v}^{n-\mathcal{k}}}\,\mathcal{Q}^{(n)}(0) \quad \ldots(7)$$

Theorem 1: *Take the continuous functions on [-$\mathcal{r}$, 0] is $\mathcal{K}(\mathfrak{t})$, $\mathcal{K}'(\mathfrak{t})$ and the (ZMAT) is $\mathbb{Z}_{MA}(\mathcal{v},\mathcal{s})$ for $\mathcal{g}(\mathfrak{t})$,then the general solution of (1)-(2).*

$$\mathcal{Q}(\mathfrak{t}) = \Lambda^{-1}\left[\frac{\mathcal{g}(\mathcal{v},\mathcal{s})+\Im(\mathcal{v},\mathcal{s})}{\mho(\mathcal{v},\mathcal{s})}\right] \quad \ldots(8)$$

$$\Im(\mathcal{v},\mathcal{s}) = \frac{1}{\mathcal{s}\,\mathcal{v}}\,\mu + \left[\frac{1}{\mathcal{s}^2\,\mathcal{v}^2} + \frac{\mathcal{o}}{\mathcal{s}\,\mathcal{v}} + \frac{\mathcal{p}}{\mathcal{s}\,\mathcal{v}}\,\mathrm{e}^{-\frac{\mathcal{r}}{\mathcal{s}\mathcal{v}}}\right] \quad \ldots(9)$$

$$\mho(\mathcal{v},\mathcal{s}) = \frac{1}{\mathcal{s}^2\,\mathcal{v}^2} + \frac{\mathcal{o}}{\mathcal{s}\,\mathcal{v}} + \left(\frac{\mathcal{p}}{\mathcal{s}\,\mathcal{v}} + \mathcal{\ell}\right)\mathrm{e}^{-\frac{\mathcal{r}}{\mathcal{s}\mathcal{v}}} + \mathcal{q} \quad \ldots(10)$$

$$\overline{\mathcal{K}(\mathcal{v},\mathcal{s})} = \frac{1}{\mathcal{s}\,\mathcal{v}}\int_{-\mathcal{r}}^{0}\mathrm{e}^{-\frac{(t+\mathcal{r})}{\mathcal{s}\mathcal{v}}}\,\mathcal{K}(\mathfrak{t})\,\mathrm{d}\mathfrak{t} \quad \ldots(11)$$

$$\overline{\overline{\mathcal{K}(\mathcal{v},\mathcal{s})}} = \frac{1}{\mathcal{s}\,\mathcal{v}}\int_{-\mathcal{r}}^{0}\mathrm{e}^{-\frac{(t+\mathcal{r})}{\mathcal{s}\mathcal{v}}}\,\mathcal{K}'(\mathfrak{t})\,\mathrm{d}\mathfrak{t} \quad \ldots(12)$$

Proof: By (ZMAT) used to obtained solution for (1)-(2), must to be offered derivatives function by (ZMAT) presented in (5)-(6) then:

$$\Lambda\{\mathcal{Q}'(\mathfrak{t})\} = \frac{1}{\mathcal{s}\,\mathcal{v}}\,\mathbb{Z}_{\mathrm{MA}}(\mathcal{v},\mathcal{s}) - \frac{1}{\mathcal{s}\,\mathcal{v}}\,\mathcal{Q}(0) \quad \ldots(13)$$

$$\Lambda\{\mathcal{Q}''(\mathfrak{t})\} = \frac{1}{\mathcal{s}^2\,\mathcal{v}^2}\,\mathbb{Z}_{\mathrm{MA}}(\mathcal{v},\mathcal{s}) - \frac{1}{\mathcal{s}^2\,\mathcal{v}^2}\,\mathcal{Q}(0) - \frac{1}{\mathcal{s}\,\mathcal{v}}\,\mu \quad \ldots(14)$$

Take (ZMAT) of $\mathcal{Q}(\mathfrak{t}-\mathcal{r})$ and let $\mathfrak{t} = \mathcal{X} + \mathcal{r}$ find:

$$\Lambda\{\mathcal{Q}(\mathfrak{t}-\mathcal{r})\} = \frac{1}{\mathcal{s}\,\mathcal{v}}\int_{0}^{\infty}\mathcal{Q}(\mathfrak{t}-\mathcal{r})\;\mathrm{e}^{-\frac{\mathfrak{t}}{\mathcal{s}\mathcal{v}}}\,\mathrm{d}\mathfrak{t} \quad \ldots(15)$$

$$= \frac{1}{\mathcal{s}\,\mathcal{v}}\int_{-\mathcal{r}}^{0}\mathrm{e}^{-\frac{(\mathcal{X}+\mathcal{r})}{\mathcal{s}\mathcal{v}}}\,\mathcal{K}(\mathcal{X})\,\mathrm{d}\mathcal{X} + \frac{\mathrm{e}^{-\frac{\mathcal{r}}{\mathcal{s}\mathcal{v}}}}{\mathcal{s}\,\mathcal{v}}\int_{0}^{\infty}\mathrm{e}^{-\frac{\mathcal{X}}{\mathcal{s}\mathcal{v}}}\,\mathcal{Q}(\mathcal{X})\,\mathrm{d}\mathcal{X} \quad \ldots(16)$$

$$\rightarrow \Lambda\{\mathcal{Q}(\mathfrak{t}-\mathcal{r})\} = \overline{\mathcal{K}(\mathcal{v},\mathcal{s})} + \mathrm{e}^{-\frac{\mathcal{r}}{\mathcal{s}\mathcal{v}}}\Lambda\{\mathcal{Q}(\mathfrak{t})\} \quad \ldots(17)$$

By the same way Take (ZMAT) of $\mathcal{Q}'(\mathfrak{t}-\mathcal{r})$ get:

$$\Lambda\{\mathcal{Q}'(\mathfrak{t}-\mathcal{r})\} = \frac{1}{\mathcal{s}\,\mathcal{v}}\int_{0}^{\infty}\mathcal{Q}'(\mathfrak{t}-\mathcal{r})\;\mathrm{e}^{-\frac{\mathfrak{t}}{\mathcal{s}\mathcal{v}}}\,\mathrm{d}\mathfrak{t} \quad \ldots(18)$$

$$= \frac{1}{\mathcal{s}\,\mathcal{v}}\int_{-\mathcal{r}}^{0}\mathrm{e}^{-\frac{(\mathcal{X}+\mathcal{r})}{\mathcal{s}\mathcal{v}}}\,\mathcal{K}'(\mathcal{X})\,\mathrm{d}\mathcal{X} + \frac{\mathrm{e}^{-\frac{\mathcal{r}}{\mathcal{s}\mathcal{v}}}}{\mathcal{s}\,\mathcal{v}}\int_{0}^{\infty}\mathrm{e}^{-\frac{\mathcal{X}}{\mathcal{s}\mathcal{v}}}\,\mathcal{Q}'(\mathcal{X})\,\mathrm{d}\mathcal{X} \quad \ldots(19)$$

$$\rightarrow \Lambda\{\mathcal{Q}'(\mathfrak{t}-\mathcal{r})\} = \overline{\overline{\mathcal{K}(\mathcal{v},\mathcal{s})}} + \mathrm{e}^{-\frac{\mathcal{r}}{\mathcal{s}\mathcal{v}}}\Lambda\{\mathcal{Q}'(\mathfrak{t})\} \quad \ldots(20)$$

Know, find (ZMAT) for (1) produce:

$$\Lambda\{\mathcal{Q}''(\mathfrak{t})\} + \mathcal{o}\Lambda\{\mathcal{Q}'(\mathfrak{t})\} + \mathcal{p}\Lambda\{\mathcal{Q}'(\mathfrak{t}-\mathcal{r})\} + \mathcal{q}\Lambda\{\mathcal{Q}(\mathfrak{t})\} + \mathcal{\ell}\Lambda\{\mathcal{Q}(\mathfrak{t}-\mathcal{r})\} = \Lambda\{\mathcal{g}(\mathfrak{t})\} \quad \ldots(21)$$

After some steps and applied inverse of (ZMAT) consequence (8).

Corollary 1: *See off the following:*

$$\mathcal{Q}'(\mathfrak{t}) = \mathcal{g}\big(\mathfrak{t}, \mathcal{Q}(\mathfrak{t}), \mathcal{Q}(\mathfrak{t}-\mathcal{r})\big) = \mathfrak{Z}\,\mathcal{Q}(\mathfrak{t}-\mathcal{r}), \mathfrak{t} \geq 0 \quad \ldots(22)$$

$$\mathcal{Q}(\mathfrak{t}) = \mathcal{K}(\mathfrak{t}), -\mathcal{r} \leq \mathfrak{t} \leq 0 \quad \ldots(23)$$

When applying (ZMAT) to locate the general solution of (22)-(23) as:

$$\mathcal{Q}(\mathfrak{t}) = \Lambda^{-1}\left[\frac{\mathfrak{J}(\mathcal{v},\mathcal{s})}{\mathfrak{U}(\mathcal{v},\mathcal{s})}\right] \quad \ldots(24)$$

Where

$$\mathfrak{J}(\mathcal{v},\mathcal{s}) = \frac{1}{\mathcal{s}\,\mathcal{v}}\,\mu + \mathfrak{Z}\,\overline{\mathcal{K}(\mathcal{v},\mathcal{s})} \quad \ldots(25)$$

$$\mathfrak{U}(\mathcal{v},\mathcal{s}) = \frac{1}{\mathcal{s}\,\mathcal{v}} - \mathfrak{Z}\,\mathrm{e}^{-\frac{\mathcal{r}}{\mathcal{s}\mathcal{v}}} \quad \ldots(26)$$

$$\overline{\mathcal{K}(\mathcal{v},\mathcal{s})} = \frac{1}{\mathcal{s}\,\mathcal{v}}\int_{-\mathcal{r}}^{0} \mathrm{e}^{-\frac{(\mathfrak{t}+\mathcal{r})}{\mathcal{s}\mathcal{v}}}\,\mathcal{K}(\mathfrak{t})\,\mathrm{d}\mathfrak{t} \quad \ldots(27)$$

Proof: Applying (ZMAT) of (22) as:

$$\Lambda\{\mathcal{Q}'(\mathfrak{t})\} = \mathfrak{Z}\,\Lambda\{\mathcal{Q}(\mathfrak{t}-\mathcal{r})\} \quad \ldots(28)$$

By using (13) for the left side and put (15) for another side when $\mathfrak{t} = \mathcal{X} + \mathcal{r}$. Then obtained the (EX) of (24).

5. Applications

1. *Consider the 2nd order linear (DDE):*

$$\mathcal{Q}''(\mathfrak{t}) - 3\,\mathcal{Q}'(\mathfrak{t}) + \mathcal{Q}'(\mathfrak{t}-\mathcal{r}) + 2\mathcal{Q}(\mathfrak{t}) - \mathcal{Q}(\mathfrak{t}-\mathcal{r}) = 0, \mathfrak{t} > 0 \quad \ldots(29)$$

$$\mathcal{Q}(\mathfrak{t}) = \mathrm{e}^{\mathfrak{t}}, -1 \leq \mathfrak{t} \leq 0;\ \mathcal{Q}'(0) = 1 \quad \ldots(30)$$

Sol: Applying (Theorem (4.1)) for (29)-(30) find:

$$\mathcal{g}(\mathcal{v},\mathcal{s}) = 0 \quad \ldots(31)$$

$$\mathfrak{J}(\mathcal{v},\mathcal{s}) = \frac{1}{\mathrm{sv}} + \left[\frac{1}{\mathcal{s}^2\,\mathcal{v}^2} - \frac{3}{\mathcal{s}\,\mathcal{v}} + \frac{\mathrm{e}^{-\frac{1}{\mathcal{s}\mathcal{v}}}}{\mathcal{s}\,\mathcal{v}}\right] \quad \ldots(32)$$

$$\mathfrak{U}(\mathcal{v},\mathcal{s}) = \frac{1}{\mathrm{s}^2\,\mathrm{v}^2} - \frac{3}{\mathcal{s}\,\mathcal{v}} + \left(\frac{1}{\mathrm{sv}} - 1\right)\mathrm{e}^{-\frac{1}{\mathcal{s}\mathcal{v}}} + 2 \quad \ldots(33)$$

$$\rightarrow \mathcal{Q}(\mathfrak{t}) = \Lambda^{-1}\left[\frac{\mathcal{g}(\mathcal{v},\mathcal{s}) + \mathfrak{J}(\mathcal{v},\mathcal{s})}{\mathfrak{U}(\mathcal{v},\mathcal{s})}\right] = \Lambda^{-1}\left[\frac{(1-2\mathcal{s}\,\mathcal{v}+\mathrm{sve}^{-\frac{1}{\mathcal{s}\mathcal{v}}})}{(1-2\mathrm{sv}+\mathcal{s}\,\mathcal{v}\,\mathrm{e}^{-\frac{1}{\mathcal{s}\mathcal{v}}})(1-\mathcal{s}\,\mathcal{v})}\right] = \mathrm{e}^{\mathfrak{t}} \quad \ldots(34)$$

2. *Consider the 1st order linear (DDE):*

$$\mathcal{Q}'(\mathfrak{t}) = -\mathcal{Q}(\mathfrak{t}-1),\ \mathfrak{t} \geq 0 \quad \ldots(35)$$

$$\mathcal{Q}(\mathfrak{t}) = 1, -1 \leq \mathfrak{t} \leq 0 \quad \ldots(36)$$

Sol: By (Corollary (1)) (36)-(37) get:

$$\mathfrak{J}(\mathcal{v},\mathcal{s}) = \frac{1}{\mathcal{s}\,\mathcal{v}} + \mathrm{e}^{-\frac{1}{\mathcal{s}\mathcal{v}}} - 1 \quad \ldots(37)$$

$$\mathfrak{U}(\mathcal{v},\mathcal{s}) = \frac{1}{\mathcal{s}\,\mathcal{v}} + \mathrm{e}^{-\frac{1}{\mathcal{s}\mathcal{v}}} \quad \ldots(38)$$

$$\mathcal{Q}(\mathfrak{t}) = \Lambda^{-1}\left[\frac{\frac{1}{\mathcal{s}\,\mathcal{v}}+\mathrm{e}^{-\frac{1}{\mathcal{s}\mathcal{v}}}-1}{\frac{1}{\mathcal{s}\,\mathcal{v}}+\mathrm{e}^{-\frac{1}{\mathcal{s}\mathcal{v}}}}\right] = \Lambda^{-1}\left[1-\frac{\mathcal{s}\,\mathcal{v}}{1+\mathcal{s}\,\mathcal{v}\,\mathrm{e}^{-\frac{1}{\mathcal{s}\mathcal{v}}}}\right] \quad \ldots(39)$$

$$=1\text{-}\mathcal{s}\,\mathcal{v}\left[1-\mathcal{s}\,\mathcal{v}\mathrm{e}^{-\frac{1}{\mathcal{s}\mathcal{v}}}+(\mathcal{s}\mathcal{v})^2\mathrm{e}^{-\frac{2}{\mathcal{s}\mathcal{v}}}-(\mathcal{s}\mathcal{v})^3\mathrm{e}^{-\frac{3}{\mathcal{s}\mathcal{v}}}+\cdots+(-1)^{\mathrm{n}}(\mathcal{s}\mathcal{v})^{\mathrm{n}}\mathrm{e}^{-\frac{\mathrm{n}}{\mathcal{s}\mathcal{v}}}\right] \quad \ldots(40)$$

When take inverse of (ZMAT) proposed the (EX) as:

$$=1-\mathfrak{t}+2!\,(\mathfrak{t}-1)^2-3!\,(\mathfrak{t}-2)^3+\cdots+(-1)^{\mathrm{n}}\mathrm{n}!\,(\mathfrak{t}-\mathrm{n}-1)^{\mathrm{n}} \quad \ldots(41)$$

When $$\Lambda^{-1}\left[(-1)^{\mathrm{n}}(\mathcal{s}\mathcal{v})^{\mathrm{n}}\mathrm{e}^{-\frac{\mathrm{n}}{\mathcal{s}\mathcal{v}}}\right] = (-1)^{\mathrm{n}}\mathrm{n}!\,(\mathfrak{t}-\mathrm{n}-1)^{\mathrm{n}} \quad \ldots(42)$$

5. Discussion and Conclusion

The major goal informed (ZMAT) to present the general solution of linear (1st and 2nd) order (DDEs) by providing a theorem, basic function informed in Table 1, and the derivative of (ZMAT). Moreover, two applications were proposed to understand this technique. In addition, show that the techniques are powerful and effective since they get the (EX) without the need for any companion numerical method to solve linear (1st and 2nd) order (DDEs).

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