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Separation axioms via ideal in R_{i^*}-open sets

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Abstract

In this study, a novel category of sets, termed R_{i^*}-open sets are presented. Within the context of ideal "topological spaces", Different types of type R_{i^*}-Continuous functions are also presented, and the relationships between them are studied. This also includes the introduction and study of generalizations of the concept of separation axioms within topological spaces, using the concept of open sets of type R_{i^*} to generalize new classes of separation axioms within ideal topological sp. An examination of these types and their relationships is conducted, and some properties of these concepts are examined.

Subject Classification: *54A05, 54D10.*

Keywords: *Ideal topological space, R_{i^*}-open set, Separation axioms, R_{i^*}-T_0-space.*

1. Introduction

The author is considered the first to formulate the concept of 'ideal', which later became one of the foundational pillars of set theory [1]. An ideal in a t-space It is a non-empty set of subsets in the year 1945 [1]. A study on a sets operator was introduced () $*: P(X) \rightarrow P(X)$ Reportedly operates locally at F via T In their study, the authors explored topological properties through the framework of ideals [2-3]. Using the concept of an ideal, several researchers have investigated a new class of separation axioms and putted the notion αg Ị-open to develop new subclasses of separation axioms in ideal spaces. The newly introduced subclasses are; αg Ị-T 0-sp., αg Ị-T 1-space, αg Ị-T 2-space. Furthermore, new Notions of convergence in ideal spaces are developed by

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means of the αg Ị-open. [4]. Further studies progressed following the introduction of nano soft topology, leading to the emergence of various types of open sets within ideal topological space [5-6]. Moreover, the soft topological properties were extended to both soft ideal topological spaces and Nano soft topological structure [7-9]. Furthermore, applications on ideal topological spaces were explored, and concepts such as separation axioms and connectedness were introduced [10-12]. Many properties of Nano ideal spaces are studied also new classes of sets are defined and many or the properties of these sets were discussed so other studies were developed on these sets and their topological properties based on the concept of the ideal [13-15]. In addition, research has been conducted on a new class of topological which is finer than [16-17]. There are many studies conducted to study topological concepts using the Grill concept, which is the dual of the ideal. And studies Separation axioms with grill-topological open set, Also On grill-semi-P-separation axioms [18-19].

2. Definitions

Definition 2.1: For any ideal top. spaces (Ҳ, Ƭ, Ị), a subset A of the Ҳ is known to be R − open set via ideal and denoted by R_{i^*}- open set if ∃ H ∈ Ţ − {Ҳ, ∅} S. T, A − CL^* (A ∩ H) ∈ Ị , which complement of R_{i^*}- open sets is described as R_{i^*}- closed sets.

Definition 2.2: A function Ƒ: (Ҳ, Ƭ, Ị) ⟶ (Ƴ, ţ, Ï) is regarded as:

i. $\boldsymbol{R_{i^*}}$ -Open function, if for every Ǔ ∈ $\boldsymbol{R_{i^*}}$O(Ҳ), then Ƒ(Ǔ) ∈ $\boldsymbol{R_{i^*}}$O(Ƴ).
ii. $\boldsymbol{R_{i^*}}'$- open function, if for every Ǔ ∈ Ƭ, then Ƒ(Ǔ) ∈ $\boldsymbol{R_{i^*}}$O (Ƴ).
iii. $\boldsymbol{R_{i^*}}''$ −open function, if for every Ǔ ∈ $\boldsymbol{R_{i^*}}$O(Ҳ)), then Ƒ(Ǔ) ∈ ţ.

3. Separation Axioms Via Ideal in $\boldsymbol{R_{i^*}}$ - Open Sets

This study aims to construct generalizations of separation axioms of topological spaces by using the concept of R_{i^*} - Open sets, leading to the introduction of new classes of separation axioms of ITSP.

Definition 3.1: The (ITSP) (Ҳ, Ƭ, Ị) is referred to as R_{i^*}-T_0-space iff for any two distinct point $\sigma_1 \neq \sigma_2$ in X, there is an R_{i^*}-O set contains one of the elements while excluding the other.

Proposition 3.2: If (Ҳ, Ƭ) is a $\mathbf{T_0}$ –sp. then (Ҳ, Ƭ, Ị) is an $\boldsymbol{R_{i^*}}$-$\mathbf{T_0}$-sp.

Proof: Consider two distinct elements σ_1 and σ_2 in X. Since (X, Ţ) is a $\mathbf{T_0}$-sp, ∃Ǔ which contains exactly one of the two elements but not the other. By Definition 3.1 then Ư represents a $\boldsymbol{R_{i^*}}$O. set that contains one of the two elements but not the other, thus (X, Ţ, Ị) is an $\boldsymbol{R_{i^*}}$-$\mathbf{T_0}$-sp.

In general, the converse of Proposition (3.3) fails, as illustrated by the as described below ex.

Example 3.3: In the (ITSP) (X, Ƭ, Ị), let X= $\{\sigma_1, \sigma_2, \sigma_3\}$, Ţ= {X, $\{\sigma_1\}$, $\{\sigma_2, \sigma_3\}$, ϕ}, Į= {ϕ, $\{\sigma_1\}$}, then $\boldsymbol{R_{i^*}}$O(X) = ℙ(X). notice that (X, T, Į) is an $\boldsymbol{R_{i^*}}$-Ţ$_0$-space but not $\mathbf{T_0}$-space since $\sigma_2 \neq \sigma_3$ such that there is no open set U that contains exactly one of them only.

Definition 3.4: The (ITSP) (Ẋ, T, Į) is known as $\boldsymbol{R_{i^*}}$ -$\mathbf{T_1}$-space iff for all pair in distinct points σ_1 , σ_2 in X, there two sets exist Ц, V which are $\boldsymbol{R_{i^*}}$-O, for which $\sigma_1 \in$ (Ц − V) together with $\sigma_2 \in$ (V − Ц).

Example 3.5: In the ITSP (Ẋ, T, Į), let Ẋ= $\{\sigma_1, \sigma_2, \sigma_3\}$, T= {Ẋ, $\{\sigma_1\}$, $\{\sigma_2, \sigma_3\}$, ϕ}, Į={$\boldsymbol{\phi}$, $\{\sigma_2\}$, $\{\sigma_3\}$, $\{\sigma_2, \sigma_3\}$}, then $\boldsymbol{R_{i^*}}$O(X)= $\mathcal{P}$(X). Therefore (Ẋ, T, Į) is an $\boldsymbol{R_{i^*}}$- $\mathbf{T_1}$-sp.

Proposition 3.6: If (Ẋ, T) is $\mathbf{T_1}$-sp. then (Ẋ, T, Į) is an $\boldsymbol{R_{i^*}}$-$\mathbf{T_1}$-sp.

Proof: Consider σ_1 and σ_2 are two different elements of Ẋ, since (Ẋ, T) is a Ţ$_1$-space, Consequently, there exist two sets. Ц and V, s.t $\sigma_1 \in$ (Ц−V) and $\sigma_2 \in$ (V−Ц). By Proposition (3.2), then Ц and V are $\boldsymbol{R_{i^*}}$- open sets whenever $\sigma_1 \in$ (Ц−V) and $\sigma_2 \in$ (V−Ц), thus (Ẋ, T, Į) is a $\boldsymbol{R_{i^*}}$-$\mathbf{T_1}$-sp. the reverse implication of Proposition, in general, not true refer to Example (3.3).

Proposition 3.7: If (Ẋ, T, Į) is an $\boldsymbol{R_{i^*}}$-$\mathbf{T_1}$-sp. consequently $\boldsymbol{R_{i^*}}$-$\mathbf{T_0}$-space.

Proof: Consider that σ_1 and σ_2 elements that are not equal in Ẋ, since (Ẋ, T, Į) is an $\boldsymbol{R_{i^*}}$-$\mathbf{T_1}$-sp., it follows that there ∃ Ц and V are two $\boldsymbol{R_{i^*}}$ - open sets s.t $\sigma_1 \in$ (Ц − V), $\sigma_2 \in$ (V − Ц). Then there is $\boldsymbol{R_{i^*}}$- open set Ц containing exactly one of them consequently (Ẋ, T, Į) is an $\boldsymbol{R_{i^*}}$-$\mathbf{T_0}$-sp.

Generally, the reverse implication of Prop. (3.7) is not necessarily valid, as illustrated the ex. below.

Example 3.8: In the (X, Ţ, Į), let X= $\{\sigma_1 , \sigma_2, \sigma_3\}$, Ţ= {X, $\{x_1\}$, ϕ}, Į= {$\{\sigma_2\}$, ϕ}. So, $\boldsymbol{R_{i^*}}$O(X) = {X, $\{\sigma_1\}$, $\{\sigma_2\}$, $\{\sigma_1, \sigma_2\}$, $\{\sigma_1, \sigma_3\}$, ϕ}, hence (X, Ţ, Į) is an $R_{?}$-$\mathbf{T_0}$-sp. but is not $\boldsymbol{R_{i^*}}$-Ţ$_1$-space, since $\sigma_1, \sigma_3 \in$ X and $\sigma_1 \neq \sigma_3$ but there are not two $\boldsymbol{R_{i^*}}$-O sets, U and V s.t $\sigma_1 \in$ (U − V), $\sigma_3 \in$ (V − U).

Definition 3.9: The ITSP (Ẋ, T, Į) is referred to as an $\boldsymbol{R_{i^*}}$-Ţ$_2$-sp. if for any are two distinct elements $\sigma_1 \neq \sigma_2$ in Ẋ, two mutually disjoint, $\boldsymbol{R_{i^*}}$- open sets Ц and V, wherever $\sigma_1 \in$ Ц, $\sigma_2 \in$ V, and Ц ∩ V = ϕ.

Proposition 3.10: If (Ẋ, T) is a $\mathbf{T_2}$-sp .It follows that (Ẋ, T, Į) is an $\boldsymbol{R_{i^*}}$-$\mathbf{T_2}$-sp.

Proof: We assume that σ_1 and σ_2 are two different elements of Ẋ, since (Ẋ, T) is a $\mathbf{T_2}$-sp., Consequently, there are two open sets Ц, V, s.t $\sigma_1 \in$ Ц, $\sigma_2 \in$ Vand Ц ∩ V = ϕ, by Proposition (3.2), then Ц and V are $\boldsymbol{R_{i^*}}$- open sets satisfy the condition, $\sigma_1 \in$ Ц and $\sigma_2 \in$ V and Ц∩ V= ϕ, so (Ẋ, T, Į) is an $\boldsymbol{R_{i^*}}$-$\mathbf{T_2}$-space.

Proposition 3.11: If the ITSP. (Ẋ, T, Į) is an $\boldsymbol{R_{i^*}}$-$\mathbf{T_2}$-sp .then it is an $\boldsymbol{R_{i^*}}$-T$_1$-sp.

Proof: Let us assume that σ_1 and σ_2 be two different elements of X whenever $\sigma_1 \neq \sigma_2$, inasmuch as (X, T, I) is a $\boldsymbol{R_{i^*}}$-T_2-sp., we can find U, V are $\boldsymbol{R_{i^*}}$- open s.t that $\sigma_1 \in$ U and $\sigma_2 \in$ V and U $\cap$ V= ϕ, we can find $\boldsymbol{R_{i^*}}$-open sets U and V, meet a condition $\sigma_1 \in$ (U $-$ V) and $\sigma_2 \in$ (V $-$ U). So (X, T, I) is an $\boldsymbol{R_{i^*}}$-T_1-sp.

Remark 3.12: Take (X, T) is a $\mathbf{T}_i$-sp. for each i= {0, 1, 2}, then the ITSP (X, T, I) is an $\boldsymbol{R_{i^*}}$-T_i-sp. ∀i= {0, 1, 2}. In general, the converse claim in Remark 3.12 is not necessarily true, As demonstrated by the ex. below.

Example 3.14: Let X= $\{\sigma_1, \sigma_2, \sigma_3\}$, T= $\{X, \{\sigma_3\}, \phi\}$, and I= $\mathcal{P}$(X), consequently $\boldsymbol{R_{i^*}}$O(X)= $\mathcal{P}$(X). It is apparent that (X, T, I) is a $\boldsymbol{R_{i^*}}$-$\mathbf{T}T_i$-space for every i= {0, 1, 2}, which is not T_i-sp. for every i= {0. 1. 2}, As shown in Figure 1.

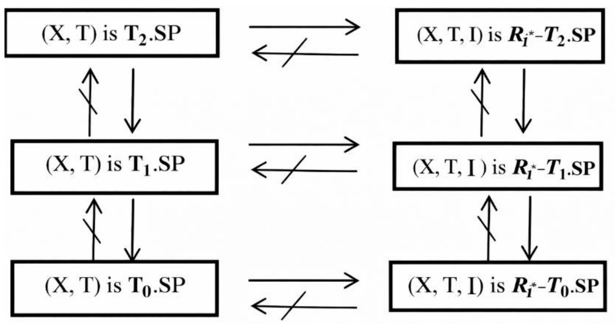

Figure 1
It explains the relationships between the concepts above

Conclusion

In this work, new R_{i^*}-open sets and related R_{i^*}-continuous functions were introduced in ideal topological spaces. Moreover, generalized separation axioms based on these sets were studied, and several relationships and properties were established.

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