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The ZMA projected differential transform method for solving linear and non-linear partial differential equations

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Abstract

In this project, present a novel technique to solve linear and non-linear (L and NL) of partial differential equations (PDEs) with simple steps to present active and accurate results take less time compare with other researcher techniques to find the exact solutions (ES). These techniques performed by mixture of ZMA-transform (ZMAT) and projected differential transformed method (PDTM) demonstrated as ZMA projected differential transformed method (ZMAPDTM). Also, to more understand this technique inserted some properties assisted of greeted algorithm with steps to solve various problems.

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Keywords: *Projected differential transformed method, ZMA-transform, ZMA projected differential transformed method, Integral transform, Differential transform method, PDEs.*

1. Introduction

The many researchers recognized for solving more problems of (PDEs) for different branches in mathematics, phenomena, physics and various scientific fields informed in [1-3]. The numerous integral transform (IT) widely used for solving various applications for the types of (PDEs) improved by contributions authors and discussed in papers [4-13]. In inclusive nature nonlinear and nonlinearity exists anywhere, the survey in this project of a perfect and simple techniques use to find exact or approximate solutions for (NLPDEs) like (A domain decomposition, Variational iteration, Homotopy perturbation, Laplace decomposition, Sumudu decomposition method, and Elzaki decomposition and ZMA decomposition) methods [14-20]. The major goal in this work was proposed the novel techniques said (ZMAPDTM) to find the (ES) for (L and NL) of (PDEs) through using constructed algorithm for this method and

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informed several applications to show that effective result obtained by (ZMAPDTM).

2. Methodology

2.1 *Fundamental Idea of (ZMAT)*

In this section, proposed the (ZMAT) of the function in [11] as:

$$\mathcal{Z}_{\mathcal{MA}}(\hbar,\varsigma) = \Lambda\{\mathcal{F}(\mathfrak{t})\} = \frac{1}{\varsigma}\int_0^{\infty}\mathcal{F}(\hbar\,\mathfrak{t})\; e^{-\frac{\mathfrak{t}}{\varsigma}}\,d\mathfrak{t}\,,\mathfrak{t}>0\,,\hbar \text{ is the parameter} \tag{1}$$

The inverse for (ZMAT) presented as:

$$\mathcal{F}(\mathfrak{t}) = \Lambda^{-1}\{\mathcal{Z}_{\mathcal{MA}}(\hbar,\varsigma)\} \tag{2}$$

Know, in [13] informed by the (ZMAT) find the partial derivative of the following:

$$\Lambda_2\left\{\frac{\partial\mathcal{F}(\eta,\mathfrak{t})}{\partial\,\mathfrak{t}}\right\} = \frac{1}{\varsigma\,\hbar}\,\mathcal{Z}_{\mathcal{MA}}(\gamma,\hbar;\varsigma) - \frac{1}{\varsigma\,\hbar}\,\mathcal{Z}_{\mathcal{MA}}(\gamma,0;,\varsigma) \tag{3}$$

$$\Lambda_2\left\{\frac{\partial\mathcal{F}(\eta,\mathfrak{t})}{\partial\eta}\right\} = \frac{d}{d\eta}\,[\mathcal{Z}_{\mathcal{MA}}(\gamma,\hbar;\varsigma)] \tag{4}$$

$$\Lambda_2\left\{\frac{\partial^2\mathcal{F}(\eta,\mathfrak{t})}{\partial\,\mathfrak{t}^2}\right\} = \frac{1}{(\varsigma\,\hbar)^2}\,\mathcal{Z}_{\mathcal{MA}}(\gamma,\hbar;\varsigma) - \frac{1}{(\varsigma\,\hbar)^2}\,\mathcal{Z}_{\mathcal{MA}}(\gamma,0;,\varsigma) - \frac{1}{\varsigma\,\hbar}\,\frac{\partial\mathcal{Z}_{\mathcal{MA}}(\gamma,0;,\varsigma)}{\partial\,\mathfrak{t}} \tag{5}$$

$$\Lambda_2\left\{\frac{\partial^2\mathcal{F}(\eta,\mathfrak{t})}{\partial\eta^2}\right\} = \frac{d^2}{d\eta^2}\,[\mathcal{Z}_{\mathcal{MA}}(\gamma,\hbar;\varsigma)] \tag{6}$$

In [11] take (ZMAT) of regular functions as shown in Table 1:

Table 1

A regular functions by (ZMAT)

S. No.	$\mathcal{F}(\mathfrak{t})$	$\Lambda\{\mathcal{F}(\mathfrak{t})\}=\mathcal{Z}_{\mathcal{MA}}(\hbar,\varsigma)$
1	*1*	*1*
2	$\mathfrak{t}$	$\hbar\,\varsigma$
3	$\mathfrak{t}^2$	$2!\,(\hbar\,\varsigma)^2$
4	$\mathfrak{t}^{\mathcal{k}}, \mathcal{k}\in\mathcal{K}$	$\mathcal{k}!\,(\hbar\,\varsigma)^{\mathcal{j}}$
5	$\mathfrak{t}^{\mathcal{k}}, \mathcal{k}>-1$	$\Gamma(\mathcal{k}+1)\,(\hbar\,\varsigma)^{\mathcal{k}}$
6	$e^{-\mathfrak{C}\,\mathfrak{t}}$	$\dfrac{1}{1+\,\hbar(\mathfrak{C}\varsigma)}$
7	$cos\mathfrak{C}\,\mathfrak{t}$	$\dfrac{1}{1+\,\hbar^2(\mathfrak{C}\varsigma)^2}$
8	$sin\mathfrak{C}\,\mathfrak{t}$	$\dfrac{\hbar(\mathfrak{C}\varsigma)}{1+\,\hbar^2(\mathfrak{C}\varsigma)^2}$
9	$cosh\mathfrak{C}\,\mathfrak{t}$	$\dfrac{1}{1-\,\hbar^2(\mathfrak{C}\varsigma)^2}$
10	$sinh\mathfrak{C}\,\mathfrak{t}$	$\dfrac{\hbar(\mathfrak{C}\varsigma)}{1-\,\hbar^2(\mathfrak{C}\varsigma)^2}$

2.2 *Preliminaries of the (PDTM)*

In this task, extend the differential transform method to the (PDTM) through realized of the function $\mathcal{F}(\eta_1, \eta_2, \dots, \eta_m)$ as:

$$\mathcal{F}(\eta_1, \eta_2, \dots, \eta_{m-1}, \mathcal{G}) = \frac{1}{\mathcal{G}!}\left[\frac{\partial^{\mathcal{G}}\mathcal{F}(\eta_1,\eta_2,\dots,\eta_m)}{\partial \eta_m^{\mathcal{G}}}\right]_{\eta_m=0} \tag{7}$$

Where $\mathcal{F}(\eta_1, \eta_2, \dots, \eta_m)$, $\mathcal{F}(\eta_1, \eta_2, \dots, \eta_{m-1}, \mathcal{G})$ is the standard function and projected transform function respectively, then write the differential inverse transform of (7) as:

$$\mathcal{F}(\eta_1, \eta_2, \dots, \eta_m) = \sum_{\mathcal{G}=0}^{\infty} \mathcal{F}(\eta_1, \eta_2, \dots, \eta_m)(\eta - \eta_0)^{\mathcal{G}} \tag{8}$$

In the following informed the requisite theorems of (PDTM):

Theorems 1: *If* $\wp$ *is constant,* $m \neq -1, -2, \dots$.

1) $$\begin{aligned} &\mathfrak{P}(\eta_1, \eta_2, \dots, \eta_m) = w(\eta_1, \eta_2, \dots, \eta_m) \pm \mathfrak{z}(\eta_1, \eta_2, \dots, \eta_m) \\ &\rightarrow \mathfrak{P}(\eta_1, \eta_2, \dots, \eta_{m-1}, \mathcal{G}) = w(\eta_1, \eta_2, \dots, \eta_{m-1}, \mathcal{G}) \pm \mathfrak{z}(\eta_1, \eta_2, \dots, \eta_{m-1}, \mathcal{G}) \end{aligned} \tag{9}$$

2) $$\begin{aligned} &\mathfrak{P}(\eta_1, \eta_2, \dots, \eta_m) = \wp\, w(\eta_1, \eta_2, \dots, \eta_m) \\ &\rightarrow \mathfrak{P}(\eta_1, \eta_2, \dots, \eta_{m-1}, \mathcal{G}) = \wp\, w(\eta_1, \eta_2, \dots, \eta_{m-1}, \mathcal{G}) \end{aligned} \tag{10}$$

3) $$\mathfrak{P}(\eta_1, \eta_2, \dots, \eta_m) = \frac{dw(\eta_1,\eta_2,\dots,\eta_m)}{d\eta_m}$$

$$\rightarrow \mathfrak{P}(\eta_1, \eta_2, \dots, \eta_{m-1}, \mathcal{G}) = (\mathcal{G}+1)w(\eta_1, \eta_2, \dots, \eta_{m-1}, \mathcal{G}+1) \tag{11}$$

4) $$\mathfrak{P}(\eta_1, \eta_2, \dots, \eta_m) = \frac{d^m w(\eta_1,\eta_2,\dots,\eta_m)}{d\eta_m^m} \tag{12}$$

$$\rightarrow \mathfrak{P}(\eta_1, \eta_2, \dots, \eta_{m-1}, \mathcal{G}) = \frac{(\mathcal{G}+m)!}{\mathcal{G}!} w(\eta_1, \eta_2, \dots, \eta_{m-1}, \mathcal{G}+m)$$

5) $$\begin{aligned} &\mathfrak{P}(\eta_1, \eta_2, \dots, \eta_m) = w(\eta_1, \eta_2, \dots, \eta_m)\mathfrak{z}(\eta_1, \eta_2, \dots, \eta_m) \\ &\rightarrow \mathfrak{P}(\eta_1, \eta_2, \dots, \eta_{m-1}, \mathcal{G}) = \sum_{\mathcal{J}=0}^{\mathcal{G}} \mathfrak{P}(\eta_1, \eta_2, \dots, \eta_{m-1}, \mathcal{J})\mathfrak{z} \\ &(\eta_1, \eta_2, \dots, \eta_{m-1}, \mathcal{G}-\mathcal{J}) \end{aligned} \tag{13}$$

6) $$\begin{aligned} &\mathfrak{P}(\eta_1, \eta_2, \dots, \eta_m) = w_1(\eta_1, \eta_2, \dots, \eta_m)w_2(\eta_1, \eta_2, \dots, \eta_m) \dots \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad w_m(\eta_1, \eta_2, \dots, \eta_m) \\ &(\eta_1, \eta_2, \dots, \eta_{m-1}, \mathcal{G}) = \sum_{\mathcal{G}_{m-1}=0}^{\mathcal{G}} \sum_{\mathcal{G}_{m-2}=0}^{\mathcal{G}_{m-1}} \cdots \cdots \sum_{\mathcal{G}_2=0}^{\mathcal{G}_3} \sum_{\mathcal{G}_1=0}^{\mathcal{G}_2} \\ &\mathfrak{P}w_1(\eta_1, \eta_2, \dots, \eta_{m-1}, \mathcal{G}_1)w_2(\eta_1, \eta_2, \dots, \eta_{m-1}, \mathcal{G}_2 - \mathcal{G}_1) \\ &\times \dots w_{\eta-1}(\eta_1, \eta_2, \dots, \eta_{m-1}, \mathcal{G}_{\eta-1} - \mathcal{G}_{\eta-2})w_{\eta}(\eta_1, \eta_2, \dots, \eta_{m-1}, \mathcal{G} - \mathcal{G}_{\eta-1}) \end{aligned} \tag{14}$$

7) $$\mathfrak{P}(\eta_1, \eta_2, \dots, \eta_m) = \eta_1^{\mathfrak{G}_1}\eta_2^{\mathfrak{G}_2} \dots \dots \eta_m^{\mathfrak{G}_m}$$

$$\mathfrak{P}(\eta_1, \eta_2, \dots, \eta_{m-1}, \mathcal{G}) = \mathfrak{E}(\eta_1, \eta_2, \dots, \eta_{m-1}, \mathfrak{G}_m - \mathcal{G}) = \begin{cases} 1 & \mathcal{G} = \mathfrak{G}_m \\ 0 & \mathcal{G} \neq \mathfrak{G}_m \end{cases} \tag{15}$$

2.3 *Derivation the (ZMAPDTM) for solving (L and NL) of the (PDEs)*

Consider the universal (NL) non-homogenous (PDE) with the premier conditions of the form:

$$\mathfrak{D}\mathfrak{A}(\eta,\mathfrak{t}) + \mathfrak{K}\,\mathfrak{A}(\eta,\mathfrak{t}) + \mathcal{N}\,\mathfrak{A}(\eta,\mathfrak{t}) = \mathfrak{S}(\eta,\mathfrak{t}) \tag{16}$$

$$\mathfrak{A}(\eta,0) = \mathfrak{H}(\eta),\ \mathfrak{A}_{\mathfrak{t}}(\eta,0) = \mathfrak{X}(\eta) \tag{17}$$

$\mathfrak{D}$: is the (L) differential factor of order two.,$\mathfrak{K}$: is the (L) differential factor $<$ order $\mathfrak{D}$.$\mathcal{N}$: is the (NL) differential factor.$\mathfrak{S}(\eta,\mathfrak{t})$: is the origin expression.

Now, by (ZMAT) of (16) present:

$$\Lambda_2[\mathfrak{D}\mathfrak{A}(\eta,\mathfrak{t})] + \Lambda_2\,[\mathfrak{K}\,\mathfrak{A}(\eta,\mathfrak{t})] + \Lambda_2[\mathcal{N}\,\mathfrak{A}(\eta,\mathfrak{t})] = \Lambda_2\,[\mathfrak{S}(\eta,\mathfrak{t})] \tag{18}$$

$$\rightarrow \Lambda_2[\mathfrak{A}(\eta,\mathfrak{t})] = (\hbar\,\mathfrak{t})^2\,\Lambda_2\,[\mathfrak{S}(\eta,\mathfrak{t})] + \mathfrak{H}(\eta) + (\hbar\,\mathfrak{t})\mathfrak{X}(\eta) + \Lambda_2\,[\mathfrak{K}\,\mathfrak{A}(\eta,\mathfrak{t}) + \mathcal{N}\,\mathfrak{A}(\eta,\mathfrak{t})] \tag{19}$$

Employing the inverse for (ZMAT) of (19) as:

$$\mathfrak{A}(\eta,\mathfrak{t}) = \mathfrak{T}(\eta,\mathfrak{t}) - \Lambda_2^{-1}\,[(\hbar\,\mathfrak{t})^2\,\Lambda_2\,[\mathfrak{K}\,\mathfrak{A}(\eta,\mathfrak{t}) + \mathcal{N}\,\mathfrak{A}(\eta,\mathfrak{t})] \tag{20}$$

$$\mathfrak{T}(\eta,\mathfrak{t}) = \Lambda_2^{-1}[(\hbar\,\mathfrak{t})^2\,\Lambda_2\,[\mathfrak{S}(\eta,\mathfrak{t})] + \mathfrak{H}(\eta) + (\hbar\,\mathfrak{t})\mathfrak{X}(\eta)] \tag{21}$$

Now, putting the (PDTM) as:

$$\mathfrak{A}(\eta,\mathfrak{n}+1) = -\Lambda_2^{-1}\,[(\hbar\,\mathfrak{t})^2\,\Lambda_2\,[\,\mathcal{A}_{\mathfrak{n}} + \mathcal{B}_{\mathfrak{n}}]\,,\mathfrak{A}(\eta,,0) = \mathfrak{T}(\eta,\mathfrak{t}) \tag{22}$$

Wherever $\mathcal{A}_{\mathfrak{n}}, \mathcal{B}_{\mathfrak{n}}$ are (PDT) of $\mathfrak{K}\,\mathfrak{A}(\eta,,\mathfrak{t}), \mathcal{N}\,\mathfrak{A}(\eta,\mathfrak{t})$.

After solving (22) obtained:

$$\begin{aligned}
\mathfrak{A}(\eta,0) &= \mathfrak{T}(\eta,\mathfrak{t})\\
\mathfrak{A}(\eta,1) &= -\Lambda_2^{-1}\,[(\hbar\,\mathfrak{t})^2\,\Lambda_2\,[\,\mathcal{A}_0 + \mathcal{B}_0]\\
\mathfrak{A}(\eta,2) &= -\Lambda_2^{-1}\,[(\hbar\,\mathfrak{t})^2\,\Lambda_2\,[\,\mathcal{A}_1 + \mathcal{B}_1]\\
\mathfrak{A}(\eta,3) &= -\Lambda_2^{-1}\,[(\hbar\,\mathfrak{t})^2\,\Lambda_2\,[\,\mathcal{A}_2 + \mathcal{B}_3]
\end{aligned} \tag{23}$$

Lastly, the (ES) of (16)-(17) get:

$$\mathfrak{A}(\eta,\mathfrak{t}) = \mathfrak{A}(\eta,0) + \mathfrak{A}(\eta,1) + \mathfrak{A}(\eta,2) + \mathfrak{A}(\eta,3) + \cdots \tag{24}$$

From above, constructed the algorithm to more understand the new techniques for solving (L and NL) of (PDEs) as:

Algorithm of (ZMAPDTM)

a) Taking (ZMAT) of (L and NL) for (PDEs).
b) Using partial derivative informed in part (2.1) with premier conditions of problems.
c) Employ inverse of result solutions obtained in step(b).
d) Applying (PDTM) of the result obtained in step (c).
e) Obtained the (ES) of the (L and NL) for (PDEs).

3. Applications

Application 1: Take the (L) of the (PDEs) as

$$\frac{\partial\mathfrak{A}}{\partial\eta} = 2\frac{\partial\mathfrak{A}}{\partial\mathfrak{t}} + \mathfrak{A}, \mathfrak{A}(\eta,0) = 6\mathrm{e}^{-3\eta} \tag{25}$$

Solution: Put (ZMAT) of (25) obtain:

$$\Lambda_2\left[\frac{\partial \mathfrak{A}}{\partial \eta}\right] = 2\Lambda_2\left[\frac{\partial \mathfrak{A}}{\partial \mathfrak{t}}\right] + \Lambda_2[\mathfrak{A}] \tag{26}$$

$$\rightarrow \Lambda_2[\mathfrak{A}(\eta, \mathfrak{t})] = 6e^{-3\eta} + \left[\frac{(\hbar\, \mathfrak{s})}{2}\, \Lambda_2\, [\mathcal{A}_\mu - \mathcal{B}_\mu]\right. \tag{27}$$

Now, by inverse (ZMAT) of (27) find:

$$\mathfrak{A}(\eta, \mu + 1) = \Lambda_2^{-1}\left[\frac{(\hbar\, \mathfrak{s})}{2}\, \Lambda_2\, [\mathcal{A}_\mu - \mathcal{B}_\mu]\right., \mathfrak{A}(\eta, 0) = 6e^{-3\eta} \tag{28}$$

Where $\mathcal{A}_\mu = \frac{\partial \mathfrak{A}(\eta, \mu)}{\partial \eta}$, $\mathcal{B}_\mu = \mathfrak{A}(\eta, \mu)$ are (PDT) of $\frac{\partial \mathfrak{A}}{\partial \eta}$, $\mathfrak{A}$.

$$\rightarrow \mathfrak{A}(\eta, \mathfrak{t}) = \mathfrak{A}(\eta, 0) + \mathfrak{A}(\eta, 1) + \mathfrak{A}(\eta, 2) + \mathfrak{A}(\eta, 3) + \cdots = 6\, e^{-(3\eta + 2\mathfrak{t})} \tag{29}$$

Application 2: Look the (L) of the (PDEs) as

$$\frac{\partial^2 \mathfrak{A}}{\partial \eta^2} + \frac{\partial^2 \mathfrak{A}}{\partial \mathfrak{t}^2} = 0\,, \mathfrak{A}(\eta, 0) = 0, \mathfrak{A}_\mathfrak{t}(\eta, 0) = \cos\eta \tag{30}$$

Solution: Put (ZMAT) of (30) obtain:

$$\Lambda_2\left[\frac{\partial^2 \mathfrak{A}}{\partial \eta^2}\right] + \Lambda_2\left[\frac{\partial^2 \mathfrak{A}}{\partial \mathfrak{t}^2}\right] = 0 \tag{31}$$

$$\rightarrow \Lambda_2[\mathfrak{A}(\eta, \mathfrak{t})] = (\hbar\, \mathfrak{s})\cos\eta + [(\hbar\, \mathfrak{s})^2\, \Lambda_2\, \left[\frac{\partial^2 \mathfrak{A}}{\partial \eta^2}\right]] \tag{32}$$

Now, by inverse (ZMAT) of (32) find:

$$\mathfrak{A}(\eta, \mu + 1) = -\Lambda_2^{-1}\, [(\hbar\, \mathfrak{s})^2\, \Lambda_2\, [\mathcal{A}_\mu]], \mathfrak{A}(\eta, 0) = 0\,, \mathfrak{A}_\mathfrak{t}(\eta, 1) = \mathfrak{t}\cos\eta \tag{33}$$

Where $\mathcal{A}_\mu = \frac{\partial^2 \mathfrak{A}(\eta, \mu)}{\partial \eta^2}$,is (PDT) of $\frac{\partial^2 \mathfrak{A}}{\partial \eta^2}$.

$$\rightarrow \mathfrak{A}(\eta, \mathfrak{t}) = \mathfrak{A}(\eta, 0) + \mathfrak{A}(\eta, 1) + \mathfrak{A}(\eta, 2) + \mathfrak{A}(\eta, 3) + \cdots = \cos\eta \sinh\mathfrak{t} \tag{34}$$

Application 3: Look the (NL) of the (PDEs) as

$$\frac{\partial \mathfrak{A}}{\partial \mathfrak{t}} = \left(\frac{\partial \mathfrak{A}}{\partial \eta}\right)^2 + \mathfrak{A}\, \frac{\partial^2 \mathfrak{A}}{\partial \mathfrak{t}^2}\,, \mathfrak{A}(\eta, 0) = \eta^2. \tag{35}$$

Solution: Put (ZMAT) of (35) obtain:

$$\Lambda_2\left[\frac{\partial \mathfrak{A}}{\partial \mathfrak{t}}\right] = \Lambda_2\, \left[\left(\frac{\partial \mathfrak{A}}{\partial \eta}\right)^2 + \mathfrak{A}\, \frac{\partial^2 \mathfrak{A}}{\partial \mathfrak{t}^2}\right] \tag{36}$$

$$\rightarrow \Lambda_2[\mathfrak{A}(\eta, \mathfrak{t})] = \eta^2 + [(\hbar\, \mathfrak{s})\, \Lambda_2\, \left[\left(\frac{\partial \mathfrak{A}}{\partial \eta}\right)^2 + \mathfrak{A}\, \frac{\partial^2 \mathfrak{A}}{\partial \mathfrak{t}^2}\right]] \tag{37}$$

Now, by inverse (ZMAT) of (37) find:

$$\mathfrak{A}(\eta, \mu + 1) = \Lambda_2^{-1}\, [(\hbar\, \mathfrak{s})\, \Lambda_2\, [\mathcal{A}_\mu + \mathcal{B}_\mu]]\,, \mathfrak{A}(\eta, ,0) = \eta^2 \tag{38}$$

Where $\mathcal{A}_\mu = \sum_{\mu=0}^{\mathcal{C}\mathcal{J}} \frac{\partial \mathfrak{A}(\eta, ,\mu)}{\partial \eta} \frac{\partial \mathfrak{A}(\eta, \mathcal{C}\mathcal{J} - \mu)}{\partial \eta}, \mathcal{B}_\mu = \sum_{\mu=0}^{\mathcal{C}\mathcal{J}} \mathfrak{A}(\eta, , \mu) \frac{\partial^2 \mathfrak{A}(\eta, \mathcal{C}\mathcal{J} - \mu)}{\partial \eta^2}$,

are (PDT) of $\left(\frac{\partial \mathfrak{A}}{\partial \eta}\right)^2$, $\mathfrak{A}\, \frac{\partial^2 \mathfrak{A}}{\partial \mathfrak{t}^2}$.

$$\rightarrow \mathfrak{A}(\eta, \mathfrak{t}) = \mathfrak{A}(\eta, 0) + \mathfrak{A}(\eta, 1) + \mathfrak{A}(\eta, 2) + + \cdots = \frac{\eta^2}{1 - 6\mathfrak{t}} \tag{39}$$

Application 4: Look the (NL) of the (PDEs) as

$$\frac{\partial \mathfrak{A}}{\partial \mathfrak{t}} = 2\,\mathfrak{A}\left(\frac{\partial \mathfrak{A}}{\partial \mathfrak{y}}\right)^2 + \mathfrak{A}^2\,\frac{\partial^2 \mathfrak{A}}{\partial \mathfrak{t}^2}\,,\ \mathfrak{A}(\mathfrak{y},0) = \frac{\mathfrak{y}+1}{2}. \tag{40}$$

Solution: Put (ZMAT) of (40) obtain:

$$\Lambda_2\left[\frac{\partial \mathfrak{A}}{\partial \mathfrak{t}}\right] = \Lambda_2\left[2\mathfrak{A}\left(\frac{\partial \mathfrak{A}}{\partial \mathfrak{y}}\right)^2 + \mathfrak{A}^2\,\frac{\partial^2 \mathfrak{A}}{\partial \mathfrak{t}^2}\right] \tag{41}$$

$$\rightarrow \Lambda_2[\mathfrak{A}(\mathfrak{y},\mathfrak{t})] = \frac{\mathfrak{y}+1}{2} + \left[(\mathfrak{h}\,\mathfrak{s})\,\Lambda_2\left[2\mathfrak{A}\left(\frac{\partial \mathfrak{A}}{\partial \mathfrak{y}}\right)^2 + \mathfrak{A}^2\,\frac{\partial^2 \mathfrak{A}}{\partial \mathfrak{t}^2}\right]\right] \tag{42}$$

Now, by inverse (ZMAT) of (42) find:

$$\mathfrak{A}(\mathfrak{y},\mathfrak{m}+1) = \Lambda_2^{-1}\left[(\mathfrak{h}\,\mathfrak{s})\,\Lambda_2\left[2\mathcal{A}_{\mathfrak{m}} + \mathcal{B}_{\mathfrak{m}}\right]\right],\ \mathfrak{A}(\mathfrak{y},,0) = \frac{\mathfrak{y}+1}{2} \tag{43}$$

$$\mathcal{A}_{\mathfrak{m}} = \sum_{\mathcal{G}=0}^{\mathcal{C}\mathcal{J}} \sum_{\mathfrak{m}=0}^{\mathcal{G}} \mathfrak{A}(\mathfrak{y},,\mathfrak{m})\,\frac{\partial \mathfrak{A}(\mathfrak{y},\mathcal{C}\mathcal{J}-\mathfrak{m})}{\partial \mathfrak{y}}\,\frac{\partial \mathfrak{A}(\mathfrak{y},\mathcal{C}\mathcal{J}-\mathcal{G})}{\partial \mathfrak{y}}, \mathcal{B}_{\mathfrak{m}} = \sum_{\mathcal{G}=0}^{\mathcal{C}\mathcal{J}} \sum_{\mathfrak{m}=0}^{\mathcal{G}}$$

$$\mathfrak{A}(\mathfrak{y},,\mathfrak{m})\mathfrak{A}(\mathfrak{y},,\mathcal{C}\mathcal{J}-\mathfrak{m})\,\frac{\partial^2 \mathfrak{A}(\mathfrak{y},\mathcal{C}\mathcal{J}-\mathcal{G})}{\partial \mathfrak{y}^2}\text{, are (PDT) of } \mathfrak{A}\left(\frac{\partial \mathfrak{A}}{\partial \mathfrak{y}}\right)^2, \mathfrak{A}^2\,\frac{\partial^2 \mathfrak{A}}{\partial \mathfrak{t}^2}.$$

$$\rightarrow \mathfrak{A}(\mathfrak{y},\mathfrak{t}) = \mathfrak{A}(\mathfrak{y},0) + \mathfrak{A}(\mathfrak{y},1) + \mathfrak{A}(\mathfrak{y},2) + +\cdots = \frac{(\mathfrak{y}+1)}{2\sqrt{1-\mathfrak{t}}} \tag{44}$$

5. Discussion and Conclusion

In this procedure, preformed anew techniques is said (ZMAPDTM) for solving (L and NL) of PDEs and used Table 1 to obtained the (ES) with sample steps without any discretization variables. The novel techniques give efficient and attractive result show that by solving several applications.

References

[1] M. D. Raisinghania, *Advanced Differential Equations.* New Delhi, India: S. Chand & Co. Ltd. (1995).

[2] A. Jeffrey, *Advanced Engineering Mathematics*. San Diego, CA, USA: Harcourt/Academic Press (2002).

[3] K. A. Stroud and D. J. Booth, *Engineering Mathematics*. New York, USA: Industrial Press, Inc. (2001).

[4] L. M. Upadhyaya, "Introducing the Upadhyaya integral transform," *Bull. Pure Appl. Sci., Sect. E: Math. Stat.*, vol. 38E, no. 1, pp. 471–510 (2019), doi: 10.5958/2320-3226.2019.00051.1.

[5] S. Aggarwal and R. Chauhan, "A comparative study of Mohand and Aboodh transforms," *International Journal of Research in Advent Technology*, vol. 7, no. 1, pp. 520–529 (2019), doi: 10.32622/IJRAT.712019107.

[6] E. Kuffi, E. S. Abbas, and S. O. Mebayet, "Al-Tememe transform in solving developed missile automatic control problem," *International*

Journal of Engineering & Technology, vol. 7, no. 4, pp. 6565–6567 (2018), doi: 10.14419/ijet.

[7] H. Jafari, "A new general integral transform for solving integral equations," *Journal of Advanced Research,* vol. 32, pp. 133–138 (2021).

[8] M. M. A. Mahgoub, "The new integral transform 'Mohand Transform'," *Advances in Theoretical and Applied Mathematics*, vol. 12, pp. 113–120 (2017).

[9] H. Jafari and S. Aggarwal, "Upadhyaya integral transform: A tool for solving non-linear Volterra integral equations," *Mathematics and Computational Sciences,* vol. 5, no. 2, pp. 63–71 (2024).

[10] M. Higazy and S. Aggarwal, "Sawi transformation for system of ordinary differential equations with application," *Ain Shams Engineering Journal*, vol. 12, pp. 3173–3182 (2021).

[11] Z. M. Alwan, "ZMA-transform method," *Journal of Interdisciplinary Mathematics,* vol. 24, no. 7, pp. 1841–1849 (2021), doi: 10.1080/09720502.2021.1963520.

[12] Z. M. Alwan, "Solution homogeneous and non-homogeneous fractional ordinary differential equations by ZMA-transform," *Journal of Interdisciplinary Mathematics,* vol. 27, no. 4, pp. 781–785 (2024), doi: 10.47974/JIM-1871.

[13] S. Kumar, "A new analytical modelling for fractional telegraph equation via Laplace transform," *Applied Mathematical Modelling*, vol. 38, no. 13, pp. 3154–3163 (2014), doi: 10.1016/j.apm.2013.11.035.

[14] G. Adomian, *Solving Frontier Problems of Physics: The Decomposition Method*, vol. 60, Fundamental Theories of Physics. Dordrecht, Netherlands: Kluwer Academic Publishers (1994), doi: 10.1007/978-94-015-8289-6.

[15] J. H. He, "Homotopy perturbation technique," *Computer Methods in Applied Mechanics and Engineering*, vol. 178, pp. 257–262 (1999).

[16] J. H. He, "Variational iteration method—a kind of nonlinear analytical technique: Some examples," *Int. J. Non. Linear. Mech.,* vol. 34, pp. 699–708 (1999).

[17] M. A. Gondal, A. S. Arife, M. Khan, and I. Hussain, "An efficient numerical method for solving linear and nonlinear partial differential equations by combining homotopy analysis and transform method," *World Applied Sciences Journal*, vol. 15, pp. 1786–1791 (2011).

[18] S. A. Khuri, "A Laplace decomposition algorithm applied to a class of nonlinear differential equations," *Journal of Applied Mathematics,* vol. 1, pp. 141–155 (2001).

[19] Y. Khan, "An effective modification of the Laplace decomposition method for nonlinear equations," *International Journal of Nonlinear Sciences and Numerical Simulation*, vol. 10, pp. 1373–1376 (2009).

[20] A. Yildirim, "HES homotopy perturbation method for solving the space- and time-fractional telegraph equations," *International Journal of Computational Mathematics*, vol. 87, no. 13, pp. 2998–3006 (2010).

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