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Some effects of particle attributes on a novel type of generalization rectangular-partial metric space across a fuzzy soft set

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Abstract

In this paper, a new class in the fuzzy soft mathematics field is introduced called the fuzzy soft b-Rectangular partial metric space, which is a generalization concept of fuzzy soft metric space and the fuzzy soft partial metric space with some properties in this space, as the F_{ss}-closed, F_{ss}-open ball. and F_{ss}-converge sequence. Furthermore, establish some essential theorems that link this space with fuzzy soft b-Rectangular metric space (F_{ss}-bRMS).

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1. Introduction

Functional Analysis (FA) is an important and fundamental branch of theoretical mathematics, as it is considered a network that connects all branches of mathematics, and without it, no branch can flourish. It helps analyze the concepts of complex systems and find practical solutions in the real world. It is a gem that attracts researchers to it [1–6]. Ambiguity and uncertainty were mathematically modeled using fuzzy set theory, which allows elements to belong partially with degrees of membership ranging from 0 to 1. Traditional theories lack the flexibility to handle ambiguous and fuzzy data. The theory of soft sets was developed to overcome this problem due to its flexibility in handling such data [8]. The idea of eigen distance inequalities approaching zero was introduced as a starting point for establishing partial metric spaces [9]. The existence of fixed points in complete partial metric spaces for set-valued

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mappings was later established by generalizing Banach contraction under rational conditions [10]. Furthermore, several fixed-point theorems under various contraction effects in partial b-metric spaces with nodal values were developed [11]. Due to the flexibility of fuzzy soft sets in processing ambiguous data, this field has become an important area of interest for researchers. Fuzzy soft sets were further developed by introducing the concept of membership under uncertainty in decision-making processes [12, 13]. Additional studies have also investigated the properties and applications of fuzzy semi-normal operators as a new generalization of soft fuzzy Hilbert spaces.

In the paper, the concept of a fuzzy soft b-rectangular partial metric space is introduced, and some basic definitions related to this space are discussed. In addition, some theorems that relate b-rectangular metric spaces with b-Rectangular partial metric spaces have been generalized too fuzzy soft.

Preliminaries

In this part, let us first discuss some basic definitions of fuzzy soft.

Definition 2.1: [7]: Let U be a universal set, then the fuzzy set $\tilde{X}$ can be defined by an ordered pair $\widetilde{X}$={($(u, \mu_{\tilde{X}}(u))$: $u \in U, \mu_{\tilde{X}}(u) \in I$)} where $\mu_{\tilde{X}}(u)$ is degree membership of u in $\widetilde{X}$ and I = [0,1].

Definition 2.2: [8]: Let G:A→ $P(U)$ is mapping, $P(U)$ is a power set of U and $A \subseteq E$ where E is a collection of parameters Then soft set of U can be defined as $G_A = \{(e, G_A(e): G_A(e) \in P(U), e \in E\}$ and denoted by ordered pair (G, A).

Definition 2.3: [12]: Let G:A→ I^U is mapping and fuzzy soft collection is $\{e \in A, G_A(e) \in I^U\}$ then (G,A) is said to fuzzy soft of U and denoted F_{ss}.

Definition 2.4: [10]:

1- If $\mu G(e) = 1$ for all $e \in A$ then $F_{ss} - (G, A)$ is said to be absolute $-F_{ss}$, symbolized by $\widetilde{C_A}$.
2- If $\mu G(e) = \widetilde{0}$ for all $e \in A$ then $F_{ss} - (G, A)$ is said to be null$-F_{ss}$, symbolized by $\widetilde{\emptyset}$.

Definition 2.5: Where U be common universal set of distinct $F_{ss} - (G_1, A)$ and (G_2, B)

1- If $A \subseteq B$, and $G_1(e) \subseteq G_2(e)$, $\mu G_1(e) \leq \mu G_2(e)$ for all $e \in A$, can be written as $(G_1, A) \subseteq (G_2, B)$.
2- If $(G_1, A) \subseteq (G_2, B)$ and$(G_2, B) \subseteq (G_1, A)$ then $F_{ss} - (G_1, A)$ and (G_2, B) are equal and denoted by (G_1, A) = (G_2, B).

Definition 2.6: If $\upsilon \in U$ and e$\in A$, such that:

$$\mu G(e) = \begin{cases} \rho \text{ if } \upsilon = \upsilon_0 \; and \; e = e_0 \in A \\ 0 \; if \; u \in U - \{\upsilon_0\} \; or \; e \in A - \{e_0\} \; where \; \rho \in (0,1] \end{cases}$$

then $\mathrm{F}_{ss} - (G, A)$ is said to be F_{ss} −point and symbolized by $\upsilon_{\mu G(e)}$.

Definition 2.7:

1- If for e all $\in A$, $G(e) \subseteq G_1(e)$ then F_{ss} −point $\upsilon_{\mu G(e)}$ is said to be is said to be belong to (G_1, A) and denoted by $\upsilon_{\mu G(e)} \in (G_1, A)$.

2- If $\upsilon^1 = \upsilon^2$, $\mu G_1(e_1) = \mu G_2(e_2)$, then F_{ss} −point ${\upsilon^1}_{\mu G(e_1)}$ is equal to ${\upsilon^2}_{\mu G(e_2)}$.

Definition 2.8: The intersection of $\mathrm{F}_{ss} - (G_1, A)$ and (G_2, B) is$\mathrm{F}_{ss} - (G_3, A)$ such that:

$$\mu G_{3\,(e)}(\upsilon) = \begin{Bmatrix} \mu G_{1\,(e)}(\upsilon), if e \in A - B, \upsilon \in U \\ \mu G_{2\,(e)}(\upsilon), if e \in B - A, \upsilon \in U \\ min[\mu G_{1\,(e)}(\upsilon), \mu G_{2\,(e)}(\upsilon)] if e \in A \cap B, \upsilon \in U \end{Bmatrix}$$

Where $\upsilon \in U$,B$\cap$ A=C and e$\in C$.

Definition 2.9: Let $\tilde{d} : \mathrm{F}_{ss}(U) \times \mathrm{F}_{ss}(U) \to R^+(A)$ be a function where $\mathrm{F}_{ss}(U)$ is non empty F_{ss} of U then $\tilde{d}$ said to fuzzy soft metric on $\mathrm{F}_{ss}(U)$ if for all ${\upsilon^1}_{\mu G(e_1)}, {\upsilon^2}_{\mu G(e_2)}, {\upsilon^3}_{\mu G(e_3)} \in \mathrm{F}_{ss}(U)$, the following conditions are satisfied:

1- $\tilde{d}({\upsilon^1}_{\mu G(e_1)}, {\upsilon^2}_{\mu G(e_2)}) \cong \tilde{d}({\upsilon^2}_{\mu G(e_2)}, {\upsilon^1}_{\mu G(e_1)})$

2- $\tilde{d}({\upsilon^1}_{\mu G(e_1)}, {\upsilon^2}_{\mu G(e_2)}) \geqq 0$

3- $\tilde{d}({\upsilon^1}_{\mu G(e_1)}, {\upsilon^2}_{\mu G(e_2)}) \cong 0 \leftrightarrow {\upsilon^1}_{\mu G(e_1)} \cong {\upsilon^2}_{\mu G(e_2)}$

4- $\tilde{d}({\upsilon^1}_{\mu G(e_1)}, {\upsilon^3}_{\mu G(e_3)}) \leqq [\tilde{d}({\upsilon^1}_{\mu G(e_1)}, {\upsilon^2}_{\mu G(e_2)}) + \tilde{d}({\upsilon^2}_{\mu G(e_2)}, {\upsilon^3}_{\mu G(e_3)})]$

and $(\mathrm{F}_{ss}(U), \tilde{d})$ is fuzzy soft metric space and denoted by $\mathrm{F}_{ss} - MS$.

Definition 2.10: Let $\tilde{d}^{rb} : \mathrm{F}_{ss}(U) \times \mathrm{F}_{ss}(U) \to R^+(A)$ be function where $\mathrm{F}_{ss}(U)$ is non empty F_{ss} of U and S real number s.t S ≥ 1 then $\tilde{d}$ said to fuzzy soft b-Rectangular metric on $\mathrm{F}_{ss}(U)$ if for all ${\upsilon^1}_{\mu G(e_1)}$, ${\upsilon^2}_{\mu G(e_2)}$, ${\upsilon^3}_{\mu G(e_3)} \in \mathrm{F}_{ss}(U)$, the following conditions are satisfy:

1- $\tilde{d}^{rb}({\upsilon^1}_{\mu G(e_1)}, {\upsilon^2}_{\mu G(e_2)}) \cong \tilde{d}^{rb}({\upsilon^2}_{\mu G(e_2)}, {\upsilon^1}_{\mu G(e_1)})$

2- $\tilde{d}^{rb}({\upsilon^1}_{\mu G(e_1)}, {\upsilon^2}_{\mu G(e_2)}) \geqq 0$

3- $\tilde{d}^{rb}({\upsilon^1}_{\mu G(e_1)}, {\upsilon^2}_{\mu G(e_2)}) \cong 0 \leftrightarrow {\upsilon^1}_{\mu G(e_1)} \cong {\upsilon^2}_{\mu G(e_2)}$

4- $\tilde{d}^{rb}({\upsilon^1}_{\mu G(e_1)}, , {\upsilon^2}_{\mu G(e_2)}) \leqq S\, [\tilde{d}^{rb}({\upsilon^1}_{\mu G(e_1)}, {\upsilon^3}_{\mu G(e_3)}) +$
$\tilde{d}^{rb}({\upsilon^3}_{\mu G(e_3)}, {\upsilon^4}_{\mu G(e_4)}) + \tilde{d}^{rb}({\upsilon^4}_{\mu G(e_4)}, {\upsilon^2}_{\mu G(e_2)})]$

and $\left(\mathrm{F}_{ss}(U), \tilde{d}^{rb}\right)$ is fuzzy soft b-Rectangular metric space and denoted by $\mathrm{F}_{ss} - bRMS$.

Fuzzy Soft b- Rectangular Partial Metric Space

In this part the definition of concept b-Rectangular partial Metric and b-rectangular metric space is studied on Fuzzy Soft with discusses some theorem in this space.

Definition 3.1: Let $\tilde{d}_p^{rb}: \mathrm{F}_{ss}(U) \times \mathrm{F}_{ss}(U) \rightarrow R^{+}(A)$ be a function where $\mathrm{F}_{ss}(U)$ is non empty F_{ss} of U and S real number s.t S ≥ 1, then for all ${\upsilon^1}_{\mu G(e_1)}$, ${\upsilon^2}_{\mu G(e_2)}$, ${\upsilon^3}_{\mu G(e_3)} \in \mathrm{F}_{ss}(U)$, $\tilde{d}_p^{rb}$ is called fuzzy soft b-Rectangular partial metric on $\mathrm{F}_{ss}(U)$ if the following conditions are held:

1- $\tilde{d}_p^{rb}\left({\upsilon^1}_{\mu G(e_1)}, {\upsilon^2}_{\mu G(e_2)}\right) \cong \tilde{d}_p^{rb}\left({\upsilon^2}_{\mu G(e_2)}, {\upsilon^1}_{\mu G(e_1)}\right)$

2- $\tilde{d}_p^{rb}\left({\upsilon^2}_{\mu G(e_2)}, {\upsilon^2}_{\mu G(e_2)}\right) \cong \tilde{d}_p^{rb}\left({\upsilon^1}_{\mu G(e_1)}, {\upsilon^1}_{\mu G(e_1)}\right) \cong \tilde{d}_p^{rb}\left({\upsilon^1}_{\mu G(e_1)}, {\upsilon^2}_{\mu G(e_2)}\right)$ if and only if ${\upsilon^1}_{\mu G(e_1)} \cong {\upsilon^2}_{\mu G(e_2)}$.

3- $\tilde{d}_p^{rb}\left({\upsilon^1}_{\mu G(e_1)}, {\upsilon^2}_{\mu G(e_2)}\right) \gtrsim \tilde{d}_p^{rb}\left({\upsilon^1}_{\mu G(e_1)}, {\upsilon^1}_{\mu G(e_1)}\right)$

4- $\tilde{d}_p^{rb}\left({\upsilon^1}_{\mu G(e_1)}, {\upsilon^2}_{\mu G(e_2)}\right) \lesssim S[\tilde{d}_p^{rb}\left({\upsilon^1}_{\mu G(e_1)}, {\upsilon^3}_{\mu G(e_3)}\right) + \tilde{d}_p^{rb}\left({\upsilon^3}_{\mu G(e_3)}, {\upsilon^4}_{\mu G(e_4)}\right)$
$+\tilde{d}_p^{rb}\left({\upsilon^4}_{\mu G(e_4)},, {\upsilon^2}_{\mu G(e_2)}\right) - \tilde{d}_p^{rb}\left({\upsilon^3}_{\mu G(e_3)}, {\upsilon^3}_{\mu G(e_3)}\right) - \tilde{d}_p^{rb}\left({\upsilon^4}_{\mu G(e_4)}, {\upsilon^4}_{\mu G(e_4)}\right)]$

and $\left(\mathrm{F}_{ss}(U), \tilde{d}_p^{rb}\right)$ is fuzzy soft b-Rectangular partial metric space and denoted by $\mathrm{F}_{ss} - bRPMS$.

Definition 3.2: Let$\left(\mathrm{F}_{ss}(U), \tilde{d}_p^{rb}\right)$ be $\mathrm{F}_{ss} - bRPMS$and $\langle {\upsilon^n}_{\mu G(e_n)} \rangle$ be any sequence in$\left(\mathrm{F}_{ss}(U), \tilde{d}_p^{rb}\right)$ then $\langle {\upsilon^n}_{\mu G(e_n)} \rangle$ is said to convergent to $\upsilon_{\mu G(e)}$ in $\mathrm{F}_{ss}(U)$ if

$$\tilde{d}_p^{rb}\left({\upsilon^n}_{\mu G(e_n)}, \upsilon_{\mu G(e)}\right) \cong \tilde{d}_p^{rb}\left(\upsilon_{\mu G(e)}, \upsilon_{\mu G(e)}\right) \cong \tilde{d}_p^{rb}\left({\upsilon^n}_{\mu G(e_n)}, {\upsilon^n}_{\mu G(e_n)}\right).$$

Definition 3.3: In $\left(\mathrm{F}_{ss}(U), \tilde{d}_p^{rb}\right)$ a sequence $\langle {\upsilon^n}_{\mu G(e_n)} \rangle$ is said to$\mathrm{F}_{ss} -$ Cauchy sequence if $\tilde{d}_p^{rb}\left({\upsilon^n}_{\mu G(e_n)}, {\upsilon^m}_{\mu G(e_m)}\right)$ exists and finite.

Definition 3.4: A $\mathrm{F}_{ss} - bRPMS$ is said to complete if every F_{ss} −Cauchy sequence in$\left(\mathrm{F}_{ss}(U), \tilde{d}_p^{rb}\right)$ is convergent.

Definition 3.5: Let$\bar{\zeta}$ be a F_{ss} real number and $\left(\mathrm{F}_{ss}(U), \tilde{d}_p^{rb}\right)$ is $\mathrm{F}_{ss} - bRPMS$we can defined F_{ss} −open ball in $\left(\mathrm{F}_{ss}(U), \tilde{d}_p^{rb}\right)$ is defined by $\left\{\upsilon_{\mu G(a)} \in \mathrm{F}_{ss}(U): \tilde{d}_p^{rb}\left(\upsilon_{\mu G(a)}, \upsilon_{\mu G(e)}\right) \tilde{<} \bar{\zeta}\right\}$ and denoted by ${\tilde{d}_p^{rb}}_{\bar{\zeta}}\left(\upsilon_{\mu G(e)}\right)$, and F_{ss} −closed ball is defined $\left\{\upsilon_{\mu G(a)} \in \mathrm{F}_{ss}(U): \tilde{d}_p^{rb}\left(\upsilon_{\mu G(a)}, \upsilon_{\mu G(e)}\right) \lesssim \bar{\zeta}\right\}$ denoted by ${\tilde{d}_p^{rb}}_{\bar{\zeta}}\left[\upsilon_{\mu G(e)}\right]$.

Definition 3.6: In a $\mathrm{F}_{ss}-bRPMS$ a $\mathrm{F}_{ss}-(G,A)$ is said to be F_{ss} −open if for each F_{ss} −point $\upsilon_{\mu G(e)}$ of (G,A) ∃ F_{ss} −open ball $\tilde{d}^{rb}_{p\,\bar{\zeta}}(\upsilon_{\mu G(e)}) \tilde{\subseteq} (G,A)$.

Definition 3.7: The F_{ss} −point $\upsilon_{\mu G(e)}$ is $\mathrm{F}_{ss}-$ limit point of (G,A) in $\mathrm{F}_{ss}-bRPMS$ if and only if for every $\tilde{0} < \tilde{r} \in R^+$ (A) we have $\tilde{d}^{rb}_{p\,\bar{\zeta}}(\upsilon_{\mu G(e)}) \tilde{\cap} (G,A) \neq \tilde{\emptyset}$.

Definition 3.8: In a $\mathrm{F}_{ss}-bRPMS$ a sequences $\langle \upsilon^n{}_{\mu G(e_n)} \rangle$ is said to be bonded if there exists F_{ss} real number $\bar{\zeta}$ such that $\upsilon^n{}_{\mu G(e_n)} \tilde{\in} \tilde{d}^{rb}_{p\,\bar{\zeta}}(\upsilon_{\mu G(e)})$ and $\left|\mu G(\upsilon^n{}_{\mu G(e_n)})(S) - \mu G(e)(S)\right| < \tilde{\varepsilon}$ where $\tilde{\varepsilon} \tilde{\in} (\tilde{0}, \tilde{1})$ for all $n \in N$.

Theorem 3.9: *Let* $(\mathrm{F}_{ss}(U), \tilde{d}^{rb}_p)$ *be* $\mathrm{F}_{ss}-bRPMS$, *then* $\tilde{\tilde{d}}_{br}(\upsilon^1{}_{\mu G(e_1)}, \upsilon^2{}_{\mu G(e_2)})$ $=2\tilde{d}^{rb}_p(\upsilon^1{}_{\mu G(e_1)}, \upsilon^2{}_{\mu G(e_2)}) - \tilde{d}^{rb}_p(\upsilon^1{}_{\mu G(e_1)}, \upsilon^1{}_{\mu G(e_1)}) - \tilde{d}^{rb}_p(\upsilon^2{}_{\mu G(e_2)}, \upsilon^2{}_{\mu G(e_2)})$ *for each* $\upsilon^1{}_{\mu G(e_1)}, \upsilon^2{}_{\mu G(e_2)}, \upsilon^3{}_{\mu G(e_3)}$ *in* $\mathrm{F}_{ss}(U)$ *is* $\mathrm{F}_{ss}-bRMS$ *on* $\mathrm{F}_{ss}(U)$.

Proof: For all $\upsilon^1{}_{\mu G(e_1)}, \upsilon^2{}_{\mu G(e_2)}, \upsilon^3{}_{\mu G(e_3)} \in \mathrm{F}_{ss}(U)$ and by definition of $\tilde{\tilde{d}}_{br}$ and $\tilde{d}^{rb}_p$. It is easy to prove that $\tilde{\tilde{d}}_{br}$ satisfies (1), (2), (3) conditions of $\mathrm{F}_{ss}-bRMS$. Now to prove the forth condition

$$\tilde{d}^{rb}(\upsilon^1{}_{\mu G(e_1)}, \upsilon^2{}_{\mu G(e_2)}) \leq S[\tilde{d}^{rb}(\upsilon^1{}_{\mu G(e_1)}, \upsilon^3{}_{\mu G(e_3)}) + \tilde{d}^{rb}(\upsilon^3{}_{\mu G(e_3)}, \upsilon^4{}_{\mu G(e_4)}) + \tilde{d}^{rb}(\upsilon^4{}_{\mu G(e_4)}, \upsilon^2{}_{\mu G(e_2)}) - \tilde{d}^{rb}(\upsilon^3{}_{\mu G(e_3)}, \upsilon^3{}_{\mu G(e_3)}) - \tilde{d}^{rb}(\upsilon^4{}_{\mu G(e_4)}, \upsilon^4{}_{\mu G(e_4)})] \quad \ldots(1)$$

$$\tilde{d}^{rb}(\upsilon^1{}_{\mu G(e_1)}, \upsilon^2{}_{\mu G(e_2)}) = 2\tilde{d}^{rb}_p(\upsilon^1{}_{\mu G(e_1)}, \upsilon^2{}_{\mu G(e_2)}) - \tilde{d}^{rb}_p(\upsilon^1{}_{\mu G(e_1)}, \upsilon^1{}_{\mu G(e_1)}) - \tilde{d}^{rb}_p(\upsilon^2{}_{\mu G(e_2)}, \upsilon^2{}_{\mu G(e_2)}) \quad \ldots(2)$$

We repeat the same step above on $\tilde{d}^{rb}(\upsilon^1{}_{\mu G(e_1)}, \upsilon^3{}_{\mu G(e_3)})$, $\tilde{d}^{rb}(\upsilon^3{}_{\mu G(e_3)}, \upsilon^4{}_{\mu G(e_4)})$ and $\tilde{d}^{rb}(\upsilon^4{}_{\mu G(e_4)}, \upsilon^2{}_{\mu G(e_2)})$ then we get an (3), (4), (5). Substituting (2) in the left side of the inequality (1) and (3), (4), (5) in the right side, we get the desired results.

Theorem 3.10: *Let* $(\mathrm{F}_{ss}(U), \tilde{d}^{rb}_p)$ *be* $\mathrm{F}_{ss}-bRPMS$ *and let* $\langle \upsilon^n{}_{\mu G(e_n)} \rangle$ *be* $\mathrm{F}_{ss}-$ *sequences in* $\mathrm{F}_{ss}(U)$ *then* $\langle \upsilon^n{}_{\mu G(e_n)} \rangle$ *is converges to* $\upsilon_{\mu G(e)}$ *in* $(\mathrm{F}_{ss}(U), \tilde{d}^{rb})$ *that is* $\tilde{d}^{rb}(\upsilon^n{}_{\mu G(e_n)}, \upsilon_{\mu G(e)}) = 0$ *if and only if* $\tilde{d}^{rb}_p(\upsilon^n{}_{\mu G(e_n)}, \upsilon_{\mu G(e)}) = \tilde{d}^{rb}_p(\upsilon^n{}_{\mu G(e_n)}, \upsilon^n{}_{\mu G(e_n)}) = \tilde{d}^{rb}_p(\upsilon_{\mu G(e)}, \upsilon_{\mu G(e)})$.

Proof: Let $\langle \upsilon^n{}_{\mu G(e_n)} \rangle$ converges to $\upsilon_{\mu G(e)}$ in $(\mathrm{F}_{ss}(U), \tilde{d}^{rb})$ that is mean $\tilde{d}^{rb}(\upsilon^n{}_{\mu G(e_n)}, \upsilon_{\mu G(e)}) = 0$ by theorem (3.9) and By take the limit as $n \to \infty$ then $\tilde{d}^{rb}(\upsilon^n{}_{\mu G(e_n)}, \upsilon_{\mu G(e)}) = lim[_{n\to\infty} 2\tilde{d}^{rb}_p(\upsilon^n{}_{\mu G(e_n)}, \upsilon_{\mu G(e)}) - \tilde{d}^{rb}_p(\upsilon^n{}_{\mu G(e_n)}, \upsilon^n{}_{\mu G(e_n)}) - \tilde{d}^{rb}_p(\upsilon_{\mu G(e)}, \upsilon_{\mu G(e)})] = 0$ Since $\tilde{d}^{rb}_p(\upsilon^n{}_{\mu G(e_n)}, \upsilon^n{}_{\mu G(e_n)})$ converges as

$n \to \infty$ then $lim_{n\to\infty}\, \tilde{d}_p^{rb}\big({\upsilon^n}_{\mu G(e_n)}, \upsilon_{\mu G(e)}\big) = \tilde{d}_p^{rb}\big(\upsilon_{\mu G(e)}, \upsilon_{\mu G(e)}\big)$. Conversely, suppose that $\tilde{d}_p^{rb}\big({\upsilon^n}_{\mu G(e_n)}, {\upsilon^n}_{\mu G(e_n)}\big) = \tilde{d}_p^{rb}\big(\upsilon_{\mu G(e)}, \upsilon_{\mu G(e)}\big)$ to prove $\tilde{d}^{rb}\big({\upsilon^n}_{\mu G(e_n)}, \upsilon_{\mu G(e)}\big) = 0$ by theorem (3. 9) Since $\tilde{d}_p^{rb}\big({\upsilon^n}_{\mu G(e_n)}, \upsilon_{\mu G(e)}\big) = \tilde{d}_p^{rb}\big(\upsilon_{\mu G(e)}, \upsilon_{\mu G(e)}\big)$ then $lim[_{n\to\infty} 2\tilde{d}_p^{rb}\big(\upsilon_{\mu G(e)}, \upsilon_{\mu G(e)}\big) - \tilde{d}_p^{rb}\big({\upsilon^n}_{\mu G(e_n)}, {\upsilon^n}_{\mu G(e_n)}\big) - \tilde{d}_p^{rb}\big(\upsilon_{\mu G(e)}, \upsilon_{\mu G(e)}\big)\ ]$

$=lim[_{n\to\infty} \tilde{d}_p^{rb}\left(\tilde{d}_p^{rb}\big(\upsilon_{\mu G(e)}, \upsilon_{\mu G(e)}\big)\right) - \tilde{d}_p^{rb}\big({\upsilon^n}_{\mu G(e_n)}, {\upsilon^n}_{\mu G(e_n)}\big)]$ then $\tilde{d}_p^{rb}\big({\upsilon^n}_{\mu G(e_n)}, {\upsilon^n}_{\mu G(e_n)}\big) = \tilde{d}_p^{rb}\big(\upsilon_{\mu G(e)}, \upsilon_{\mu G(e)}\big)$ and $\tilde{d}^{rb}\big({\upsilon^n}_{\mu G(e_n)}, \upsilon_{\mu G(e)}\big) = 0$.

Theorem 3.11:

(1) *In* $\mathrm{F}_{ss} - bRPMS$*, a sequence* $\langle {\upsilon^n}_{\mu G(e_n)} \rangle$ *is* $\mathrm{F}_{ss} -$ *Cauchy sequences if and only if is* $\mathrm{F}_{ss} -$ *Cauchy sequences in* $\mathrm{F}_{ss} - bRMS$*.*

(2) $\mathrm{F}_{ss} - \mathrm{bRMS}$ is complete if and only if $\mathrm{F}_{ss} - \mathrm{bRPMS}$ is complete.

Proof:

(1) Let $\langle {\upsilon^n}_{\mu G(e_n)} \rangle$ be $\mathrm{F}_{ss} -$ Cauchy sequences in $\mathrm{F}_{ss} - bR\mathrm{PMS}$ So$\tilde{d}^{r}\big({\upsilon^n}_{\mu G(e_n)}, {\upsilon^m}_{\mu G(e_m)}\big) = \big(\upsilon_{\mu G(e)}, \upsilon_{\mu G(e)}\big)$ .Let$\tilde{\epsilon} > 0$ then there exists a natural number N_0 such that $|\tilde{d}_p^{rb}\big({\upsilon^n}_{\mu G(e_n)}, {\upsilon^m}_{\mu G(e_m)}\big) - \tilde{d}_p^{rb}(\upsilon_{\mu G(e)}, \upsilon_{\mu G(e)})| < \frac{\tilde{\epsilon}}{4}$ for all $n, m \geq N$ and $\left|\, \tilde{d}_p^{rb}\big({\upsilon^n}_{\mu G(e_n)}, {\upsilon^m}_{\mu G(e_m)}\big)\right| = |2\tilde{d}_p^{rb}({\upsilon^n}_{\mu G(e_n)}, {\upsilon^m}_{\mu G(e_m)}) - \tilde{d}_p^{rb}({\upsilon^n}_{\mu G(e_n)}, {\upsilon^n}_{\mu G(e_n)}) - \tilde{d}_p^{rb}({\upsilon^m}_{\mu G(e_m)}, {\upsilon^m}_{\mu G(e_m)})|$

$$= \left| \begin{matrix} 2\tilde{d}_p^{rb}\big({\upsilon^n}_{\mu G(e_n)}, {\upsilon^m}_{\mu G(e_m)}\big) - 2\tilde{d}_p^{rb}\big(\upsilon_{\mu G(e)}, \upsilon_{\mu G(e)}\big) \\ -\tilde{d}_p^{rb}\big({\upsilon^n}_{\mu G(e_n)}, {\upsilon^n}_{\mu G(e_n)}\big) + \tilde{d}_p^{rb}\big(\upsilon_{\mu G(e)}, \upsilon_{\mu G(e)}\big) - \\ \tilde{d}_p^{rb}\big({\upsilon^m}_{\mu G(e_m)}, {\upsilon^m}_{\mu G(e_m)}\big) + \tilde{d}_p^{rb}\big(\upsilon_{\mu G(e)}, \upsilon_{\mu G(e)}\big) \end{matrix} \right|$$

$$\leq \left|\tilde{d}_p^{rb}\big({\upsilon^n}_{\mu G(e_n)}, {\upsilon^m}_{\mu G(e_m)}\big) - \tilde{d}_p^{rb}\big(\upsilon_{\mu G(e)}, \upsilon_{\mu G(e)}\big)\right| + \left|\tilde{d}_p^{rb}\big({\upsilon^n}_{\mu G(e_n)}, {\upsilon^m}_{\mu G(e_m)}\big) - \tilde{d}_p^{rb}\big(\upsilon_{\mu G(e)}, \upsilon_{\mu G(e)}\big)\right| < \tilde{\epsilon}$$

For all $n, m > N$. So $\langle {\upsilon^n}_{\mu G(e_n)} \rangle$ is $\mathrm{F}_{ss} -$ Cauchy sequences in $\mathrm{F}_{ss} - bRMS$ Conversely, Let $\langle {\upsilon^n}_{\mu G(e_n)} \rangle$ be $\mathrm{F}_{ss} -$ Cauchy sequences in $\mathrm{F}_{ss} - bRMS$ and let $\tilde{\epsilon} = \frac{\tilde{1}}{2}$ then n_0 in N such that $\tilde{d}_p^{rb}\big({\upsilon^n}_{\mu G(e_n)}, {\upsilon^m}_{\mu G(e_m)}\big) < \frac{\tilde{1}}{2}$ for all $n, m \geq n_0$ Implies $2\tilde{d}_p^{rb}\left({\upsilon^n}_{\mu G(e_n)}, {\upsilon^{n_0}}_{\mu G(e_{n_0})}\right) - \tilde{d}_p^{rb}\left({\upsilon^{n_0}}_{\mu G(e_{n_0})}, {\upsilon^{n_0}}_{\mu G(e_{n_0})}\right) - \tilde{d}_p^{rb}\big({\upsilon^n}_{\mu G(e_n)}, {\upsilon^n}_{\mu G(e_n)}\big) < \frac{\tilde{1}}{2}$ implies $\tilde{d}_p^{rb}\left({\upsilon^n}_{\mu G(e_n)}, {\upsilon^{n_0}}_{\mu G(e_{n_0})}\right) - \tilde{d}_p^{rb}\left({\upsilon^{n_0}}_{\mu G(e_{n_0})}, {\upsilon^{n_0}}_{\mu G(e_{n_0})}\right) < \frac{\tilde{1}}{2}\ \tilde{d}_p^{rb}\big({\upsilon^n}_{\mu G(e_n)}, {\upsilon^n}_{\mu G(e_n)}\big) < \tilde{d}_p^{rb}\left({\upsilon^n}_{\mu G(e_n)}, {\upsilon^{n_0}}_{\mu G(e_{n_0})}\right) < \tilde{d}_p^{rb}\left({\upsilon^{n_0}}_{\mu G(e_{n_0})}, {\upsilon^{n_0}}_{\mu G(e_{n_0})}\right) + \frac{\tilde{1}}{2}$

So $\{\tilde{d}_p^{rb}\big({\upsilon^n}_{\mu G(e_n)}, {\upsilon^n}_{\mu G(e_n)}\big)\}$ is a bounded sequences in R(A).

Hence $\tilde{d}_p^{rb}\left(\upsilon^{n_k}{}_{\mu G(e_{n_k})}, \upsilon^{n_k}{}_{\mu G(e_{n_k})}\right) = \left(\upsilon^{n_1}{}_{\mu G(e_{n_1})}, \upsilon^{n_1}{}_{\mu G(e_{n_1})}\right)$ Since $\langle \upsilon^{n}{}_{\mu G(e_n)} \rangle$ is $\mathrm{F}_{ss} -$ Cauchy sequences in $\mathrm{F}_{ss} - RbMS$ for $\epsilon > 0$ there exist $n_{\tilde{\epsilon}}$ such that $\tilde{d}_p^{rb}\left(\upsilon^{n}{}_{\mu G(e_n)}, \upsilon^{m}{}_{\mu G(e_m)}\right) < \epsilon$ for all $n, m \geq n_{\tilde{\epsilon}}$ then $\tilde{d}_p^{rb}\left(\upsilon^{n}{}_{\mu G(e_n)}, \upsilon^{n}{}_{\mu G(e_n)}\right) - \tilde{d}_p^{rb}\left(\upsilon^{m}{}_{\mu G(e_m)}, \upsilon^{m}{}_{\mu G(e_m)}\right) \leq$
$\tilde{d}_p^{rb}\left(\upsilon^{n}{}_{\mu G(e_n)}, \upsilon^{m}{}_{\mu G(e_m)}\right) - \tilde{d}_p^{rb}\left(\upsilon^{m}{}_{\mu G(e_m)}, \upsilon^{m}{}_{\mu G(e_m)}\right) <$
$\tilde{d}_p^{rb}\left(\upsilon^{n}{}_{\mu G(e_n)}, \upsilon^{m}{}_{\mu G(e_m)}\right) < \tilde{\epsilon}$

So $\{\tilde{d}_p^{rb}\left(\upsilon^{n}{}_{\mu G(e_n)}, \upsilon^{n}{}_{\mu G(e_n)}\right)\}$ is Cauchy sequences in R(A). Implies $\tilde{d}_p^{r}\left(\upsilon^{n}{}_{\mu G(e_n)}, \upsilon^{n}{}_{\mu G(e_n)}\right) = \tilde{d}_p^{rb}\left(\upsilon^{n_1}{}_{\mu G(e_{n_1})}, \upsilon^{n_1}{}_{\mu G(e_{n_1})}\right)$.

Now $\left|\tilde{d}_p^{rb}\left(\upsilon^{n}{}_{\mu G(e_n)}, \upsilon^{m}{}_{\mu G(e_m)}\right) - \tilde{d}_p^{rb}(\upsilon^{n_1}{}_{\mu G(e_{n_1})}, \upsilon^{n_1}{}_{\mu G(e_{n_1})})\right|$

$$\left| \begin{array}{l} \tilde{d}_p^{rb}\left(\upsilon^{n}{}_{\mu G(e_n)}, \upsilon^{m}{}_{\mu G(e_m)}\right) - \tilde{d}_p^{rb}\left(\upsilon^{n}{}_{\mu G(e_n)}, \upsilon^{n}{}_{\mu G(e_n)}\right) \\ + \tilde{d}_p^{rb}\left(\upsilon^{n}{}_{\mu G(e_n)}, \upsilon^{n}{}_{\mu G(e_n)}\right) - \tilde{d}_p^{rb}\left(\upsilon^{n_1}{}_{\mu G(e_{n_1})}, \upsilon^{n_1}{}_{\mu G(e_{n_1})}\right) \end{array} \right|$$

$\leq \tilde{d}_p^{rb}\left(\upsilon^{n}{}_{\mu G(e_n)}, \upsilon^{m}{}_{\mu G(e_m)}\right)$

$+ \left|\tilde{d}_p^{rb}\left(\upsilon^{n}{}_{\mu G(e_n)}, \upsilon^{n}{}_{\mu G(e_n)}\right) - \tilde{d}_p^{rb}(\upsilon^{n_1}{}_{\mu G(e_{n_1})}, \upsilon^{n_1}{}_{\mu G(e_{n_1})})\right|$

Implies $\tilde{d}_p^{r}\left(\upsilon^{n}{}_{\mu G(e_n)}, \upsilon^{m}{}_{\mu G(e_m)}\right) = \tilde{d}_p^{rb}\left(\upsilon^{n_1}{}_{\mu G(e_{n_1})}, \upsilon^{n_1}{}_{\mu G(e_{n_1})}\right)$.

Therefor $\langle \upsilon^{n}{}_{\mu G(e_n)} \rangle$ is $\mathrm{F}_{ss} -$ Cauchy sequences in $\mathrm{F}_{ss} - bRPMS$.

(2) Let $\langle \upsilon^{n}{}_{\mu G(e_n)} \rangle$ be any $\mathrm{F}_{ss} -$ Cauchy sequences in $\mathrm{F}_{ss} - bRPMS$ then $\langle \upsilon^{n}{}_{\mu G(e_n)} \rangle$ is any $\mathrm{F}_{ss} -$ Cauchy sequences in $\mathrm{F}_{ss} - bMS$ Since $\mathrm{F}_{ss} - bRMS$ is complete, there exists a F_{ss} $-$point$\upsilon_{\mu G(e)}$ in $\mathrm{F}_{ss}(U)$ such that and by theorem (3.10)$\mathrm{F}_{ss} - bRPMS\ is$ complete. Conversely, Let$\mathrm{F}_{ss} - RPMS\ is$ complete and let $\langle \upsilon^{n}{}_{\mu G(e_n)} \rangle$ be any $\mathrm{F}_{ss} -$ Cauchy sequences in $\mathrm{F}_{ss} - bRMS$ then $\langle \upsilon^{n}{}_{\mu G(e_n)} \rangle$ is $\mathrm{F}_{ss} -$ Cauchy sequences in $\mathrm{F}_{ss} - bRPMS$, since $\mathrm{F}_{ss} - bRPMS$ is complete, there exists a F_{ss} $-$point$\upsilon^{1}{}_{\mu G(e_1)}$ in $\mathrm{F}_{ss}(U)$ such that $\tilde{d}_p^{r}\left(\upsilon^{n}{}_{\mu G(e_n)}, \upsilon^{1}{}_{\mu G(e_1)}\right) = \tilde{d}_p^{r}\left(\upsilon^{1}{}_{\mu G(e_1)}, \upsilon^{1}{}_{\mu G(e_1)}\right) = \tilde{d}_p^{r}(\upsilon^{n}{}_{\mu G(e_n)}, \upsilon^{n}{}_{\mu G(e_n)})$

Now, by theorem (3.10)$\tilde{d}_p^{r}\left(\upsilon^{n}{}_{\mu G(e_n)}, \upsilon^{1}{}_{\mu G(e_1)}\right) = 0$ hence $\mathrm{F}_{ss} - bRMS$ is complete.

Conclusion

In this research, fuzzy soft partial metric space has been expanded by providing fuzzy soft b- Rectangular partial metric space, which is considered a new idea in the fuzzy soft field. The characteristics related to this space were presented, such as: the F_{ss}-closed, F_{ss}-open ball, F_{ss}- space, and F_{ss}-converge sequence, F_{ss}-limit point. In addition, some theorems that explain the relationship between this space and fuzzy soft b- Rectangular metric space have been proven.

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