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New classes of soft generalized mappings and their applications in soft separation axioms and group theory

Muthanna Mishlish *
Abdulsalam F. Talak †
A. M. F. A. Jumaili §
Department of Mathematics
College of Education for Pure Sciences
University of Anbar
Anbar
Iraq

Abstract

Study of soft set theory is extremely vital of possible implementations in the classical and nonclassical logic. Therefore, in the present study, new classes of soft generalized mappings namely; soft δ-ß-Continuous and δ-ß-Homeomorphism mappings have been introduced by employing another notion of generalized soft sets. Moreover, new structures of soft group theory and soft δ-ß-separation axioms are defined and discussed with assist of δ-ß-soft open sets. These types of generalized soft δ-ß-separation axioms would be beneficial for progress of soft topological spaces to solve the complicated issues containing uncertainties. Additionally, numerous applications related to these categories of soft generalized mappings in the soft group theory and soft δ-ß-separation axioms are discussed which may be of importance for further studies. Finally, we have shown the family of $\mathcal{S}\delta$ßr–(X, $\mathfrak{J}$, E) create the theory of soft group.

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Keywords: *Soft δ-ß-open, Soft δ-ß-homeomorphism, Soft δ-ß-continuity, Soft δ-ß-separation.*

1. Introduction

The theory of a soft set is a significant and powerful mathematical tool to study the uncertainty. It can be applied in many diverse fields of mathematical sciences include the game theory, smoothness of mappings, operations research,

* *E-mail:* **muthana.kh.m@uoanbar.edu.iq (Corresponding Author)**
† *E-mail:* **abd19u2007@uoanbar.edu.iq**
§ *E-mail:* **eps.alaamahmood.farhan@uoanbar.edu.iq**

algebra, measurement theory and other scientific fields. The theory of soft sets presented via Authors [1] as a novel of mathematical instrument to deal with uncertainties while modeling the issues with incomplete datums in physics, economic and medical sciences, as well he offered in his papers [1-2] essential outcomes of the original theory and successfully applied it to numerous trends. Additionally, demonstrated how theory of soft sets is free from the parametrization inadequacy syndrome of fuzzy set and game theory. Lately, numbers of researchers have been written regarding theory of soft sets and their implementations in different areas [3-4]. The authors [5] utilized soft sets in multicriteria decision making issues. After that, other authors [6] started the study of soft topology which is described over initial universe with fixed set of parameters. Worth mentioning authors [7] defined and characterized different structures of soft maps, such as semi-Continuous, irresolute, soft semi-open maps. For extra information about recent soft topological spaces indicate the authors to [8-11]. Motivated by above facts, the authors [12], investigated various kinds of soft separation axioms, θ-Continuity as well soft connectedness utilizing ordinary points of topology. Thereafter, other authors [13] defined the ideas of soft e-open as well e-closed sets and investigated some of their properties; also presented the categories of E-soft continuous, E-soft open, E-soft irresolute maps as well obtained some of their characterizations. Newly, novel idea of extended soft open and closed sets namely, soft-*δ-ß*-open with *δ-ß*-closed are defined via Al-Jumaili [4], in addition various essential properties related to these sorts of extended soft sets have been discussed.

Our major aim of present article is to introduce novel concepts of soft extended maps namely; soft *δ-ß*-Continuous as well soft *δ-ß*-Homeomorphism maps on soft topological spaces utilizing another notion of extended soft sets namely, soft *δ-ß*-open sets. Additionally, novel classes of extended soft δ-ß-separation axioms are defined and investigated. Also, various applications related to these kinds of soft extended maps in soft *δ-ß*-separation axioms are discussed. Finally, demonstrated that the family of $\mathcal{S}\delta\beta r - \hbar(X,\Im,E)$ form theory of a soft group.

2. Prerequisites

This section, recall next required conceptions and fundamental outcomes on soft sets theory and soft topology which will be used throughout this manuscript and play significant role in this study for verifying our outcomes.

Definition 2.1: [1] Suppose X is basic universe and $\not{E}$ is a collection of parameters, $\mathcal{P}(X)$ indicates the power of X and $\not{E} \supseteq \mathcal{H} \neq \emptyset$. $(\mathfrak{F}, \mathcal{H})$ soft set (Concisely, $\mathbb{S}$-s) over X, (s. t) $\mathfrak{F}$ a map given via $\mathfrak{F}: \mathcal{H} \longrightarrow \mathcal{P}(X)$ described via $\mathfrak{F}(\omega) \in \mathcal{P}(X)\ \forall\ \omega \in \mathcal{H}$. This means, a $\mathbb{S}$-s is parameterized collection of subsets of universe X. For $\omega \in \mathcal{H}, \mathfrak{F}(\omega)$ may be considered as ω-approximate elements of $\mathbb{S}$-s $(\mathfrak{F}, \mathcal{H})$, and if $\omega \notin \mathcal{H} \Longrightarrow \mathfrak{F}(\omega) = \emptyset$. This mean $\mathfrak{F}_{\mathcal{H}} = \{\mathfrak{F}(\omega): \omega \in \mathcal{H} \subseteq \mathbb{Q}, \mathfrak{F}: \mathcal{H} \longrightarrow \mathcal{P}(X)\}$. The collection of each $\mathbb{S}$-sets over $\mathbb{X}$ indicated via $\mathbb{SS}(X)_{\mathcal{H}}$.

Definition 2.2: [14] If $(\mathfrak{F},\mathcal{H})$ & $(\mathcal{F},\mathcal{M})$ are two $\mathbb{S}$-sets over common universe X. Therefore, $(\mathfrak{F},\mathcal{H}) \,\widetilde{\subseteq}\, (\mathcal{F},\mathcal{M})$ if $\mathcal{H} \subseteq \mathcal{M}$, & $\mathfrak{F}(\omega) \subseteq \mathcal{F}(\omega)\ \forall\ \omega \in \mathcal{H}$.

Definition 2.3: [6] Assume $\mathfrak{I}$ is the family of $\mathbb{S}$-sets, so $\mathfrak{I}$ is soft topology on X if it gratifies the next conditions:

($\boldsymbol{a}$) Φ & $\widetilde{X}$, $\in \mathfrak{I}$.

($\boldsymbol{b}$) Union of any element of $\mathbb{S}$-sets in $\mathfrak{I}$ belongs to $\mathfrak{I}$.

($\boldsymbol{c}$) Intersection of arbitrary two $\mathbb{S}$-sets in $\mathfrak{I}$ belongs to $\mathfrak{I}$.

Remark 2.4: (X, , $\mathfrak{I}$, E) is soft topological space (concisely, $\mathbb{S}$.T. S) over $\mathbb{X}$ and the members of $\mathfrak{I}$ are called $\mathbb{S}$-open sets in X.

Definition 2.5: [6] If (X, , $\mathfrak{I}$, E) is a ($\mathbb{S}$.T.S), $\mathbb{S}$-set $(\mathfrak{F},\mathcal{H})$ called $\mathbb{S}$-closed, *if* $(\mathfrak{F},\mathcal{H})^c \in \mathfrak{I}$.

Definition 2.6: [15] If (X, , $\mathfrak{I}$, E) is ($\mathbb{S}$.T.S) over X, then:

($\boldsymbol{a}$) The complement of $(\mathfrak{F},\mathcal{H})$, indicated via $(\mathfrak{F},\mathcal{H})^c$ is described via $(\mathfrak{F},\mathcal{H})^c = (\mathfrak{F}^c,\mathcal{H})$.

($\boldsymbol{b}$) The map $\mathfrak{F}^c\colon \mathcal{H} \longrightarrow \mathcal{P}(X)$ given via $\mathfrak{F}^c(\omega) = X - \mathfrak{F}(\omega)\ \forall\ \omega \in \mathcal{H}$ and $\mathfrak{F}^c$ is called $\mathbb{S}$-Complement map of $\mathfrak{F}$.

Definition 2.7: Assume (X, , $\mathfrak{I}$, E) is ($\mathbb{S}$.T.S) over X and $(\mathfrak{F},\mathcal{H})$ $\mathbb{S}$-set over X. So:

($\boldsymbol{a}$) $\mathbb{S}$ -Interior for $(\mathfrak{F},\mathcal{H})$ [16] is $\mathbb{S}$-s $Int((\mathfrak{F},\mathcal{H})) = \cup\{(\mathcal{D},\mathcal{H})\colon (\mathcal{D},\mathcal{H})$ $\mathbb{S}$-s & $(\mathcal{D},\mathcal{H}) \,\widetilde{\subset}\, (\mathfrak{F},\mathcal{H})\}$.

($\boldsymbol{b}$) $\mathbb{S}$ -Closure for $(\mathfrak{F},\mathcal{H})$ [6] is $\mathbb{S}$-s $Cl((\mathfrak{F},\mathcal{H})) = \cap\{(\mathfrak{F},E)\colon (\mathfrak{F},E)$ is $\mathbb{S}$-closed with $(\mathfrak{F},\mathcal{H}) \,\widetilde{\subset}\, (\mathfrak{F},E)\}$.

Definition 2.8: [4] $\mathbb{S}$-s $(\mathfrak{F},\mathcal{H})$ in ($\mathbb{S}$.T.S) called:

($\boldsymbol{a}$) $\mathbb{S}$ -δ-ß-open iff $(\mathfrak{F},\mathcal{H}) \,\widetilde{\subset}\, Cl(Int(Cl_\delta(\mathfrak{F},\mathcal{H})))$.

($\boldsymbol{b}$) $\mathbb{S}$-δ-ß-closed iff $(\mathfrak{F},\mathcal{H}) \,\widetilde{\supset}\, Cl(Int(Cl_\delta(\mathfrak{F},\mathcal{H})))$.

Definition 2.9: [4] Assume, (X, $\mathfrak{I}$, E) be ($\mathbb{S}$.T.S) &$(\mathfrak{F},\mathcal{H})$ be a $\mathbb{S}$-set over X. So:

($\boldsymbol{a}$) A $\mathbb{S}$-*δ*-*ß*-interior of a $\mathbb{S}$-*s* $(\mathfrak{F},\mathcal{H})$ of X is indicated via:

$\mathbb{S}\delta ß - Int(\mathfrak{F},\mathcal{H}) = \widetilde{\cup}\{(\mathbb{U},\mathcal{H})\} \,\widetilde{\subset}\, (\mathfrak{F},\mathcal{H})\colon (\mathbb{U},\mathcal{H})\}$ *is* $\mathbb{S}$-*δ*-*ß*-open of X.

($\boldsymbol{b}$) A $\mathbb{S}$-*δ*-*ß*-closure of ($\mathbb{S}$-set) $(\mathfrak{F},\mathcal{H})$ of X is indicated via*:*

$\mathbb{S}\delta ß - Cl(\mathfrak{F},\mathcal{H}) = \widetilde{\cap}\{(f,\mathcal{H})\} \,\widetilde{\supset}\, (\mathfrak{F},\mathcal{H})\colon (f,\mathcal{H})\}$ is a $\mathbb{S}$-*δ*-*ß*-closed of X.

Definition 2.10: [15] A $\mathbb{S}$-s $(\mathfrak{F},\mathcal{H})$ of ($\mathbb{S}$.T.S) (X, $\mathfrak{I}$, E) called:

($\boldsymbol{a}$) $\mathbb{S}$-Regular open if$(\mathfrak{F},\mathcal{H}) = Int(Cl(\mathfrak{F},\mathcal{H}))$.

($\boldsymbol{b}$) $\mathbb{S}$-Regular closed if$(\mathfrak{F},\mathcal{H}) = Cl(Int\ (\mathfrak{F},\mathcal{H}))]$.

Definition 2.11: [16] The $\mathbb{S}$-set $(\mathfrak{F},\mathcal{H})$ in ($\mathbb{S}$.T.S) (X, $\mathfrak{I}$, E) called $\mathbb{S}$-point of X, indicated via $\mathcal{P}_\alpha^{\mathfrak{F}}$, if for $\alpha \in \mathcal{H}$, $\mathfrak{F}(\alpha) \neq \emptyset$ & $\mathfrak{F}(\lambda) = \emptyset$, for $\lambda \notin \mathcal{H}$.

Definition 2.12: [13] A soft point $\mathcal{P}_\alpha^{\mathfrak{F}}$ in (𝕊.T.S) $(\mathrm{X}, \mathfrak{I}, \mathrm{E})$ is 𝕊-δ-cluster of 𝕊-s $(\mathcal{F},\mathcal{H})$ if ∀ 𝕊-Regular open $(\mathbb{U},\mathcal{H})$ containing $\mathcal{P}_\alpha^{\mathfrak{F}}$, $(\mathcal{F},\mathcal{H}) \widetilde{\sqcap} (\mathbb{U},\mathcal{H}) \neq \Phi$. Set of each 𝕊-δ-cluster of $(\mathcal{F},\mathcal{H})$ is said to be 𝕊-δ-closure of $(\mathcal{F},\mathcal{H})$ and is indicated via $[(\mathcal{F},\mathcal{H})]_\delta$ or $\mathbb{S}\mathrm{Cl}_\delta(\mathcal{F},\mathcal{H})$. A $\mathbb{S}-\delta$-interior of 𝕊-s $(\mathfrak{F},\mathcal{H})$ indicated via

$$\mathbb{S}-Int_\delta(\mathfrak{F},\mathcal{H}) = \left\{ \begin{matrix} \mathcal{P}_\alpha^{\mathfrak{F}} \in \mathrm{X}: for\ some\ \mathbb{S}-open\ subset\ (\mathcal{F},\mathcal{H}) of \\ \mathrm{X}, \mathcal{P}_\alpha^{\mathfrak{F}} \in (\mathcal{F},\mathcal{H}) \tilde{\subseteq} Int\big(Cl(\mathcal{F},\mathcal{H})\big) \tilde{\subseteq} (\mathfrak{F},\mathcal{H}) \end{matrix} \right\} \quad (1)$$

3. Some Applications of New Structures of Soft Separation Axioms and Soft Group Theory

This segment devoted to present novel extensions of soft generalized maps namely, 𝕊-δ-β-Continuous and 𝕊-δ-β-Homeomorphism maps utilizing another idea namely; 𝕊-δ-β-open-set, additionally novel structures of soft extended separation axioms are defined with assist of 𝕊-δ-β-open sets. Lastly, shown that the family of $\mathcal{S}\delta\beta r - \hbar\ (X, \mathfrak{I}, E)$ create 𝕊-group theory.

Definition 3.1: $\digamma: (\mathrm{X}, \mathfrak{I}_1, \mathrm{E}) \to (Y, \mathfrak{I}_2, K)$ is called 𝕊-δ-β-Continuous (concisely, 𝕊δ-β-Cont) when $\digamma^{-1}(\mathcal{F},\mathcal{H})$ is 𝕊-δ-β-open in X for each soft open $(\mathcal{F},\mathcal{H})$ in Y.

Example 3.2: Assume that $\mathrm{X} = Y = \{\ell_1, \ell_2, \ell_3\}$, $\mathrm{E} = \{c_1, c_2\}$ and let $\mathfrak{I}_1 = \{\Phi, \tilde{\mathrm{X}}, (\mathcal{F}_1, \mathrm{E}), (\mathcal{F}_2, \mathrm{E}), (\mathcal{F}_3, \mathrm{E})\}$ is 𝕊-topology defined on X and let $\mathfrak{I}_2 = \{\Phi, \tilde{Y}, (\mathcal{D}_1, \mathrm{E}), (\mathcal{D}_2, \mathrm{E})\}$ is soft topology on Y. In that case, $(\mathcal{F}_1, \mathrm{E}), (\mathcal{F}_2, \mathrm{E})$ & $(\mathcal{F}_3, \mathrm{E})$ are 𝕊-sets on X with $(\mathcal{D}_1, \mathrm{E})$ & $(\mathcal{D}_2, \mathrm{E})$ are 𝕊-sets on Y, described as follows: $\mathcal{F}_1(c_1) = \{\ell_1\}$,$\mathcal{F}_1(c_2) = \{\ell_1\}$, $\mathcal{F}_2(c_1) = \{\ell_2\}$, $\mathcal{F}_2(c_2) = \{\ell_2\}$, $\mathcal{F}_3(c_1) = \{\ell_1, \ell_2\}$, $\mathcal{F}_3(c_2) = \{\ell_1, \ell_2\}$ & $\mathcal{D}_1(c_1) = \{\ell_1\}$, $\mathcal{D}_1(c_2) = \{\ell_1\}$,$\mathcal{D}_2(c_1) = \{\ell_1, \ell_2\}$, $\mathcal{D}_2(c_2) = \{\ell_1, \ell_2\}$. Presume the mapping $\digamma: (\mathrm{X}, \mathfrak{I}_1, \mathrm{E}) \longrightarrow (Y, \mathfrak{I}_2, \mathrm{E})$ described as:$\digamma(\ell_1) = \ell_1$, $\digamma(\ell_2) = \ell_3$ with $\digamma(\ell_3) = \ell_2$, so $\digamma$ is 𝕊-δ-ß-Cont.

Definition 3.3: A (𝕊.T.S) $(\mathrm{X}, \mathfrak{I}, \mathrm{E})$ is called 𝕊-δ-ß-$\mathfrak{I}_1$-space if ∀ pair of different 𝕊-points $\mathcal{P}_\alpha^{\mathfrak{F}}$ & $\mathcal{P}_\eta^{\mathcal{G}}$ of X, ∃ 𝕊-δ-ß-open $(\mathbb{U},\mathcal{H})$ & $(\mathbb{V},\mathcal{M})$ (s. t) $\mathcal{P}_\alpha^{\mathfrak{F}} \in (\mathbb{U},\mathcal{H})$ & $\mathcal{P}_\eta^{\mathcal{G}} \in (\mathbb{V},\mathcal{M})$, $\mathcal{P}_\alpha^{\mathfrak{F}} \notin (\mathbb{V},\mathcal{M})$ & $\mathcal{P}_\eta^{\mathcal{G}} \notin (\mathbb{U},\mathcal{H})$.

Theorem 3.4: If $\digamma: (\mathrm{X}, \mathfrak{I}_1, \mathrm{E}) \longrightarrow (Y, \mathfrak{I}_2, \mathrm{E})$ is 𝕊-δ-ß-Cont injective map, Y is 𝕊-$\mathfrak{I}_1$-space, so X is 𝕊-δ-ß-$\mathfrak{I}_1$-space.

Proof: Presume Y is 𝕊-$\mathfrak{I}_1$-space. For arbitrary two different 𝕊-points $\mathcal{P}_\alpha^{\mathfrak{F}} * \mathcal{P}_\eta^{\mathcal{G}}$ of X, ∃ soft open $(\mathbb{U},\mathcal{H})$ & $(\mathbb{V},\mathcal{M})$ in Y where $\digamma(\mathcal{P}_\alpha^{\mathfrak{F}}) \in (\mathbb{U},\mathcal{H})$, $\digamma(\mathcal{P}_\eta^{\mathcal{G}}) \in (\mathbb{V},\mathcal{M})$ & $\digamma(\mathcal{P}_\alpha^{\mathfrak{F}}) \notin (\mathbb{V},\mathcal{M})$, $\digamma(\mathcal{P}_\eta^{\mathcal{G}}) \notin (\mathbb{U},\mathcal{H})$. As $\digamma$ injective𝕊-δ-ß-Cont map, get $\digamma^{-1}(\mathbb{U},\mathcal{H})$ & $\digamma^{-1}(\mathbb{V},\mathcal{M})$are 𝕊-δ-ß-open set in X. Thus, via definition 3-3, get X is 𝕊-δ-ß-$\mathcal{T}_1$-space.

Definition 3.5: A (𝕊.T.S) $(\mathrm{X}, \mathfrak{I}, \mathrm{E})$ is 𝕊-δ-ß-$\mathfrak{I}_2$ (𝕊-δ-ß-Hausdorff) space if ∀ pair of different 𝕊-points$\mathcal{P}_\alpha^{\mathfrak{F}}$ & $\mathcal{P}_\eta^{\mathcal{G}}$ in X, ∃ disjoint 𝕊-δ-ß-open $(\mathbb{U},\mathcal{H})$ & $(\mathbb{V},\mathcal{M})$ (s. t) $\mathcal{P}_\alpha^{\mathfrak{F}} \in (\mathbb{U},\mathcal{H})$ & $\mathcal{P}_\eta^{\mathcal{G}} \in (\mathbb{V},\mathcal{M})$.

Theorem 3.6: *If* $\digamma: (\mathrm{X}, \Im_1, \mathrm{E}) \longrightarrow (Y, \Im_2, \mathrm{E})$ *is* $\mathbb{S}$*-δ-ß-Cont injective map, Y is* $\mathbb{S}$*-*$\Im_2$*, so* X *is* $\mathbb{S}$*-δ-ß-*$\Im_2$*-space.*

Proof: Presume Y is $\mathbb{S}$-$\Im_2$-space. For arbitrary different $\mathbb{S}$-points $\mathcal{P}_\alpha^{\mathfrak{F}}$ & $\mathcal{P}_\eta^{\mathcal{G}}$ in X, there exists disjoint $\mathbb{S}$-open $(\mathbb{U}, \mathcal{H})$ & $(\mathbb{V}, \mathcal{M})$ of $\mathbb{Y}$ where $\digamma(\mathcal{P}_\alpha^{\mathfrak{F}}) \in (\mathbb{U}, \mathcal{H})$ & $\digamma(\mathcal{P}_\eta^{\mathcal{G}}) \in (\mathbb{V}, \mathcal{M})$. As $\digamma$ is $\mathbb{S}$-δ-ß-injective Cont, get $\digamma^{-1}(\mathbb{U}, \mathcal{H})$ & $\digamma^{-1}(\mathbb{V}, \mathcal{M})$ are disjoint $\mathbb{S}$-δ-ß-open in X. Thus, via definition 3-5, get X is $\mathbb{S}$-δ-ß-$\mathcal{T}_2$-space.

Definition 3.7: A $(\mathrm{X}, \Im, \mathrm{E})$ is $\mathbb{S}$-δ-ß-Regular if ∀ $\mathbb{S}$-closed sets $(f, \mathcal{H})$ in X and every $\mathbb{S}$-point $\mathcal{P}_\alpha^{\mathfrak{F}} \in \mathrm{X} - (f, \mathcal{H}), \exists$ disjoint $\mathbb{S}$-δ-ß-open $(\mathbb{U}, \mathcal{H})$ & $(\mathbb{V}, \mathcal{M})$ (s.t) $\mathcal{P}_\alpha^{\mathfrak{F}} \in (\mathbb{U}, \mathcal{H})$ & $(f, \mathcal{H}) \tilde{\subseteq} (\mathbb{V}, \mathcal{M})$.

Theorem 3.8: *Let* $\digamma: (\mathrm{X}, \Im_1, \mathrm{E}) \longrightarrow (Y, \Im_2, \mathrm{E})$ *be* $\mathbb{S}$*-δ-ß-Cont-closed injective map, Y is* $\mathbb{S}$*-Regular, therefore* X *is* $\mathbb{S}$*-δ-ß-Regular.*

Proof: Presume $(f, \mathcal{H})$ $\mathbb{S}$-closed of Y with $\mathcal{P}_\alpha^{\mathfrak{F}} \notin (f, \mathcal{H})$. Select $\mathcal{P}_\alpha^{\mathfrak{F}} = \digamma(\mathcal{P}_\alpha^{\mathfrak{F}})$. As, Y $\mathbb{S}$-Regular, ∃ disjoint $\mathbb{S}$-open $(\mathbb{U}, \mathcal{H})$ & $(\mathbb{V}, \mathcal{M})$ (s. t), $\mathcal{P}_\alpha^{\mathfrak{F}} \in (\mathbb{U}, \mathcal{H})$ with $\mathcal{P}_\alpha^{\mathfrak{F}} = \digamma(\mathcal{P}_\alpha^{\mathfrak{F}}) \in \mathbb{F}(\mathbb{U}, \mathcal{H})$ & $(f, \mathcal{H}) \tilde{\subseteq} \digamma(\mathbb{V}, \mathcal{M})$ (s. t), $\digamma(\mathbb{U}, \mathcal{H})$ & $\digamma(\mathbb{V}, \mathcal{M})$ disjoint $\mathbb{S}$-open sets. Consequently, obtain $\digamma^{-1}(f, \mathcal{H}) \tilde{\subseteq} (\mathbb{V}, \mathcal{M})$. Because $\digamma$ is $\mathbb{S}$-δ-ß-Cont, $\digamma^{-1}(f, \mathcal{H})$ is $\mathbb{S}$-δ-ß-closed, $\mathcal{P}_\alpha^{\mathfrak{F}} \notin \digamma^{-1}(f, \mathcal{H})$. Thus, by definition 3-7, X is $\mathbb{S}$-δ-ß-Regular.

Theorem 3.9: *If* $(f, \mathcal{H})$ *is* $\mathbb{S}$*-δ-ß-closed with* $\digamma: (\mathrm{X}, \Im_1, \mathrm{E}) \to (Y, \Im_2, \mathrm{E})$ *bijective,* $\mathbb{S}$*-Cont and* $\mathbb{S}$*-δ-ß-closed, so* $\digamma(f, \mathcal{H})$ *is* $\mathbb{S}$*-δ-ß-closed in* Y.

Proof: Presume $\digamma(f, \mathcal{H}) \tilde{\subseteq} (\mathcal{G}, \mathcal{M})$(s. t), $(\mathcal{G}, \mathcal{M})$ is $\mathbb{S}$-open in Y. As, $\digamma$ is $\mathbb{S}$-Cont, $\digamma^{-1}(\mathcal{G}, \mathcal{M})$ is $\mathbb{S}$-open including $(f, \mathcal{H})$.

Therefore, $\mathbb{S}\delta\beta - Cl(f, \mathcal{H}) \tilde{\subseteq} F^{-1}(\mathcal{G}, \mathcal{M})$ because $(f, \mathcal{H})$ is $\mathbb{S}$-*δ-ß*-closed. As F is $\mathbb{S}$-*δ-ß*-closed, $F(\mathbb{S}\delta\beta - Cl(f, \mathcal{H}))$ is $\mathbb{S}$-*δ-ß*-closed set included in the $\mathbb{S}$-open $(\mathcal{G}, \mathcal{M}), \Longrightarrow$ $\mathbb{S}\delta\beta - ClF(\mathbb{S}\delta\beta - Cl(f, \mathcal{H})) \tilde{\subseteq} (\mathcal{G}, \mathcal{M})$, for this reason $\mathbb{S}\delta\beta - ClF(f, \mathcal{H}) \tilde{\subseteq} (\mathcal{G}, \mathcal{M})$. So, $F(f, \mathcal{H})$ is $\mathbb{S}$-*δ-ß*-closed set in Y.

Definition 3.10: A ($\mathbb{S}$.T.S) $(\mathrm{X}, \Im, \mathrm{E})$ is called $\mathbb{S}$-δ-ß-Normal if for all distinct $\mathbb{S}$-closed $(f, \mathcal{H})$ and $(\mathcal{G}, \mathcal{M})$ in X, ∃ disjoint $\mathbb{S}$-δ-ß-open$(\mathbb{U}, \mathcal{H})$ and $(\mathbb{V}, \mathcal{M})$ (s.t), $(f, \mathcal{H}) \tilde{\subseteq} (\mathbb{U}, \mathcal{H})$ with $(\mathcal{G}, \mathcal{M}) \tilde{\subseteq}$ $(\mathbb{V}, \mathcal{M})$ & $(\mathbb{U}, \mathcal{H}) \tilde{\cap} (\mathbb{V}, \mathcal{M}) = \tilde{\varphi}$.

Theorem 3.11: *If* $\digamma: (\mathrm{X}, \Im_1, \mathrm{E}) \longrightarrow (Y, \Im_2, \mathrm{E})$ *is* $\mathbb{S}$*-δ-ß-Cont-closed injective map,Y is* $\mathbb{S}$*-Normal, so* X *is* $\mathbb{S}$*-δ-ß-Normal.*

Proof: Assume Y is $\mathbb{S}$-Normal, $(f, \mathcal{H})$ & $(\mathcal{G}, \mathcal{M})$ are $\mathbb{S}$-closed sets in X (s. t) $(f, \mathcal{H}) \tilde{\cap} (\mathcal{G}, \mathcal{M}) = \tilde{\emptyset}$. As $\digamma$ $\mathbb{S}$-closed injection, so $\digamma(f, \mathcal{H})$ & $\digamma(\mathcal{G}, \mathcal{M})$ are $\mathbb{S}$-closed,$\digamma(f, \mathcal{H}) \tilde{\cap} \digamma(\mathcal{G}, \mathcal{M}) = \tilde{\emptyset}$. Because Y is normal, ∃ $\mathbb{S}$-open $(\mathbb{U}, \mathcal{H})$ &

$(\mathbb{V},\mathcal{M})$ of Y (s. t) $\digamma(f,\mathcal{H}) \widetilde{\subseteq} \mathbb{U}$ & $\digamma(\mathcal{G},\mathcal{M}) \widetilde{\subseteq} \mathbb{V}$ with $\mathbb{U} \widetilde{\cap} \mathbb{V} = \widetilde{\emptyset}$. As a result, get , $(f,\mathcal{H}) \widetilde{\subseteq} \digamma^{-1}(\mathbb{U})$ & $(\mathcal{G},\mathcal{M}) \widetilde{\subseteq} \digamma^{-1}(\mathbb{V})$ with $\digamma^{-1}(\mathbb{U} \widetilde{\cap} \mathbb{V}) = \widetilde{\emptyset}$. As $\digamma$ is $\mathbb{S}$-δ-ß-Cont,$\digamma^{-1}(\mathbb{U})$ & $\digamma^{-1}(\mathbb{V})$ are $\mathbb{S}$-δ-ß-open. So via Def-3-10,X is $\mathbb{S}$-δ-ß-Normal.

Definition 3.12: The $\mathbb{S}$-subset$(f,\mathcal{H})$ in *($\mathbb{S}$.T.S)* (X, $\Im$, Ɇ) is $\mathbb{S}$-*δ-ß*-Connected (briefly,$\mathbb{S}$-*δ-ß*-Conn) $\Leftrightarrow$ $(f,\mathcal{H})$ not possible be state union of nonempty disjoint $\mathbb{S}$-*δ-ß*-open sets.

Theorem 3.13: If a mapping $\digamma$: (X, $\Im_1$, Ɇ) $\longrightarrow$ $(Y,\Im_2,$ Ɇ$)$ is a $\mathbb{S}$-*δ-ß*-Cont and surjection, $(\mathcal{G},\mathcal{H})$ is $\mathbb{S}$-*δ-ß*-Conn, so $\digamma(\mathcal{G},\mathcal{H})$ *is* $\mathbb{S}$-Conn.

Proof: Assume $\digamma(\mathcal{G},\mathcal{H})$ isn't $\mathbb{S}$-Conn. In that case, there exist a nonempty $\mathbb{S}$-open $(f,\mathcal{H})$ *&* $(\mathfrak{F},\mathcal{H})$ in Y (s. t), $\digamma(\mathcal{G},\mathcal{H}) = (f,\mathcal{H}) \widetilde{\cup} (\mathfrak{F},\mathcal{H})$. Since $\digamma$ *is* $\mathbb{S}$-*δ-ß*-Cont*,* so $\digamma^{-1}(f,\mathcal{H})$&$\digamma^{-1}(\mathfrak{F},\mathcal{H})$ are disjoint $\mathbb{S}$-*δ-ß*-open sets in X, with $(\mathcal{G},\mathcal{H}) = \digamma^{-1}[(f,\mathcal{H}) \widetilde{\cup} (\mathfrak{F},\mathcal{H})] = \digamma^{-1}(f,\mathcal{H})\widetilde{\cup}\, \digamma^{-1}(\mathfrak{F},\mathcal{H})$. It is apparent $\digamma^{-1}(f,\mathcal{H})$ and $\digamma^{-1}(\mathfrak{F},\mathcal{H})$ are $\mathbb{S}$-*δ-ß*-open of X, Consequently, $(\mathcal{G},\mathcal{H})$ isn't $\mathbb{S}$-*δ-ß*-Conn, is contradiction with assumption. Therefore, $\digamma(\mathcal{G},\mathcal{H})$ is $\mathbb{S}$-Conn.

Definition 3.14: A map $\digamma$: (X, $\Im_1$, Ɇ) → $(Y,\Im_2,$ Ɇ$)$ $\mathbb{S}$-δ-ß-Irresolute if $\digamma^{-1}(\mathcal{F},\mathcal{H})$ $\mathbb{S}$-δ-ß-open of (X, $\Im_1$, Ɇ) ∀ $\mathbb{S}$-δ-ß-open $(\mathcal{F},\mathcal{H})$ in $(Y,\Im_2,$ Ɇ$)$.

Theorem 3.15: *Every* $\mathbb{S}$*-δ-ß-Irresolute map is* $\mathbb{S}$*-δ-ß-Continuous.*

Proof: It is unpretentious, thus deleted.

Theorem 3.16: *If* $\digamma$: (X, $\Im_1$, Ɇ) $\longrightarrow$ $(Y,\Im_2,K)$ *is* $\mathbb{S}$*-δ-ß-Irresolute map and* $\mathbb{G}$: $(Y,\Im_2,K) \longrightarrow (\mathbb{Z},\Im_3,\mathcal{B})$ *be* $\mathbb{S}$*-δ-ß-Continuous map. So,* $\mathbb{G}o\digamma$: (X, $\Im_1$, Ɇ) $\longrightarrow$ $(\mathbb{Z},\Im_3,\mathcal{B})$ *is* $\mathbb{S}$*-δ-ß-Continuous map.*

Proof: It's clear and follows from their respective definitions.

Theorem 3.17: *If* $\digamma$: (X, $\Im_1$, Ɇ) $\longrightarrow$ $(Y,\Im_2,K)$ *and* $\mathbb{G}$: $(Y,\Im_2,K) \longrightarrow (\mathbb{Z},\Im_3,\mathcal{B})$ *are* $\mathbb{S}$*-δ-ß-irresolute maps. So,* $\mathbb{G}o\digamma$: (X, $\Im_1$, Ɇ) → $(\mathbb{Z},\Im_3,\mathcal{B})$ *is* $\mathbb{S}$*-δ-ß-Irresolute map.*

Proof: It's unpretentious.

Definition 3.18: A map $\digamma$: (X, $\Im_1$, Ɇ) $\longrightarrow$ $(Y,\Im_2,$ Ɇ$)$ is called:

(***a***) $\mathbb{S}$-δ-ß-Homeomorphism (Concisely, $\mathbb{S}$-δ-ß-Homeo) if $\mathbb{F}$ is $\mathbb{S}$-δ-ß-Continuous bijection map, $\digamma^{-1}(Y,\Im_2,$ Ɇ$)$ → (X, $\Im_1$, Ɇ) is $\mathbb{S}$-δ-ß-Cont.

(***b***) $\mathbb{S}$-δßr-Homeo if $\digamma$ is $\mathbb{S}$-δ-ß-Irresolute bijection, $\digamma^{-1}(Y,\Im_2,$ Ɇ$)$ $\longrightarrow$ (X, $\Im_1$, Ɇ) is $\mathbb{S}$-δ-ß-Irresolute.

Definition 3.19: With regards *($\mathbb{S}$.T.S)* (X, $\Im$, Ɇ), describe next families of maps:

(***a***) $\mathcal{S}\delta$ß $- \hbar$(X, $\Im$, Ɇ) = {$\digamma$ \ $\digamma$: (X, $\Im$, Ɇ) $\longrightarrow$ (X, $\Im$, Ɇ) is $\mathbb{S}$-δ-ß-Cont-bijection, $\digamma^{-1}$ (X, $\Im$, Ɇ) $\longrightarrow$ (X, $\Im$, Ɇ) is $\mathbb{S}$-δ-ß-Cont}.

(***b***) $\mathcal{S}\delta ß\mathscr{r} - \hbar(X, \Im, E) = \{ F \setminus F : (X, \Im, E) \longrightarrow (X, \Im, E)$ is a $\mathbb{S}$-δ-ß-Irresolute bijection map, $F^{-1}(X, \Im, E) \longrightarrow (X, \Im, E)$ is $\mathbb{S}$-δ-ß-Irresolute$\}$.

Theorem 3.20: *For* $(X, \Im, E)$, $\mathcal{S} - \hbar(X, \Im, E) \subseteq \mathcal{S}\delta ß\mathscr{r} - \hbar(X, \Im, E) \subseteq \mathcal{S}\delta ß - \hbar(X, \Im, E)$, *(s.t)* $\mathcal{S} - \hbar(X, \Im, E) = \{F \setminus F: (X, \Im, E) \longrightarrow (X, \Im, E)$ *is* $\mathbb{S}$*-Cont-bijection,* $F^{-1}(X, \Im, E) \longrightarrow (X, \Im, E)$*, is* $\mathbb{S}$*-Cont, that is* F *is* $\mathbb{S}$*-Homeo}.*

Proof: At the beginning explain that each $\mathbb{S}$-Homeo $F: (X, \Im_1, E) \longrightarrow (Y, \Im_2, E)$ is $\mathbb{S}$-δß$\mathscr{r}$-Homeo. Assume that $(\mathcal{G}, \mathcal{H}) \in \mathcal{S}\delta ß - OS(Y)$, for this reason $(\mathcal{G}, \mathcal{H}) \subseteq Cl(Int(Cl_\delta(\mathcal{G}, \mathcal{H})))$. So, $F^{-1}(\mathcal{G}, \mathcal{H}) \subseteq F^{-1}[Cl(Int(Cl_\delta(\mathcal{G}, \mathcal{H})))] = Cl(Int(Cl_\delta(F^{-1}(\mathcal{G}, \mathcal{H}))))$. Consequently $\mathbb{F}^{-1}(\mathcal{G}, \mathcal{H}) \in \mathcal{S}\delta ß - OS(X)$. As a result, F is $\mathbb{S}$-δ-ß-Irresolute. In an analogous method, we can verify F^{-1} is $\mathbb{S}$-δ-ß-Irresolute. Therefore, obtain, $\mathcal{S} - \hbar(X, \Im, E) \subseteq \mathcal{S}\delta ß\mathscr{r} - \hbar(X, \Im, E)$. Lastly, it's apparent $\mathcal{S}\delta ß\mathscr{r} - \hbar(X, \Im, E) \subseteq \mathcal{S}\delta ß - \hbar(X, \Im, E)$, since each $\mathbb{S}$-δ-ß-Irresolute map is $\mathbb{S}$-δ-ß-Cont.

Theorem 3.21: *If* $(X, \Im, E)$ *is a soft topological space,* $F: (X, \Im_1, E) \longrightarrow (Y, \Im_2, E)$ *and* $\mathbb{G}: (Y, \Im_2, E) \rightarrow (\mathbb{Z}, \Im_3, E)$ *are* $\mathbb{S}$*-δß*$\mathscr{r}$*-Homeo-maps, so the family* $\mathcal{S}\delta ß\mathscr{r} - \hbar(X, \Im, E)$ *creates a group under* $\mathbb{G}oF: (X, \Im_1, E) \longrightarrow (\mathbb{Z}, \Im_3, E)$.

Proof: Assume $F: (X, \Im_1, E) \longrightarrow (Y, \Im_2, E)$ & $\mathbb{G}: (Y, \Im_2, E) \longrightarrow (\mathbb{Z}, \Im_3, E)$ are $\mathbb{S}$-δß$\mathscr{r}$-Homeo, then $\mathbb{G}oF: (X, \Im_1, E) \longrightarrow (\mathbb{Z}, \Im_3, E)$ is $\mathbb{S}$-δß$\mathscr{r}$-Homeo. It is apparent for bijective $\mathbb{S}$-δß$\mathscr{r}$-Homeo $F: (X, \Im_1, E) \longrightarrow (Y, \Im_2, E), F^{-1}: (Y, \Im_2, E) \longrightarrow (X, \Im_1, E)$ is $\mathbb{S}$-δß$\mathscr{r}$-Homeo and $\mathcal{I}: (X, \Im_1, E) \longrightarrow (X, \Im_1, E)$ is $\mathbb{S}$-δß$\mathscr{r}$-Homeo.

A binary operation $Q: \mathcal{S}\delta\beta\mathscr{r} - \hbar(X, \Im, E) \times \mathcal{S}\delta\beta\mathscr{r} - \hbar(X, \Im, E) \longrightarrow \mathcal{S}\delta\beta\mathscr{r} - \hbar(X, \Im, E)$ is well defined via $Q(\mathcal{u}, \mathcal{v}) = \mathcal{v}o\mathcal{u}$, (s. t) $\mathcal{u}, \mathcal{v} \in \mathcal{S}\delta ß\mathscr{r} - \hbar(X, \Im, E)$ and $\mathcal{v}o\mathcal{u}$ is composition of $\mathcal{u}$ & $\mathcal{v}$. Utilizing above conditions, get $\mathcal{S}\delta\beta\mathscr{r} - \hbar(X, \Im, E)$ creates group under $\mathbb{G}oF: (X) \rightarrow (\mathbb{Z})$.

Theorem 3.22: *If* $(X, \Im, E)$ *is soft topological space, then a group* $\mathcal{S} - \hbar(X, \Im, E)$*, of each soft homeomorphism is sub-group of* $\mathcal{S}\delta ß\mathscr{r} - \hbar(X, \Im, E)$.

Proof: It is follows directly by utilizing Theorem (3.16 & 3.17).

5. Conclusion

For significance and powerful for theory of soft set as mathematical tool to study the uncertainty and to solving complex troubles in real life circumstances, the troubles in physics, economics, engineering and computer sciences, do not always include brittle information. As a result, cannot successfully utilize the classical procedure. Therefore, in this manuscript novel ideas of soft extended maps as, soft *δ-ß*-Continuous and *δ-ß*-Homomorphism are presented and investigated. Further $\mathbb{S}$-*δ-ß*-separation axioms were defined and discussed utilizing $\mathbb{S}$-δ-*ß*-open sets. As well, various applications related to these kinds of soft generalized mappings in $\mathbb{S}$-*δ-ß*-separation axioms are obtained. In the ending, we expect that the conclusions in this manuscript are beginning of novel structures and they will create theoretical fundamental for additional implementations of soft topology, additionally lead to expansion of data system with a different area of pure and applied mathematics.

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