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Some concepts on soft topological soft group space

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Abstract

This article introduces some concepts on the $soft$ topological $soft$ group (Stsg). We used a concept for the $soft$ topological $soft$ groups to construct the $b\tau_i$-$soft$ topological $soft$ groups, b-regular $soft$ topology $soft$ groups, as well as studied relationships between them.

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Keywords: *Soft group, Soft topological $soft$ group, B-open soft set, B-Closed soft set.*

1. Introduction

Previous studies investigated soft topological spaces and introduced several concepts, such as soft closed sets, soft subspaces, soft closures, and neighborhoods of a point, together with their fundamental properties [1]. Later, the notion of soft topological groups was established, and several of their structural characteristics were examined [2]. More recently, center topological groups were introduced, and various related properties were explored [3–4]. In addition, the soft separation axioms associated with b-open soft sets were developed, including investigations of soft $b\tau_i$ $(i = 1, \dots, 4)$, soft b-regular spaces, and soft b-normal spaces [5].

2. Preliminaries

Definition 2.1: [6] If $(\chi, \tau, \hat{E})$ is the soft topology spaces & $(K, \hat{E}) \tilde{\in} SS(\chi, \hat{E})$. the $(K, \hat{E})$ named b - $open$ soft sets where $(K, \hat{E}) \tilde{\subseteq} cl(in((K, \hat{E})) \tilde{\cup} in(cl$

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$((K, \hat{E}))$ while the complements named b-closed soft sets. A set for each b-$open$ soft set is denoted as $BOS(\chi, \tau, \hat{E})$ or $BOS(\chi)$ and a set for the b-closed soft set be denoted by $BCS(\chi, \tau, \hat{E})$ or $BCS(\chi)$.

Theorem 2.2: [6]

(i) *Unions for the b-$open$ soft set is b-$open$ $soft$.*

(ii) *Intersections of b-closed soft set are b-closed $soft$.*

Proposition 2.3: [6] The $open$ $soft$ sets imply the b-$open$ $soft$.

Remark 2.4: The converse of (2.3) does not always have to be true, as below:

Example 2.5: Let $\chi = \{v_1, v_2, v_3\}$, $\tilde{E} = \{e_1, e_2\}$ & $\tau = \{\tilde{\chi}, \tilde{\phi}, (K, \hat{E})\}$, where

$$K(e_1) = \{v_1, v_3\}, K(e_2) = \{v_2, v_3\}.$$

"Let $(H, \hat{E})$ defined by: $H(e_1) = \{v_1, v_2\}, H(e_2) = \{v_1\}$ is b-$open$ $soft$ set but not $open$ $soft$".

Remark 2.6: A set for the **b-*open*** $soft$ set may fail to be a soft topology as below:

Example 2.7: Let $\chi = \{v_1, v_2, v_3\}$, $\tilde{E} = \{m_1, m_2\}$ & $\tau = \{\tilde{\chi}, \tilde{\phi}, (K, \hat{E})\}$, where

$$K(m_1) = \{v_1, v_3\}, K(m_2) = \{v_2, v_3\}.$$

Since the $soft$ sets $(H, \hat{E}), (D, \hat{E})$:

$$H(m_1) = \{v_1, v_2\}, H(v_2) = \{v_1\}$$

$$D(m_1) = \{v_2, v_3\}, D(m_2) = \{v_1\}$$

Are b-$open$ sets, but their intersection isn't b - $open$ $soft$ sets.

Definition 2.8: [6] If $(K, \hat{E}) \tilde{\in} SS(\chi, \hat{E})$ & v^e be a soft point, then v^e named:

(i) b-interior for the $(K, \hat{E})$ where $\exists (D, \hat{E}) \tilde{\in} BOS(\chi)$ s.t $v^e \tilde{\in} (D, \hat{E}) \tilde{\subseteq} (K, \hat{E})$.

(ii) b-closure $soft$ points for the $(K, \hat{E})$ when $(K, \hat{E}) \tilde{\cap} (D, \hat{E}) \neq \tilde{\emptyset}$ for all $(D, \hat{E}) \tilde{\in} BOS(\chi)$.

Theorem 2.9: [6]

i. $bSint(\tilde{\chi}) = \tilde{\chi}, bSint(\tilde{\phi}) = \tilde{\phi}$ and $bScl(\tilde{\chi}) = \tilde{\chi}, bScl(\tilde{\phi}) = \tilde{\phi}$.

ii. $bSint((\mathcal{M}, \hat{E})) \tilde{\subseteq} (\mathcal{M}, \hat{E})$ & $(\mathcal{M}, \hat{E}) \subseteq bScl((\mathcal{M}, \hat{E}))$.

iii. $bSint\left((\mathcal{M}, \hat{E})\right) \tilde{\cup} bSint((D, \hat{E})) \tilde{\subseteq} bSint((\mathcal{M}, \hat{E}) \tilde{\cup} (D, \hat{E}))$ and $bScl\left((\mathcal{M}, \hat{E})\right) \tilde{\cup} bScl((D, \hat{E})) \tilde{\subseteq} bScl((\mathcal{M}, \hat{E}) \tilde{\cup} (D, \hat{E}))$.

iv. $bSint((\mathcal{M}, \hat{E}) \tilde{\cap} (D, \hat{E})) \tilde{\subseteq} bSint((\mathcal{M}, \hat{E})) \tilde{\cap} bSint((D, \hat{E}))$ and $bScl((\mathcal{M}, \hat{E}) \tilde{\cap} (D, \hat{E})) \tilde{\subseteq} bScl((\mathcal{M}, \hat{E})) \tilde{\cap} bScl((D, \hat{E}))$.

3. $b\tau_i$ $-soft$ topological $soft$ group

Definition 3.1: [2] If $(K, \hat{E})$ the $soft$ groups on G & τ the $soft$ topology on G, so $(G, \tau, K, \hat{E})$ named soft topological soft groups (Sţsg) if $(K(e), \tau_{K(e)})$" is a topological group on "$K(e)$ $\forall$ $e \in \hat{E}$, where $\tau_{K(e)}$" is the relativized topology on "$K(e)$ induced from τ_e."

Example 3.2: [7, 8] Let $G = \{i_G, (13), (23), (12), (123), (132)\}$ "be the group of permutations" on $\{1,2,3\}$, $\hat{E} = \{g_1, g_2, g_3\}$ and $(K, \hat{E})$ the soft sets, where $K(g_1) = \{i_G, (12)\}$, $K(g_2) = \{i_G, (13)\}$, $K(g_3) = \{i_G\}$.

Let "$\tau = \{\widetilde{\emptyset}, \widetilde{G}, (\mathcal{H}_1, \hat{E}), (\mathcal{H}_2, \hat{E}), (\mathcal{H}_3, \hat{E})\}$", where:

$$\mathcal{H}_1(g_1) = \{i_G\}, \mathcal{H}_1(g_2) = \{i_G\}, \mathcal{H}_1(g_3) = G.$$

$$\mathcal{H}_2(g_1) = \{(12)\}, \mathcal{H}_2(g_2) = \{(13)\}, \mathcal{H}_2(g_3) = G.$$

$$\mathcal{H}_3(g_1) = \{i_G, (12)\}, \mathcal{H}_3(g_2) = \{i_G, (13)\}, \mathcal{H}_3(g_3) = G.$$

It is clearly that $(K, \hat{E})$ is $soft$ group over G.

Now:

$$\tau_{g_1} = \{i_G, \{(12)\}, \{i_G, (12)\}, \emptyset, G\}, \tau_{\mathcal{F}_{(g_1)}} = \{i_G, \{(12)\}, \{i_G, (12)\}, \emptyset\}.$$

$$\tau_{g_2} = \{i_G, \{(13)\}, \{i_G, (13)\}, \emptyset, G\}, \tau_{K_{(g_2)}} = \{i_G, \{(13)\}, \{i_G, (13)\}, \emptyset\}.$$

$$\mathrm{T}_{e_3} = \{\emptyset, G\}, \tau_{K_{(g_3)}} = \{i_G, \emptyset\}.$$

Then $(G, \tau, K, \hat{E})$ is Sţsg.

Definition 3.3: The (Stsg) $(G, \tau, K, \hat{E})$ named $b\tau_0$ $-$Stsg iff $(K(e), \tau_{\mathcal{F}(e)})$ is $b\tau_0$ $-$space for all $e \in \hat{E}$."

Example 3.4: If $G = (Z, +), \hat{E} = \{e_1, e_2\}, \tau$ the discrete $soft$ topologies on G & $(K, \hat{E})$ the $soft$ sets where: $K(e_1) = \{0\}, K(e_2) = Z_e$ the set of even integer numbers.

So, $(K, \hat{E})$ the $soft$ group over G.

$\tau_{e_1} = \tau_{e_2} = \tau_e = \{S: S \subseteq Z\}, \tau_{K(e_1)} = \{\emptyset, \{0\}\}$, then $(K(e_1), \tau_{\mathcal{F}(e_1)})$ is a topological group.

$\tau_{K(e_2)} = \{S: S \subseteq Z_e\}$, then $(K(e_2), \tau_{K(e_2)})$ is a topological group.

Therefor $(G, \tau, K, \hat{E})$ be Stsg.

Since $(K(e_1), \tau_{K(e_1)})$& $(K(e_2), \tau_{K(e_2)})$ are $b\tau_0$ $-$space, then $(G, \tau, K, \hat{E})$is $b\tau_0$ $-$Stsg.

Proposition 3.5: Let $(G, \tau, K, \hat{E})$ be a Stsg s.t $(K(e), \tau_e)$ is $b\tau_0$ $-$space for all $e \in \hat{E}$, then $(G, \tau, K, \hat{E})$ is $b\tau_0$ $-$Stsg.

Proof: Let $(G, \tau, K, \hat{E})$ be a Stsg such that $(K(e), \tau_e)$ is $b\tau_0$ $-$space.

"Since the property of $b\tau_0$ −space is hereditary, then $(K(e), \tau_{K(e)})$ is $b\tau_0$ −space for all $e \in \hat{E}$, therefore $(G, \tau, K, \hat{E})$ is $b\tau_0$ −Stsg".

The converse of (3.5) does not always have to be true, as below:

Example 3.6: Let $G = (Z_2, +_2), \hat{E} = \{e_1, e_2\}$ & $\tau = \{\emptyset, G\}$, $K(e_1) = \{0\} = K(e_2)$, then $(K, \hat{E})$ is soft group over G".

$\tau_{e_1} = \tau_{e_2} = \{\emptyset, Z_2\}$, $\tau_{K(e_1)} = \tau_{K(e_2)} = \{\emptyset, \{0\}\}$, then $(K(e_1), \tau_{K(e_1)})$ and $(K(e_2), \tau_{K(e_2)})$ are topological groups.

Since $(K(e_1), \tau_{K(e_1)})$ and $(K(e_2), \tau_{K(e_2)})$ are $b\tau_0$ −spaces, then $(G, \tau, K, \hat{E})$ is $b\tau_0$ −Stsg, but $(K(e), \tau_e)$ isn't $b\tau_0$ −space.

Definition 3.7: A Stsg $(G, \tau, K, \hat{E})$ is called $b\tau_1$ −Stsg iff $(K(e), \tau_{K(e)})$ is $b\tau_1$ −space for all $e \in \hat{E}$.

Example 3.8: If $G = (R, +), \hat{E} = \{e_1, e_2\}$ and τ be a discrete $soft$ topology on G, $R, K(e_1) = \{0\} = K(e_2)$, then $(K, \hat{E})$ is $soft$ group over G.

$\tau_{e_1} = \tau_{e_2} = \{\mathcal{M}: \mathcal{M} \subseteq R\}$. $\tau_{K(e_1)} = \{\mathcal{M}: \mathcal{M} \subseteq R\}$, then $(K(e_1), \tau_{K(e_1)})$ is a topological group and $\tau_{K(e_2)} = \{\emptyset, \{0\}\}$, then $(K(e_2), \tau_{K(e_2)})$ is a topological group. Therefor $(G, \tau, K, \hat{E})$ be Stsg.

Since $(K(e), \tau_{K(e)})$ is $b\tau_1$ −space $\forall\, e \in \hat{E}$, then $(G, \tau, K, \hat{E})$ is $b\tau_1$ −Stsg.

Definition 3.9: A Stsg $(G, \tau, K, \hat{E})$ is called $b\tau_2$ −Stsg iff $(K(e), \tau_{K(e)})$ is $b\tau_2$ −space $\forall\, e \in \hat{E}$."

Proposition 3.10: Let $(G, \tau, K, \hat{E})$ be a Stsg where $(K(e), \tau_e)$ is $b\tau_2$ −space $\forall e \in \hat{E}$, then $(G, \tau, K, \hat{E})$ is $b\tau_2$ −Stsg".

Proof: Let $(G, \tau, K, \hat{E})$ be a Stsg such that $(K(e), \tau_e)$ is $b\tau_2$ −space, then $(K(e), \tau_{K(e)})$ is $b\tau_2$ −space $\forall e \in \hat{E}$, hence $(G, \tau, K, \hat{E})$ is $b\tau_2$ −Stsg.

The conversely of (3.10) does not always have to be true as below:

Example 3.11: If we have $G = (Z_8, +_8), \hat{E} = \{e_1, e_2\}$, & $(K, \hat{E})$ the $soft$ set s.t $K(e_1) = \{0\}, K(e_2) = \{0,4\}$, then $(K, \hat{E})$ is $soft$ group over G.

Let $\tau = \{\widetilde{\emptyset}, \widetilde{Z_8}, (\mathcal{M}_1, \hat{E}), (\mathcal{M}_2, \hat{E}), (\mathcal{M}_3, \hat{E}), (\mathcal{M}_4, \hat{E})\}$, where:

$$\mathcal{M}_1(e_1) = \{0\}, \mathcal{M}_1(e_2) = \{0\}$$

$$\mathcal{M}_2(e_1) = \{0,2,4,6\}, \mathcal{M}_2(e_2) = \{0,1,4\}$$

$$\mathcal{M}_3(e_1) = \{4\}, \mathcal{M}_3(e_2) = \{4\}$$

$$\mathcal{M}_4(e_1) = \{0,4\}, \mathcal{M}_4(e_2) = \{0,4\}$$

$$\tau_{e_1} = \{Z_8, \emptyset, \{0\}, \{0,2,4,6\}, \{4\}, \{0,4\}\}. \ \tau_{K(e_1)} = \{\{0\}, \emptyset\},$$

$\tau_{e_2} = \{Z_8, \emptyset, \{0\}, \{0,1,4\}, \{4\}, \{0,4\}\}$. $\tau_{K(e_2)} = \{\emptyset, \{0\}, \{4\}, \{0,4\}\}$, then $(K(e_1), \tau_{K(e_1)})$ and $(K(e_2), \tau_{K(e_2)})$ are topological groups. Therefor $(G, \tau, K, \hat{E})$ be Stsg.

"We can see that $(K(e), \tau_{K(e)})$ is $b\tau_2$ −space $\forall e \in \hat{E}$, but $(G, \tau, K, \hat{E})$ isn't $b\tau_2$ −Stsg".

Proposition 3.12: Every $b\tau_1$ −Stsg is $b\tau_0$ −Stsg.

Proof: Suppose $(G, \tau, K, \hat{E})$ be $b\tau_1$ −Stsg, then $(K(e), \tau_{K(e)})$ is $b\tau_1$ −space $\forall e \in \hat{E}$

Then $(K(e), \tau_{K(e)})$ is $b\tau_0$ −space $\forall e \in \hat{E}$

Hence $(G, \tau, K, \hat{E})$ be $b\tau_0$ −Stsg.

Proposition 3.13: Every $b\tau_2$ −Stsg is $b\tau_1$ −Stsg.

Proof: Suppose $(G, \tau, K, \hat{E})$ be $b\tau_2$ −Stsg, then $(K(e), \tau_{K(e)})$ is $b\tau_2$ −space $\forall e \in \hat{E}$

Then $(K(e), \tau_{K(e)})$ is $b\tau_1$ −space $\forall e \in \hat{E}$

Hence $(G, \tau, K, \hat{E})$ be $b\tau_1$ −Stsg.

Theorem 3.14: *Every $b\tau_2$ −Stsg is $b\tau_0$ −Stsg.*

Proof: By ***proposition*** (3.13) and (3.12).

Definition 3.15: A Stsg $(G, \tau, K, \hat{E})$ is called b −regular Stsg iff $(K(e), \tau_{K(e)})$ is b −regular space $\forall e \in \hat{E}$.

Proposition 3.16: A Stsg $(G, \tau, K, \hat{E})$ is b −regular if $(K(e), \tau_e)$ is b − regular spaces $\forall e \in \hat{E}$.

Proof: If $(G, \tau, K, \hat{E})$ the Stsg and $(K(e), \tau_e)$ is b −regular space $\forall\, e \in \hat{E}$, then $(K(e), \tau_{K(e)})$ is b −regular space $\forall\, e \in \hat{E}$, hence $(G, \tau, K, \hat{E})$ is b −regular Stsg".

The converse of (3.16) does not always have to be true, as below:

Example 3.17: Let $G = (Z_8, +_8), \hat{E} = \{e_1, e_2, e_3\}$, let $(K, \hat{E})$ be a $soft$ set where $K(e_1) = \{0\}, K(e_2) = K(e_3) = \{0,4\}$, then $(K, \hat{E})$ is $soft$ group over G.

Let $\tau = \{\widetilde{\emptyset}, \widetilde{Z_8}, (\mathcal{M}_1, \hat{E}), (\mathcal{M}_2, \hat{E}), (\mathcal{M}_3, \hat{E}), (\mathcal{M}_4, \hat{E})\}$, where:

$$\mathcal{M}_1(e_1) = \{0\}, \mathcal{M}_1(e_2) = \{0\}\ \mathcal{M}_1(e_3) = \{0\}$$

$$\mathcal{M}_2(e_1) = \{0,2,4,6\}, \mathcal{M}_2(e_2) = \{0,1,4\}\ \mathcal{M}_2(e_3) = Z_8$$

$$\mathcal{M}_3(e_1) = \{4\}, \mathcal{M}_3(e_2) = \{4\}\ \mathcal{M}_3(e_3) = \{4\}$$

$$\mathcal{M}_4(e_1) = \{0,4\}, \mathcal{M}_4(e_2) = \{0,4\}\ \mathcal{M}_4(e_3) = \{0,4\}$$

$$\tau_{e_1} = \{Z_8, \emptyset, \{0\}, \{0,2,4,6\}, \{4\}, \{0,4\}\}. \tau_{K(e_1)} = \{\{0\}, \emptyset\},$$

$\tau_{e_2} = \{Z_8, \emptyset, \{0\}, \{0,1,4\}, \{4\}, \{0,4\}\}$. $\tau_{K(e_2)} = \{\emptyset, \{0\}, \{4\}, \{0,4\}\}$

$\tau_{e_3} = \{Z_8, \emptyset, \{0\}, \{4\}, \{0,4\}\}$. $\tau_{K(e_3)} = \{\emptyset, \{0\}, \{4\}, \{0,4\}\}$

Then $(K(e_i), \tau_{K(e_i)})$ is a topological group $\forall\, e \in \hat{E}$. Therefor $(G, \tau, K, \hat{E})$ be b −regular Stsg, but τ_e be not b −regular space $\forall\, e \in \hat{E}$.

4. Discussion and Conclusion

We introduced a concept for the soft topology soft groups. and studied the relationships between them, such as:

1) Every $b\tau_1$ −Stsg is $b\tau_0$ −Stsg.
2) Every $b\tau_2$ −Stsg is $b\tau_1$ −Stsg.
3) Every $b\tau_2$ −Stsg is $b\tau_0$ −Stsg.

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