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Micro bitopological spaces and some generalizations

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Abstract

In this essay, address the recently found microbiological topological spaces, such as microbiological closed and open sets in bitopological spaces. Furthermore, make and justify a number from statements about these concepts. A micro bitopological space is one whose topology is determined via a group of bundles. In comparison, in a typical topological space, the topology is specified via a single set. The power of micro bitopological spaces lies in their ability to model a broad range of mathematical objects that can describe the topology of a manifold or a group. Micro bitopological spaces have numerous uses in mathematics, such as investigating the geometry of a space or the structure of a group. Microtopological spaces are also simple to manipulate and have numerous mathematical applications.

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Keywords: *Micro bitopological spaces, Micro.ƀi.closed, Micro.ƀi.open, Micro-ƀi-pre-open.*

1. Introduction

Fine topological spaces were first proposed in 2012 via Powar and Rajak [1]. In 2013 saw the introduction from nano topology via Thivagar and Richard [2]. Based on the ideas of lower approximation, upper approximation, and boundary area, nano topology. In order to extend additional open sets, may

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apply Levine's straightforward extension notion in nano topology, which be when a topology be referred to as micro topology. Nano topology has a maximum from five nano open sets and a minimum from two nano open sets, including Ư. Suppose wish to add more open sets [3]. Micro topology underlies every nano topology, and some studies have expanded on the concept of micro topology via many of their researchers mention some them [4], [5], [6], [7], [8] and below is the summary from the paper. Introduction be given in the main definitions. The idea of correlated fibrillar topological spaces, Yousif, Hussain, and Hussain [9] delved into the study of fibrewise with bitopology and general topology as well as with nano and micro. was introduced in the study. Kelly [10] developed the bitopological space study in 1963, and since then, a vast number from studies have been done to expand topological notions to bitopological settings [11]. In this study, (Ɲ, $\mathcal{T}$, $\grave{\mathcal{T}}$), and (Ϻ, Λ, $\grave{\Lambda}$) (or succinctly, Ɲ and Ϻ) always denote bitopological spaces.

Definition 1.1: [7] Let (Ư, $\mathcal{T}_{R}$ (W)) be nano topological space, and the family $ɱ_{R}$(W) = {$\mathcal{N}$ ∪ ($\mathcal{N}'$ ∩ ɱ): $\mathcal{N}$, $\mathcal{N}'$ ∈ $\mathcal{T}_{R}$ (W), ɱ ∉ $\mathcal{T}_{R}$ (W)} be a known to micro topological space on Ư with respect to W, which is indicated by the symbol (ɱ.Ʈ.ς).

Definition 1.2: [3] The ɱ.Ʈ.ς adheres to the following axioms:

I. Ư, ∅ ∈ $ɱ_{R}$(W).

II. Intersection from the elements from any finite sub collection from $ɱ_{R}$(W) be in $ɱ_{R}$(W).

III. Union from the elements from each sub collection of $ɱ_{R}$(W) be in $ɱ_{R}$(W).

The ɱ.Ʈ.ς be defined as a triplet (Ư, $\mathcal{T}_{R}$ (W), $ɱ_{R}$(W)). Each member from $ɱ_{R}$(W) be referred to as a ɱ.open which is indicated via the symbol (ɱ.open). The complement to the ɱ.open set be also known as the ɱ.closed set.

Definition 1.3: [10], [9] The term "bitopological space" which be indicated via the symbol (ƀi.Ʈ.ς) refers to a triple (Ɲ, $\mathcal{T}$, $\grave{\mathcal{T}}$), abbreviation Ɲ in which Ɲ be non.empty set and $\mathcal{T}$, $\grave{\mathcal{T}}$ are topologies on.

2. Micro-Bitopological Spaces

In this part, the micro bitopology be explained along with a number from topological traits that may be inferred from it.

Definition 2.1: Let (Ư, $\mathcal{T}_{R1,2}$(W)) be nano bitopological space, and the family $ɱ_{R1,2}$(W) = {$\mathcal{N}$ ∪ ($\mathcal{N}'$ ∩ ɱ): $\mathcal{N}$ ∈ $\mathcal{T}_{R1}$(W), $\mathcal{N}'$ ∈ $\mathcal{T}_{R2}$(W), ɱ ∉ $\mathcal{T}_{R1}$(W) and ɱ́ ∉ $\mathcal{T}_{R2}$(W) be a known to micro bitopological space on Ư with respect to W, which be indicated via the symbol ($ɱ_{R1,2}$.ƀi.Ʈ.ς).

Definition 2.2: Suppose that (Ư, $\mathcal{T}_{\text{Ʀ}1,2}(\text{W})$, $\mathfrak{m}_{\text{Ʀ}1,2}(\text{W})$), where W ⊆ Ư be $\mathfrak{m}_{\text{Ʀ}1,2}$.ƀi.Ʈ.ς, conforms to the following axioms:

I. Ư, ∅ ∈ $\mathfrak{m}_{\text{Ʀ}1,2}(\text{W})$.

II. Intersection from the elements from any finite subcollection from $\mathfrak{m}_{\text{Ʀ}1,2}(\text{W})$ be in $\mathfrak{m}_{\text{Ʀ}1,2}(\text{W})$.

III. Union from the elements from any subcollection from $\mathfrak{m}_{\text{Ʀ}1,2}(\text{W})$ be in $\mathfrak{m}_{\text{Ʀ}1,2}(\text{W})$.

The ɱ.ƀi.Ʈ.ς be defined as a quintet (Ư, $\mathcal{T}_{\text{Ʀ}1}(\text{W})$, $\mathcal{T}_{\text{Ʀ}2}(\text{W})$, $\mathfrak{m}_{\text{Ʀ}1}(\text{W})$, $\mathfrak{m}_{\text{Ʀ}2}(\text{W})$), abbreviation (Ư, $\mathcal{T}_{\text{Ʀ}1,2}(\text{W})$, $\mathfrak{m}_{\text{Ʀ}1,2}(\text{W})$). Each member from $\mathfrak{m}_{\text{Ʀ}1,2}(\text{W})$ be referred to as a $\mathfrak{m}_{\text{Ʀ}1,2}$.ƀi.open sets. The complement to the $\mathfrak{m}_{\text{Ʀ}1,2}$.ƀi.open set be known as the $\mathfrak{m}_{\text{Ʀ}1,2}$.ƀi.closed set.

Example 2.3: Suppose that Ư = {q, w, e, r}, Ư/Ʀ = {{q}, {e}, {w, r}}, $\mathcal{H}$ = {w, r} ⊆ Ư. The $\mathcal{T}_{\text{Ʀ}}(\mathcal{H})$ = {Ư, ∅, {w, r}}, let ɱ = {w} ∉ $\mathcal{T}_{\text{Ʀ}}(\mathcal{H})$. Then ɱ.ƀi.open = $\mathfrak{m}_{\text{Ʀ}}(\mathcal{H})$ = {{w}, {w, r}, Ư, ∅}. Let Ư/Ʀ́ = {{q}, {w}, {e, r}}, $\acute{\mathcal{H}}$ = {e, r} ⊆ Ư. The $\acute{\mathcal{T}}_{\acute{\text{Ʀ}}}(\acute{\mathcal{H}})$ = {Ư, ∅, {e, r}}, let ɱ́ = {q} ∉ $\acute{\mathcal{T}}_{\acute{\text{Ʀ}}}(\acute{\mathcal{H}})$. Then ɱ.ƀi.open = $\acute{\mathfrak{m}}_{\acute{\text{Ʀ}}}(\acute{\mathcal{H}})$ = {{q, e, r}, {e, r}, {q}, Ư, ∅}, and $\mathfrak{m}_{\text{Ʀ}}(\mathcal{H}, \acute{\mathcal{H}})$ = {{w}, {q}, {w, r}, {e, r}, {q, e, r}, Ư, ∅} be $\mathfrak{m}_{\text{Ʀ}}(\mathcal{H}, \acute{\mathcal{H}})$.ƀi.open sets.

Example 2.4: Suppose that Ư = {q, w, e, r}, Ư/Ʀ = {{q}, {e}, {w, r}}, $\mathcal{H}$ = {w, r} ⊆ Ư. The $\mathcal{T}_{\text{Ʀ}}(\mathcal{H})$ = {Ư, ∅, {w, r}}, let ɱ = {w} ∉ $\mathcal{T}_{\text{Ʀ}}(\mathcal{H})$. Then $\mathfrak{m}_{\text{Ʀ}}(\mathcal{H})$ = {{w}, {w, r}, Ư, ∅}, then a ɱ.ƀi.closed = {{q, e, r}, {q, e}, Ư, ∅ }. Let Ư/Ʀ́ = {{q}, {w}, {e, r}}, $\acute{\mathcal{H}}$ = {e, r} ⊆ Ư. The $\acute{\mathcal{T}}_{\acute{\text{Ʀ}}}(\acute{\mathcal{H}})$ = {Ư, ∅, {e, r}}, let ɱ́ = {q} ∉ $\acute{\mathcal{T}}_{\acute{\text{Ʀ}}}(\acute{\mathcal{H}})$. Then $\acute{\mathfrak{m}}_{\acute{\text{Ʀ}}}(\acute{\mathcal{H}})$ = {{q, e, r}, {e, r}, {q}, Ư, ∅}, then a ɱ.ƀi.closed = {{w}, {q, w}, {w, e, r}, Ư, ∅}, and $\mathfrak{m}_{\text{Ʀ}}(\mathcal{H}, \acute{\mathcal{H}})$ = { {w}, {q}, {q, w}, {q, e}, {q, e, r}, {w, e, r}, Ư, ∅} be $\mathfrak{m}_{\text{Ʀ}}(\mathcal{H}, \acute{\mathcal{H}})$.ƀi.closed sets.

Definition 2.5: Suppose that (Ư, $\mathcal{T}_{\text{Ʀ}1,2}(\mathcal{X})$, $\mathfrak{m}_{\text{Ʀ}1,2}(\mathcal{X})$) be a $\mathfrak{m}_{\text{Ʀ}1,2}$.ƀi.Ʈ.ς, a set $\mathcal{X}$ ⊆ Ư be $\mathfrak{m}_{\text{Ʀ}1,2}$.ƀi.closure be indicated via the symbol $\mathfrak{m}_{\text{Ʀ}1,2}$.ƀi.ç̥l̥($\mathcal{X}$), which be defined as $\mathfrak{m}_{\text{Ʀ}1,2}$.ƀi.ç̥l̥($\mathcal{X}$) = ∩{ $\mathcal{Y}$; $\mathcal{Y}$ be $\mathfrak{m}_{\text{Ʀ}1,2}$.ƀi.closed in Ư, and $\mathcal{X}$ ⊆ $\mathcal{Y}$ }, $\mathfrak{m}_{\text{Ʀ}1,2}$.ƀi.ç̥l̥($\mathcal{X}$).

A set $\mathcal{X}$ ⊆ Ư be $\mathfrak{m}_{\text{Ʀ}1,2}$.ƀi.interior be indicated via the notation $\mathfrak{m}_{\text{Ʀ}1,2}$.ƀi.i̥n̥ʇ($\mathcal{X}$), which has the definition $\mathfrak{m}_{\text{Ʀ}1,2}$.ƀi.i̥n̥ʇ($\mathcal{X}$) = ∪{Ʊ: Ʊ be $\mathfrak{m}_{\text{Ʀ}1,2}$.ƀi.open in Ư and Ʊ ⊆ $\mathcal{X}$}.

It is clear that $\mathfrak{m}_{\text{Ʀ}1,2}$.ƀi.ç̥l̥($\mathcal{X}$) (resp., $\mathfrak{m}_{\text{Ʀ}1,2}$.ƀi.i̥n̥ʇ($\mathcal{X}$)) be smallest $\mathfrak{m}_{\text{Ʀ}1,2}$.ƀi.closed (resp., biggest $\mathfrak{m}_{\text{Ʀ}1,2}$.ƀi.open) subset from Ư that includes (resp., conversely, is contained in) $\mathcal{X}$. A $\mathfrak{m}_{\text{Ʀ}1,2}$.ƀi.ç̥l̥($\mathcal{X}$) (resp., $\mathfrak{m}_{\text{Ʀ}1,2}$.ƀi.i̥n̥ʇ($\mathcal{X}$), depending) if and only if $\mathcal{X}$ = $\mathfrak{m}_{\text{Ʀ}1,2}$.ƀi.ç̥l̥($\mathcal{X}$) (resp., $\mathfrak{m}_{\text{Ʀ}1,2}$.ƀi.i̥n̥ʇ($\mathcal{X}$)).

Definition 2.6: For each pair of $\mathfrak{m}_{R1,2}$.ƀi sets Ȧ, and ß in $\mathfrak{m}_{R1,2}$.ƀi.Ʈ.ς (Uʼ, $\mathcal{T}_{R1,2}(\mathcal{X})$, $\mathfrak{m}_{R1,2}(\mathcal{X})$):

I. Ȧ ⊂ ß, viz., $\mathfrak{m}_{R1,2}$.ƀi.iņŧ(Ȧ) ⊂ $\mathfrak{m}_{R1,2}$.ƀi.iņŧ(ß), and $\mathfrak{m}_{R1,2}$.ƀi.ḉļ(Ȧ) ⊂ $\mathfrak{m}_{R1,2}$.ƀi.ḉļ(ß).

II. $\mathfrak{m}_{R1,2}$.ƀi.ḉļ($\mathfrak{m}_{R1,2}$.ƀi.ḉļ(Ȧ)) = $\mathfrak{m}_{R1,2}$.ƀi.ḉļ(Ȧ), and $\mathfrak{m}_{R1,2}$.ƀi.iņŧ($\mathfrak{m}_{R1,2}$.ƀi.iņŧ(Ȧ)) = $\mathfrak{m}_{R1,2}$.ƀi.iņŧ(Ȧ).

III. $\mathfrak{m}_{R1,2}$.ƀi.iņŧ(Ȧ ∩ ß) ⊂ $\mathfrak{m}_{R1,2}$.ƀi.iņŧ(Ȧ) ∩ $\mathfrak{m}_{R1,2}$.ƀi.iņŧ(ß).

IV. $\mathfrak{m}_{R1,2}$.ƀi.iņŧ(Ȧ) ∪ $\mathfrak{m}_{R1,2}$.ƀi.iņŧ(ß) ⊂ $\mathfrak{m}_{R1,2}$.ƀi.iņŧ(Ȧ ∪ ß).

V. $\mathfrak{m}_{R1,2}$.ƀi.ḉļ(Ȧ) ∪ $\mathfrak{m}_{R1,2}$.ƀi.ḉļ(ß) ⊂ $\mathfrak{m}_{R1,2}$.ƀi.ḉļ(Ȧ ∪ß).

VI. $\mathfrak{m}_{R1,2}$.ƀi.ḉļ(Ȧ ∩ ß) ⊂ $\mathfrak{m}_{R1,2}$.ƀi.ḉļ (Ȧ) ∩ $\mathfrak{m}_{R1,2}$.ƀi.ḉļ(ß).

VII. $\mathfrak{m}_{R1,2}$.ƀi.ḉļ($Ȧ^c$) = [$\mathfrak{m}_{R1,2}$.ƀi.iņŧ(Ȧ)]^c.

VIII. $\mathfrak{m}_{R1,2}$.ƀi. iņŧ($Ȧ^c$) = [$\mathfrak{m}_{R1,2}$.ƀi.ḉļ(Ȧ)]^c.

3. Micro-ƀi-pre-open sets

Defining and studying micro.ƀi.pre-open sets in bitopological spaces be covered in this section. Some of their traits are similar to those of micro.ƀi.open sets.

Definition 3.1: Suppose that (Uʼ, $\mathcal{T}_{R1,2}$(Å), $\mathfrak{m}_{R1,2}$(Å)) be a $\mathfrak{m}_{R1,2}$.ƀi.Ʈ.ς, and a set Å ⊆ Uʼ, thereafter Å be known as $\mathfrak{m}_{R1,2}$.ƀi.pre.open which be represented via the symbol ($\mathfrak{m}_{R1,2}$.ƀi. Ᵽ.open) if Å ⊆ $\mathfrak{m}_{R1,2}$.ƀi.iņŧ ($\mathfrak{m}_{R1,2}$.ƀi.ḉļ(Å)), and $\mathfrak{m}_{R1,2}$.ƀi.pre.closed if $\mathfrak{m}_{R1,2}$.ƀi.ḉļ($\mathfrak{m}_{R1,2}$.ƀi.iņŧ(Å)) ⊆ Å.

Example 3.2: Suppose that Uʼ = {q, r, t, p}, Uʼ/Ɽ = {{r}, {p}, {q, t}}, $\mathcal{H}$ = {t, p} ⊆ Uʼ. The $\mathcal{T}_{R}(\mathcal{H})$ = {Uʼ, ∅, {r}, {t, p}}, let $\mathfrak{m}$ = {t} ∉ $\mathcal{T}_{R}(\mathcal{H})$. Then $\mathfrak{m}_{R}(\mathcal{H})$ = {{t}, {t, p}, {p}, {q, t, p}, {q, t}, Uʼ, ∅}. Let Uʼ/Ɽ́ = {{q}, {t}, {r, p}}, $\acute{\mathcal{H}}$ = {r, p} ⊆ Uʼ. The $\acute{\mathcal{T}}_{\acute{R}}(\acute{\mathcal{H}})$ = {Uʼ, ∅, {r, p}}, let $\acute{\mathfrak{m}}$ = {q} ∉ $\acute{\mathcal{T}}_{\acute{R}}(\acute{\mathcal{H}})$. Then $\acute{\mathfrak{m}}_{\acute{R}}(\acute{\mathcal{H}})$ = {{q}, {r, p}, {q, r, p}, Uʼ, ∅}, and $\mathfrak{m}_{R}(\mathcal{H}, \acute{\mathcal{H}})$ = {{q}, {r, p}, {q, r, p}, {t}, {t, p}, {p}, {q, t, p}, {q, t}, Uʼ, ∅}. Evidently $\mathcal{A}$ = {q, t} be $\mathfrak{m}_{R1,2}$.ƀi. Ᵽ.open.

Theorem 3.3: *Each* $\mathfrak{m}_{R1,2}$*.ƀi.open is* $\mathfrak{m}_{R1,2}$*.ƀi. Ᵽ.open set.*

Proof: Suppose that Ȧ be $\mathfrak{m}_{R1,2}$.ƀi.open, thereafter Ȧ ⊆ $\mathfrak{m}_{R1,2}$.ƀi.iņŧ($\mathfrak{m}_{R1,2}$.ƀi.iņŧ(Ȧ)). Because $\mathfrak{m}_{R1,2}$.ƀi.iņŧ($\mathfrak{m}_{R1,2}$.ƀi.iņŧ(Ȧ)) ⊆ $\mathfrak{m}_{R1,2}$.ƀi.iņŧ($\mathfrak{m}_{R1,2}$.ƀi.ḉļ(Ȧ)), thus Ȧ ⊆ $\mathfrak{m}_{R1,2}$.ƀi.iņŧ($\mathfrak{m}_{R1,2}$.ƀi.ḉļ(Ȧ)). Therefore, Ȧ be $\mathfrak{m}_{R1,2}$.ƀi. Ᵽ.open set.

The following example demonstrate that the theorem (3.3) be not reversible.

Example 3.4: Suppose that Uʼ = {r, j, k}, Uʼ/Ɽ = {{k}, {r, j}, $\mathcal{H}$ = {r, j} ⊆ Uʼ. The $\mathcal{T}_{R}(\mathcal{H})$ = {Uʼ, ∅, {r, j}}, let $\mathfrak{m}$ = {k} ∉ $\mathcal{T}_{R}(\mathcal{H})$. Then $\mathfrak{m}_{R}(\mathcal{H})$ = {{k}, {r, j}, Uʼ, ∅}. Let Uʼ/Ɽ́ = {{r, j}, {k}}, $\acute{\mathcal{H}}$ = {j} ⊆ Uʼ. The $\acute{\mathcal{T}}_{\acute{R}}(\acute{\mathcal{H}})$ = {Uʼ, ∅, {j}}, let $\acute{\mathfrak{m}}$

= {r} ∉ $\acute{\mathcal{T}}_{\acute{R}}(\acute{\mathcal{H}})$. Then $\acute{ɱ}_{\acute{R}}(\acute{\mathcal{H}})$ = {{r}, {j}, {r, j}, U̇, ∅}, and $ɱ_{R}(\mathcal{H},\acute{\mathcal{H}})$ = {{k}, {r}, {j}, {r, j}, U̇, ∅}. Evidently $\mathcal{A}$ = {r, k} be $ɱ_{R1,2}$.ƀi. Ᵽ.open, but not $ɱ_{R1,2}$.ƀi.open.

Remark 3.5: A finite intersection from $ɱ_{R1,2}$.ƀi. Ᵽ.open not necessary $ɱ_{R1,2}$.ƀi. Ᵽ.open sets.

Example 3.6: In Example (3.4), the sets {r, j, k} and {j} are $ɱ_{R1,2}$.ƀi. Ᵽ.open, but {r, j, k} ∩ {j} = {j} be not $ɱ_{R1,2}$.ƀi. Ᵽ.open set.

Definition 3.7: Suppose that (U̇, $\mathcal{T}_{R1,2}(\mathcal{X})$, $ɱ_{R1,2}(\mathcal{X})$) be a $ɱ_{R1,2}$.ƀi.Ʈ.ς, the element å ∈ Ȧ be known as $ɱ_{R1,2}$.ƀi.Ᵽ.interior point from Ȧ, If there are $ɱ_{R1,2}$.ƀi.Ᵽ.open set Ɲ; å ∈ Ɲ ⊂ Ȧ.

Definition 3.8: Aset from all $ɱ_{R1,2}$.ƀi.Ᵽ.interior points from Ȧ be known as $ɱ_{R1,2}$.ƀi.pre.interior from Ȧ,which be represented via the symbol ($ɱ_{R1,2}$.ƀi. Ᵽ.i̥n̥ƚ(Ȧ)).

Definition 3.9: Suppose that (U̇, $\mathcal{T}_{R1,2}(\mathcal{X})$, $ɱ_{R1,2}(\mathcal{X})$) be a $ɱ_{R1,2}$.ƀi.Ʈ.ς, a set $\mathcal{X}$ ⊆ U̇ be $ɱ_{R1,2}$.ƀi.pre.closure, be indicated via the symbol $ɱ_{R1,2}$.ƀi.Ᵽ.ç̥l̥($\mathcal{X}$), which be defined as $ɱ_{R1,2}$.ƀi.Ᵽ.ç̥l̥($\mathcal{X}$) = ∩{ $\mathcal{Y}$; $\mathcal{Y}$ be $ɱ_{R1,2}$.ƀi.Ᵽ.closed in U̇, and $\mathcal{X}$ ⊆ $\mathcal{Y}$ } $ɱ_{R1,2}$.ƀi.Ᵽ.ç̥l̥($\mathcal{X}$).

Remark 3.10:

(I) A $ɱ_{R1,2}$.ƀi.Ᵽ. ç̥l̥ ($\mathcal{X}$), also $ɱ_{R1,2}$.ƀi.Ᵽ. closed set.

(II) A $ɱ_{R1,2}$.ƀi.Ᵽ. ç̥l̥ ($\mathcal{X}$) be smallest $ɱ_{R1,2}$.ƀi.Ᵽ. closed set in $\mathcal{X}$.

Theorem 3.11: *Each* $ɱ_{R1,2}$.*ƀi.closed is* $ɱ_{R1,2}$.*ƀi.Ᵽ.closed set.*

Proof: Suppose that Ȧ be $ɱ_{R1,2}$.ƀi.closed, now have $ɱ_{R1,2}$.ƀi.ç̥l̥($ɱ_{R1,2}$.ƀi.ç̥l̥(Ȧ)) ⊆ Ȧ. Because $ɱ_{R1,2}$.ƀi.ç̥l̥($ɱ_{R1,2}$.ƀi.i̥n̥ƚ(Ȧ)) ⊆ $ɱ_{R1,2}$.ƀi.ç̥l̥($ɱ_{R1,2}$.ƀi.ç̥l̥(Ȧ)) ⊆ Ȧ. Hence, Ȧ is $ɱ_{R1,2}$.ƀi. .Ᵽ.closed set.

The following example demonstrate that the theorem (3.11) be not reversible.

Example 3.12: Suppose that U̇ = {q, w, e}, U̇/Ʀ = {{q}, {e, w}, $\mathcal{H}$ = {q} ⊆ U̇. The $\mathcal{T}_{R}(\mathcal{H})$ = {U̇, ∅, {q}}, let ɱ = {w} ∉ $\mathcal{T}_{R}(\mathcal{H})$. Then $ɱ_{R}(\mathcal{H})$ = {{q}, {w}, {q, w}, U̇, ∅}, then a ɱ.ƀi.closed = {{w, e}, {q, e}, {e}, U̇, ∅ }. Let U̇/Ʀ́ = {{w, e}, {q}}, $\acute{\mathcal{H}}$ = {q} ⊆ U̇. The $\acute{\mathcal{T}}_{\acute{R}}(\acute{\mathcal{H}})$ = {U̇, ∅, {q}}, let ɱ́ = {q, w} ∉ $\acute{\mathcal{T}}_{\acute{R}}(\acute{\mathcal{H}})$. Then $\acute{ɱ}_{\acute{R}}(\acute{\mathcal{H}})$ = {{q, w}, {q}, U̇, ∅}, then a ɱ.ƀi. closed = {{e}, {w, e}, U̇, ∅}, then $ɱ_{R}(\mathcal{H},\acute{\mathcal{H}})$ = {{q}, {w}, {q, w}, U̇, ∅}, and $ɱ_{R}(\mathcal{H},\acute{\mathcal{H}})$.closed =

{{w, e}, {q, e}, {e}} Evidently $\mathcal{A}$ = {q, w} be $ɱ_{Ʀ1,2}$.ƀi. Ᵽ. closed, but not $ɱ_{Ʀ1,2}$.ƀi. closed.

Theorem 3.13: *Ά be* $ɱ_{Ʀ1,2}$*.ƀi.Ᵽ. closed iff Ά =* $ɱ_{Ʀ1,2}$*.ƀi.Ᵽ.ç̥ḽ(Ά).*

Proof: $ɱ_{Ʀ1,2}$.ƀi.Ᵽ. ç̥ḽ(Ά) = ∩{ $\mathcal{Y}$; $\mathcal{Y}$ be $ɱ_{Ʀ1,2}$.ƀi.Ᵽ. closed set, and Ά ⊆ $\mathcal{Y}$}, if Ά be $ɱ_{Ʀ1,2}$.ƀi.Ᵽ. closed, thereafter Ά is a member from the preceding collection, and any member includes Ά. Therefore, the intersection be Ά, and $ɱ_{Ʀ1,2}$.ƀi.Ᵽ.ç̥ḽ(Ά) = Ά. The other direction, Ά = $ɱ_{Ʀ1,2}$.ƀi.Ᵽ.ç̥ḽ(Ά), therefore Ά be $ɱ_{Ʀ1,2}$.ƀi.Ᵽ. closed set.

4. Discussion and Conclusion

There are many different ways that topological spaces have been described over time. When transparency is taken into account, several interesting problems arise. In this article, discussed $ɱ_{Ʀ1,2}$.ƀi.open sets, $ɱ_{Ʀ1,2}$.ƀi.closed sets, and in $ɱ_{Ʀ1,2}$.bitopological spaces. Numerous branches from mathematics and associated sciences value its importance. This will be developed upon and additional applications added in later study. Its importance is significant in a variety of mathematical and related scientific domains. On this and adding certain uses will be future research's expansion.

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