Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 29 (2026), No. 5-A, pp. 1119–1125
DOI : 10.47974/JIM-2291

New categories of generalized soft mappings in soft topological spaces

Hamad Mohammed Salih *
Department of Mathematics
College of Sciences
University of Anbar
Anbar
Iraq

Alaa M. F. Al-Jumaili †
Department of Mathematics
College of Education for Pure Sciences
University of Anbar
Anbar
Iraq

Abstract

The presenting work studies novel extensions for the soft maps, which are soft δ-ß-open, soft δ-ß-continuous, by employing another idea of generalized soft sets, namely, soft δ-ß-open sets. Investigated relations between soft δ-ß-continuity and various forms of soft continuity, such as soft (α, semi, pre, ß) continuity. Using appropriate counterexamples, we have shown that noncoincidence exists among these diverse kinds of generalized soft maps. Various interesting characterizations related to these classes of extended soft mappings are obtained.

Subject Classification: *Primary 54C08, Secondary 54A10.*

Keywords: *S-δ-ß-open set, S-δ-ß-open map, S-δ-ß-closed map, S-δ-ß-continuity, S-δ-ß-irresolutness.*

1. Introduction

Although soft sets are important mathematical tools for studying uncertainties and solving complex problems in physics and computer science, they do not always capture brittle information and, as a result, cannot successfully employ the classical procedure due to the diverse kinds of uncertainties encountered in these problems. At the beginning, the authors in [1] introduced the concept of soft sets and, in [1-2], presented the essential results of

* *E-mail:* **hamadm1969@uoanbar.edu.iq (Corresponding Author)**
† *E-mail:* **eps.alaamahmood.farhan@uoanbar.edu.iq**

the original theory and effectively applied it to Riemann integration, game theory, and so on. Recently, many articles have been published on this theory and its implementations across diverse domains [3-4]. Soft sets have been applied in multicriteria decision-making, leading to the development of several related concepts and applications [5]. Later, soft topology and the characterizations of soft openness were investigated [6], [7]. Furthermore, many studies contributed to the development of algebraic structures in soft set theory [6–10]. Various forms of soft maps were also characterized in subsequent works [11], [12]. Other authors [13-14] described b-soft and α-soft open sets correspondingly. Later, the authors [15] defined e-soft open; we refer the authors to [16-19]. Recently, the authors introduced novel notions, namely soft δ-ß-closed and soft δ-ß-open, in [16].

Our main aim is to present novel ideas of soft extended maps, namely soft δ-ß-open, soft δ-ß-Cont, and soft δ-ß-Irresolute maps, by utilizing another notion of extended soft sets, and to obtain a number of their characterizations. Also, we have discussed the relationships among soft δ-β-Continuity with some forms of soft continuous maps.

2. Preliminaries

In this segment, the required results for extended soft open sets, which will be used throughout this article, have been presented.

Definition 2.1: [1] Assume Ɱ is an incipient comprehensive, $\mathcal{K}$ sets for the parameter with $\mathcal{P}(Ɱ)$ indicates a power for the Ɱ with $\emptyset \neq \mathcal{A} \subseteq \mathcal{K}.(\mathfrak{U}, \mathcal{K})$ the soft sets (concisely, S.s) on Ɱ, wherever $\mathcal{F}$ be given via $\mathfrak{U}: \mathcal{A} \longrightarrow \mathcal{P}(Ɱ)$.

Definition 2.2: [6] Assume Ꜧ be family for the (S.s) over Ɱ, so Ꜧis S-Topology if it gratifies the next presuppositions: (a) Φ & $\widetilde{Ɱ}$ ∈ Ꜧ. (b) Intersections for two (S.s) belong to Ꜧ. (c) Union of arbitrary elements of (S.s) belong to the Ꜧ.

Remark 2.3: $(Ɱ, Ꜧ, \mathcal{K})$ be soft topologicals spaces (in brief, S. Ꜧ.S) over Ɱ, elements of Ꜧ are soft open sets (briefly, (S. o. s)) in Ɱ.

Definition 2.4: [20] The map $\mathfrak{U}^c: \mathcal{A} \longrightarrow \mathcal{P}(Ɱ)$ give via $\mathfrak{U}^c\,(\varepsilon) = Ɱ - \mathfrak{U}(\varepsilon)\ \forall\ \varepsilon \in \mathcal{A}$ and $\mathfrak{U}^c$ named soft complements map for $\mathfrak{U}$.

Definition 2.5: Soft subset $(\mathfrak{U}, \mathcal{A})$ of $(S.$ Ꜧ.S) $(Ɱ, Ꜧ, \mathcal{K})$ is called:

(a) S – Pre - open [21] where $(\mathfrak{U}, \mathcal{A}) \tilde{\subseteq} int(cl(\mathfrak{U}, \mathcal{A}))$.

(b) S- semi - open [10] where $(\mathfrak{U}, \mathcal{A}) \tilde{\subseteq} cl(int(\mathfrak{U}, \mathcal{A}))$.

(c) S – α – open [14] when $(\mathfrak{U}, \mathcal{A}) \tilde{\subseteq} int(cl(int(\mathfrak{U}, \mathcal{A})))$.

(d) S – ß - open [22] when $(\mathfrak{U}, \mathcal{A}) \tilde{\subseteq} c\mathrm{l}(int(c\mathrm{l}(\mathfrak{U}, \mathcal{A})))$.

(e) S - E - open [18] if $\{(\mathfrak{U}, \mathcal{A}) \tilde{\subseteq} [int(c\mathrm{l}_\delta(\mathfrak{U}, \mathcal{A})) \tilde{\cup}\ c\mathrm{l}(Iint_\delta(\mathfrak{U}, \mathcal{A}))]\}$.

(f) S-δ-ß-open [3] if $(\mathfrak{U}, \mathcal{A}) \tilde{\subseteq} \mathrm{Cl}(Int(Cl_\delta(\mathfrak{U}, \mathcal{A})))$.

Definition 2.6: [11] A soft set $(\mathfrak{U},\mathcal{A})$ in (S. Ɥ.S) $(\text{Ɱ},\text{Ɥ},\mathrm{E})$ a soft point, indicated via $\mathcal{P}_{\alpha}^{\mathfrak{U}}$, if for $\alpha\in\mathcal{A},\mathfrak{U}(\alpha)\neq\emptyset$ & $\mathfrak{U}(ß)=\emptyset, ß\notin\mathcal{A}$.

Definition 2.7: [15] A soft point $\mathcal{P}_{\alpha}^{\mathfrak{U}}$ in (S. Ɥ.S) S-δ-cluster for $(\mathcal{G},\mathcal{A})$ if ∀ S-Regular $(\mathcal{U},\mathcal{A})$ including $\mathcal{P}_{\alpha}^{\mathfrak{U}}$, $(\mathcal{G},\mathcal{A})\tilde{\cap}(\mathcal{U},\mathcal{A})\neq\Phi$. S-δ-interior indicated $SInt_{\delta}(\mathfrak{U},\mathcal{A})=\{\mathcal{P}_{\alpha}^{\mathfrak{U}}\in\text{Ɱ}: (\mathcal{G},\mathcal{A}),\ \mathcal{P}_{\alpha}^{\mathfrak{U}}\in(\mathcal{G},\mathcal{A})\tilde{\subseteq} Int\big(Cl(\mathcal{G},\mathcal{A})\big)\tilde{\subseteq}(\mathfrak{U},\mathcal{A})\}$.

3. Some New Kinds of Generalized Soft Maps

Definition 3.1: $\digamma: (\text{Ɱ},\text{Ɥ}_1,\mathcal{W})\rightarrow(\text{Ʀ},\text{Ɥ}_2,\mathcal{B})$ named:

(a) *S*-Semi continuous (shortly, Cont) [12] *if* $\digamma^{-1}(\mathfrak{U},\mathcal{K})$ *S*-Semi open of $(\text{Ɱ},\text{Ɥ}_1,\mathcal{W})$ ∀ (*S*. o. s) $(\mathfrak{U},\mathcal{K})$ in $(\text{Ʀ},\text{Ɥ}_2,\mathcal{B})$.

(b) *S*-Pre-Cont [14] *if* $\digamma^{-1}(\mathfrak{U},\mathcal{K})$is S-Pre-open of $(\text{Ɱ},\text{Ɥ}_1,\mathcal{W})$ ∀ (*S*. o. s) $(\mathfrak{U},\mathcal{K})$ of $(\text{Ʀ},\text{Ɥ}_2,\mathcal{B})$.

(c) *S*-*α*-Cont [14] *if* $\digamma^{-1}(\mathfrak{U},\mathcal{K})$be the *S* – *α* - open for the $(\text{Ɱ},\text{Ɥ}_1,\mathcal{W})$ ∀ (*S*. o. s) $(\mathfrak{U},\mathcal{K})$ of $(\text{Ʀ},\text{Ɥ}_2,\mathcal{B})$.

(d) *S*-*ß*-Cont *if* $\digamma^{-1}(\mathfrak{U},\mathcal{K})$is *S*-*ß*-open of Ɱ ∀ (*S*. o. s) $(\mathfrak{U},\mathcal{K})$ of Ʀ.

(e) *S*-*E*-Cont [15] *if* $\digamma^{-1}(\mathfrak{U},\mathcal{K})$is *S*-*E*-open of Ɱ ∀ (*S*. o. s) $(\mathfrak{U},\mathcal{K})$of Ʀ.

(f) *S*-*δ*-*ß*-Cont *if* $\digamma^{-1}(\mathfrak{U},\mathcal{K})$is *S*-*δ*-*ß*-open in Ɱ ∀ (*S*. o. s) $(\mathfrak{U},\mathcal{K})$ of Ʀ.

Remark 3.2: Relationships between *S*-*δ*-*ß*-Cont maps and other corresponding categories of recognized *S*-Cont maps are shown in Figure 1 below:

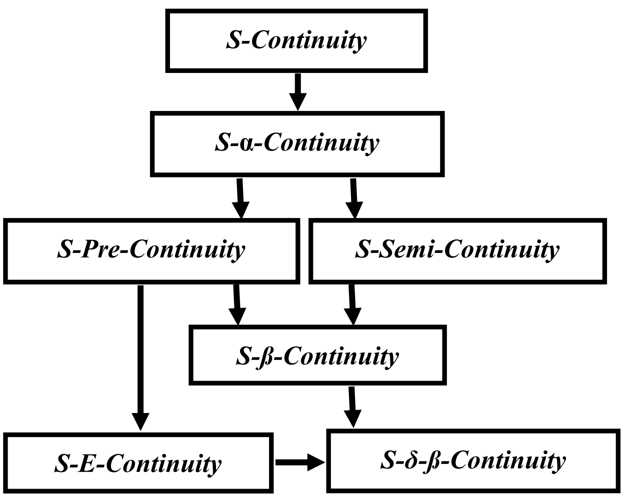

Figure 1
Relations among *S*-*δ*-*ß*-Cont-maps and various kinds of soft Cont-maps

The opposite of these inclusions is not correct; see [19-22] and the next examples:

Example 3.3: Suppose that $\text{Ɱ} = \text{Ɽ} = \{\hbar_1, \hbar_2, \hbar_3\}, \mathcal{W} = \{w_1, w_2\}$ and $\text{Ħ}_1 = \{\Phi, \widetilde{\text{Ɱ}}, (\mathfrak{U}_1, \mathcal{W}), (\mathfrak{U}_2, \mathcal{W}), (\mathfrak{U}_3, \mathcal{W})\}$ is (S. Ħ) regard to Ɱ and $\text{Ħ}_2 = \{\Phi, \widetilde{\text{Ɱ}}, (\mathcal{D}_1, \mathcal{W}), (\mathcal{D}_2, \mathcal{W})\}$ is (S. Ħ) with regard to Ɽ. So, $(\mathfrak{U}_1, \mathcal{W}), (\mathfrak{U}_2, \mathcal{W})$ and $(\mathfrak{U}_3, \mathcal{W})$ are soft sets on Ɱ and $(\mathcal{D}_1, \mathcal{W})$ & $(\mathcal{D}_2, \mathcal{W})$ are soft sets on Ɽ, Ƒ be the soft δ-β-Cont, but it is not *S*- pre-Cont, as $\text{Ƒ}^{-1}(\mathcal{D}_2) = \{\{\hbar_1, \hbar_2\}, \{\hbar_1, \hbar_2\}\}$, isn't soft pre-open on Ɱ.

Example 3.4: Assume $\text{Ɱ} = \text{Ɽ} = \{\hbar_1, \hbar_2, \hbar_3\}, \mathcal{W} = \{w_1, w_2\}$ & soft topologie on Ɱ is soft, indiscreet & soft discreet on Ɽ. If $\text{Ƒ}: (\text{Ɱ}, \text{Ħ}_1, \mathcal{W}) \to (\text{Ɽ}, \text{Ħ}_2, \mathcal{W})$ described:$\text{Ƒ}(\hbar_1) = \hbar_2$, $\text{Ƒ}(\hbar_2) = \hbar_1$and $\text{Ƒ}(\hbar_3) = \hbar_3$, consequently Ƒ is S-$\delta$-$\beta$-Cont, but not *S*-Semi-Cont.

Definition 3.5: $\text{Ƒ}: (\text{Ɱ}, \text{Ħ}_1, \mathcal{W}) \to (\text{Ɽ}, \text{Ħ}_2, \mathcal{W})$ be the *S*-δ-β-irresolutes *where* $\text{Ƒ}^{-1}(\mathfrak{U}, \mathcal{K})$ be the *S*-δ-β-open of $(\text{Ɱ}, \text{Ħ}_1, \mathcal{W})$ ∀ *S*-δ-β-open $(\mathfrak{U}, \mathcal{K})$ in $(\text{Ɽ}, \text{Ħ}_2, \mathcal{B})$.

Theorem 3.6: The next statements are equivalents for $\text{Ƒ}: (\text{Ɱ}, \text{Ħ}_1, \mathcal{W}) \to (\text{Ɽ}, \text{Ħ}_2, \mathcal{B})$:

(i) Ƒ is *S*-δ-β-Cont.

(ii) ∀ *S*-singleton $\mathcal{P}_\alpha^{\mathfrak{U}} \in \text{Ɱ}$ & ∀ (S. o. s) $(\mathfrak{U}, \mathcal{K})$ of Ɽ (s. t) $\text{Ƒ}(\mathcal{P}_\alpha^{\mathfrak{U}}) \widetilde{\subseteq} (\mathfrak{U}, \mathcal{K})$, there exist *S*-$\delta$-$\beta$-open $(\mathcal{D}, \mathcal{K})$ in Ɱ (s. t) $\mathcal{P}_\alpha^{\mathfrak{U}} \in (\mathcal{D}, \mathcal{K})$ & $\text{Ƒ}((\mathcal{D}, \mathcal{K})) \widetilde{\subseteq} (\mathfrak{U}, \mathcal{K})$.

(iii) $\text{Ƒ}^{-1}(\mathfrak{U}, K) = Cl(Int(\mathrm{Cl}_\delta(\text{Ƒ}^{-1}(\mathfrak{U}, K))))$ ∀ (S. o. s) $(\mathfrak{U}, K)$ of Ɽ.

(iv) Inverses for S-closed of Ɽ *be the S*-δ-β-closed.

(v) $Cl(Int(Cl_\delta(\text{Ƒ}^{-1}(\mathfrak{U}, \mathcal{K})))) \widetilde{\subseteq} \text{Ƒ}^{-1}(CI(\mathfrak{U}, \mathcal{K}))$ ∀ soft$(\mathfrak{U}, \mathcal{K}) \widetilde{\subseteq}$ Ɽ.

(vi) $\text{Ƒ}[\mathrm{Cl}(Int(Cl_\delta(\mathcal{D}, \mathcal{K})))] \widetilde{\subseteq} \mathrm{CI}(\text{Ƒ}(\mathcal{D}, \mathcal{K}))$ ∀ soft$(\mathcal{D}, \mathcal{K})$ of Ɱ.

Theorem 3.7: *Each soft δ-β-irresolute map is S-δ-β-Cont-map.*

Proof: It's obvious and follows from their respective definitions.

Theorem 3.8: If $\text{Ƒ}: (\text{Ɱ}, \text{Ħ}_1, \mathcal{W}) \to (\text{Ɽ}, \text{Ħ}_2, \mathcal{B})$ the *S*-δ-β-Cont mapping and $\mathcal{G}: (\text{Ɽ}, \text{Ħ}_2, \mathcal{B}) \to (\mathcal{Z}, \text{Ħ}_3, \mathcal{L})$ a *S*-Cont map. We have $\mathcal{G}o\text{Ƒ}: (\text{Ɱ}, \text{Ħ}_1, \mathcal{W}) \longrightarrow (\mathcal{Z}, \text{Ħ}_3, \mathcal{L})$ be the *S*-δ-β-Cont-map.

Proof: It's clear and follows from their respective definitions, thus omitted.

Theorem 3.9: *If* $\text{Ƒ}: (\text{Ɱ}, \text{Ħ}_1, \mathcal{W}) \to (\text{Ɽ}, \text{Ħ}_2, \mathcal{B})$ *the S-δ-β-irresolute map and* $\mathcal{G}: (\text{Ɽ}, \text{Ħ}_2, \mathcal{B}) \to (\mathcal{Z}, \text{Ħ}_3, \mathcal{L})$ *a S-δ-β-Cont-map. Then, a map* $\mathcal{G}o\text{Ƒ}: (\text{Ɱ}, \text{Ħ}_1, \mathcal{W}) \longrightarrow (\mathcal{Z}, \text{Ħ}_3, \mathcal{L})$ *is S-δ-β-Cont-map.*

Proof: Assume $(\mathfrak{U}, \mathcal{K})$ is (*S*. o. s) of $\mathcal{Z}$. At this instant, $(\mathcal{G}o\text{Ƒ})^{-1}(\mathfrak{U}, \mathcal{K}) = (\text{Ƒ}^{-1}o\mathcal{G}^{-1})(\mathfrak{U}, \mathcal{K}) = \text{Ƒ}^{-1}(\mathcal{G}^{-1}(\mathfrak{U}, \mathcal{K}))$. Since $\mathcal{G}$ is *S*-δ-β-Cont, $\mathcal{G}^{-1}(\mathfrak{U}, \mathcal{K})$ is *S*-δ-β-open & so $(\mathcal{G}o\text{Ƒ})^{-1}(\mathcal{F}, \mathcal{K})$ = Ƒ^{-1}(S-δ-β-open) of $\mathcal{G}$. Except, Ƒ being δ-β-irresolute, so$(\mathcal{G}o\text{Ƒ})^{-1}(\mathfrak{U}, \mathcal{K})$ is *S*-δ-β-open. As a result $\mathcal{G}o\text{Ƒ}$ is *S*-δ-β-Cont.

Theorem 3.10: *If* $\digamma: (\mathrm{M}, \mathrm{H}_1, \mathcal{W}) \longrightarrow (\mathrm{R}, \mathrm{H}_2, \mathcal{B})$ *&* $\mathcal{G}: (\mathrm{R}, \mathrm{H}_2, \mathcal{B}) \longrightarrow (\mathcal{Z}, \mathrm{H}_3, \mathcal{L})$ *are S-δ-ß-irresolute maps. so* $\mathcal{G}o\digamma: (\mathrm{M}, \mathrm{H}_1, \mathcal{W}) \longrightarrow (\mathcal{Z}, \mathrm{H}_3, \mathcal{L})$ *be the soft δ-ß-irresolute-map.*

***Proof*:** It is unpretentious, thus omitted.

Definition 3.11: *The map* $\digamma: (\mathrm{M}, \mathrm{H}_1, \mathcal{W}) \rightarrow (\mathrm{R}, \mathrm{H}_2, \mathcal{B})$ *be the:*

(a) *S-δ-ß-open if the image of all (S. o. s) is S-δ-ß-open of* R.

(b) *S-δ-ß-closed if the image of all (S. c. s) of* M *is S-δ-ß-closed of* R.

Theorem 3.12*:* *If* $\digamma: (\mathrm{M}, \mathrm{H}_1, \mathcal{W}) \rightarrow (\mathrm{R}, \mathrm{H}_2, \mathcal{B})$ *is a soft closed map and* $\mathcal{G}: (\mathrm{R}, \mathrm{H}_2, \mathcal{B}) \rightarrow (\mathcal{Z}, \mathrm{H}_3, \mathcal{L})$ *a S-δ-ß-closed map. so* $\mathcal{G}o\digamma: (\mathrm{M}, \mathrm{H}_1, \mathcal{W}) \longrightarrow (\mathcal{Z}, \mathrm{H}_3, \mathcal{L})$ *be the S-δ-ß-closed map.*

***Proof*:** It is evident and follows from their respective definitions, thus canceled.

Theorem 3.13: $\digamma: (\mathrm{M}, \mathrm{H}_1, \mathcal{W}) \rightarrow (\mathrm{R}, \mathrm{H}_2, \mathcal{B})$ *be the S-δ-ß-Closed iff every soft set* $(\mathcal{A}, \mathcal{B})$ *of* R *and all (S. o. s)* $(\mathfrak{U}, \mathcal{K})$ *where* $\digamma^{-1}(\mathcal{A}, \mathcal{B}) \tilde{\sqsubseteq} (\mathfrak{U}, \mathcal{K})$, *there is a S-δ-ß-open* $(\mathcal{D}, \mathcal{B})$ *of* R *where* $(\mathcal{A}, \mathcal{B}) \tilde{\sqsubseteq} (\mathcal{D}, \mathcal{B})$ *&* $\digamma^{-1}(\mathcal{D}, \mathcal{B}) \tilde{\sqsubseteq} (\mathfrak{U}, \mathcal{K})$.

Proof: Presume $\digamma$ *is S-δ-ß*-closed, and $(\mathcal{A}, \mathcal{B})$ a soft set of R, $(\mathfrak{U}, \mathcal{K})$ is (S. o. s) of M, (s. t) $\digamma^{-1}(\mathcal{A}, \mathcal{B}) \tilde{\sqsubseteq} (\mathfrak{U}, \mathcal{K})$. In that case $(\mathcal{D}, \mathcal{B}) = \left(\digamma((\mathfrak{U}, \mathcal{K})^c)\right)^c$ is *S-δ-ß*-open of R (s. t) $(\mathcal{A}, \mathcal{B}) \tilde{\sqsubseteq} (\mathcal{D}, \mathcal{B})$ with $\digamma^{-1}(\mathcal{D}, \mathcal{B}) \tilde{\sqsubseteq} (\mathfrak{U}, \mathcal{K})$.

Contrarily, presume $(\mathfrak{U}, \mathcal{M})$ is (S. c. s) of M. In that case $\digamma^{-1}\left(\digamma((\mathfrak{U}, \mathcal{M})^c)\right) \tilde{\sqsubseteq} (\mathfrak{U}, \mathcal{M})^c$, with $(\mathfrak{U}, \mathcal{M})^c$ *is* (S. o. s). Via supposition, there is *S-δ-ß*-open $(\mathcal{D}, \mathcal{B})$ of R (s. t) $\left(\digamma((\mathfrak{U}, \mathcal{K})^c)\right)^c \tilde{\sqsubseteq} (\mathcal{D}, \mathcal{B})$&$\digamma^{-1}(\mathcal{D}, \mathcal{B}) \tilde{\sqsubseteq} (\mathfrak{U}, \mathcal{M})$, consequently $(\mathfrak{U}, \mathcal{M}) \tilde{\sqsubseteq} \digamma^{-1}(\mathcal{D}, \mathcal{B})$.

Therefore, $(\mathcal{D}, \mathcal{B})^c \tilde{\sqsubseteq} \digamma(\mathcal{D}, \mathcal{B}) \tilde{\sqsubseteq} \digamma\left(\digamma^{-1}(\mathcal{D}, \mathcal{B})\right)^c) \tilde{\sqsubseteq} (\mathcal{D}, \mathcal{B})$ which implies that $\digamma(\mathfrak{U}, \mathcal{M}) = (\mathcal{D}, \mathcal{B})^c$. As, $(\mathcal{D}, \mathcal{B})^c$ is *S-δ-ß*-closed, $\digamma(\mathfrak{U}, \mathcal{M})$ is *S-δ-ß*-closed set.

Theorem 3.14: *If* $\digamma: (\mathrm{M}, \mathrm{H}_1, \mathcal{W}) \longrightarrow (\mathrm{R}, \mathrm{H}_2, \mathcal{B})$ *and we have* $\mathcal{G}: (\mathrm{R}, \mathrm{H}_2, \mathcal{B}) \longrightarrow (\mathcal{Z}, \mathrm{H}_3, \mathcal{L})$ *are maps s.t* $\mathcal{G}o\digamma: (\mathrm{M}, \mathrm{H}_1, \mathcal{W}) \longrightarrow (\mathcal{Z}, \mathrm{H}_3, \mathcal{L})$ *be the S-δ-ß-closed maps, then the next statement are satisfying:*

(a) *If* $\digamma$ *is S-Cont surjective map, so* $\mathcal{G}$ *is S-δ-ß-Closed.*

(b) *If* $\mathcal{G}$ *is S-δ-ß-Irresolute injective, so* $\digamma$ *is S-δ-ß-Closed.*

Proof (a)***:*** Assume $(\mathcal{A}, \mathcal{B})$ is a (S. c. s) of R. Then, $\digamma^{-1}(\mathcal{A}, \mathcal{B})$ is (S. c. s) in M because $\digamma$ S-Cont. As, $\mathcal{G}o\digamma$ is *S-δ-ß*-closed, $(\mathcal{G}o\digamma)\left(\digamma^{-1}(\mathcal{A}, \mathcal{B})\right) = \mathcal{G}(\mathcal{A}, \mathcal{B})$ *S-δ-ß*-closed in $\mathcal{Z}$. Consequently, $\mathcal{G}: (\mathrm{R}, \mathrm{H}_2, \mathcal{B}) \rightarrow (\mathcal{Z}, \mathrm{H}_3, \mathcal{L})$ *S-δ-ß*-closed map.

Proof (b)*:* Presume $(\mathfrak{U}, \mathcal{K})$ is (S. o. s) of M. Then, $(\mathcal{G}o\digamma)(\mathfrak{U}, \mathcal{K})$ is *S-δ-ß*-closed set of $\mathcal{Z}$, thus $\mathcal{G}^{-1}(\mathcal{G}o\digamma)(\mathfrak{U}, \mathcal{K}) = \digamma(\mathfrak{U}, \mathcal{K})$ is *S-δ-ß*-closed of R. Because, $\mathcal{G}$ is *S-δ-ß*-Irresolute and inj-map. Consequently,$\digamma$ is *S-δ-ß*-closed.

4. Conclusion

The major reason of current manuscript is to investigate and study novel ideas of extended soft maps as, S-δ-β-open, S-δ-β-Cont and S-δ-β-Irresolute maps, as well their relationships with other forms of S-Continuous maps have been discussed. Also, several of their characterizations have been established. Expect that the conclusions in our article are beginning of original structures and they constitute the notional basis for additional implementations of soft topology.

References

[1] D. A. Molodtsov, "Soft set theory — First results," *Computers & Mathematics with Applications*, vol. 37, no. 4-5, pp. 19–31 (1999), doi: 10.1016/S0898-1221(99)00056-5.

[2] D. Molodtsov, V. Y. Leonov, and D. V Kovkov, "Soft Sets Technique and Its Application," *Fuzzy Systems and Soft Computing*, vol. 1, no. 1, pp. 8–39 (2006).

[3] A. M. Al-Jumaili, "New generalizations of soft sets in soft topological spaces and their applications," *J. Interdiscip. Math,* vol. 27, no. 4, pp. 705–713 (2024), doi: 10.47974/JIM-1755.

[4] A. M. F. AL. Jumaili, "Other Certain Classes of Generalized Neutrosophic Soft Separation Structures," *International Journal of Neutrosophic Science*, vol. 22, no. 1, pp. 77-85 (2023).

[5] P. K. Maji, A. R. Roy, and R. Biswas, "An application of soft sets in a decision-making problem," *Computers & Mathematics with Applications*, vol. 44, no. 8–9, pp. 1077–1083 (2002).

[6] M. Shabir and M. Naz, "On soft topological spaces," *Computers & Mathematics with Applications*, vol. 61, no. 7, pp. 1786–1799 (2011).

[7] S. Hussain and B. Ahmad, "Some properties of soft topological spaces," *Computers & Mathematics with Applications,* vol. 62, no. 11, pp. 4058–4067 (2011).

[8] Y. B. Jun, H. S. Kim, and J. Neggers, "Pseudo d-algebras," *Information Sciences,* vol. 179, no. 11, pp. 1751–1759 (2009).

[9] Y. B. Jun, K. J. Lee, and A. Khan, "Soft ordered semigroups," *Mathematical Logic Quarterly,* vol. 56, no. 1, pp. 42–50 (2010), doi: 10.1002/malq.200810030.

[10] B. Chen, "Soft semi-open sets and related properties in soft topological spaces," *Appl. Math. Inf. Sci,* vol. 7, no. 1, pp. 287–294 (2013).

[11] Y. Yumak and A. K. Kaymakcı, “Soft β-open sets and their applications,” *Journal of New Theory*, vol. 4, pp. 80–89 (2015).

[12] M. Akdag and A. Ozkan, “Soft b-open sets and soft b-continuous functions,” *Mathematical Sciences*, vol. 8, no. 2, p. 124 (2014).

[13] M. Akdag and A. Ozkan, “Soft α-open sets and soft α-continuous functions,” *Abstract and Applied Analysis,* vol. 2014, no. 1, Art. no. 891341 (2014).

[14] A. Mukherjee and B. Debnath, “On soft e-Open sets and soft e-Continuity in soft Topological space,” *Journal of New Theory,* vol. 15, pp. 1–18 (2017).

[15] Y. Y. Yousif, L. A. Hussain, and M. A. Hussain, “Fibrewise soft bitopological spaces,” *Journal of Interdisciplinary Mathematics,* vol. 24, no. 7, pp. 1925–1934 (2021).

[16] H. J. Ali and S. I. Mahmood, “New Generalizations of Soft LC-Spaces,” *Iraqi Journal of Science*, vol. 63, no. 11, pp. 4890–4900 (2022).

[17] N. A. Nadhim, R. N. Majeed, and H. J. Ali, “Soft K (sc)-spaces,” *Journal of Interdisciplinary Mathematics,* vol. 22, no. 8, pp. 1463–1470 (2019).

[18] Y. Y. Yousif and M. A. Ghafel, “Fibrewise soft ideal topological space,” *J. Phys. Conf. Ser.,* vol. 1003, no. 1 (2018).

[19] M. I. Ali, F. Feng, X. Liu, W. K. Min, and M. Shabir, “On some new operations in soft set theory,” *Computers & Mathematics with Applications,* vol. 57, no. 9, pp. 1547–1553 (2009).

[20] I. Arockiarani and A. A. Lancy, “Generalized soft gβ-closed sets and soft gsβ-closed sets in soft topological spaces,” *International Journal of Mathematical Archive,* vol. 4, no. 2, pp. 1–7 (2013).

[21] P. Yadav, R. Singh, and K. Khurana, “A Review on soft topological spaces,” *Psychol. Educ.*, vol. 57, no. 9, pp. 1430–1442 (2020).

[22] I. Zorlutuna, M. Akdag, W. K. Min, and S. Atmaca, “Remarks on soft topological spaces,” A*nnals of fuzzy Mathematics and Informatics*, vol. 3, no. 2, pp. 171–185 (2012).

Received May, 2025