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Fuzzy non-adjacency vertex topological spaces associated with undirected fuzzy graphs

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Abstract

This study introduces the novel fuzzy topology for undirect fuzzy graphs, that named "fuzzy non-adjacent vertex topology". This topology clearly highlights non-adjacency relations, which are frequently eclipsed by traditional adjacency-centric models in fuzzy graph theory. The model builds on a relation that forms a sub-base for the new fuzzy topology by taking the vertex set and, for each vertex, specifying its non-adjacent ties to all other vertices in the fuzzy graph. After creating this new fuzzy topology and presenting the main examples of specific types of undirected fuzzy graphs, we studied and analyzed a number of properties associated with it and discussed them in an important context.

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Keywords: *Undirected fuzzy graphs, Fuzzy non-adjacency vertex topology, Fuzzy non-adjacency vertex topological spaces.*

1. Introduction

A fuzzy graph is a useful mathematical tool that enables users to represent fuzzy relationships among elements in a flexible and precise manner [1, 2]. The nature of fuzziness is well-suited to any environment, making it ideal in situations where it is difficult to define relationships in a traditional manner, thereby facilitating problem prediction and resolution. Since it provides a mathematical framework that allows dealing with complex phenomena more flexibly than classical models that rely on specific relationships. Fuzzy graphs can be used in many fields, including engineering and social sciences, due to

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their ability to estimate changing and uncertain relationships. Sources [3-6] provide examples of topological spaces within $graph$ theory.

Chang [7] laid the groundwork for fuzzy topological spaces in 1968. Several years later, in 2021, Atef, El-Atik, and Nawar [8] extended this idea by proposing “fuzzy topological graphs,” which apply fuzzy-graph theory to a designated class of fuzzy sets. Because these two frameworks appear widely across numerous applications, clarifying the connection between fuzzy topological spaces and fuzzy graphs has become an important line of inquiry.

In 2022, Alzubaidi and Dammak [9] Introduced graphic topologies of the fuzzy graphs. Subsequently, in 2023, Gholap and Nikumbh [10] developed fuzziest vertex and edge topologies on simply connected fuzzy graphs, utilizing adjacency relations, and also defined fuzzy neighborhood topology on simple fuzzy graphs.

Building on this, in 2024, researchers in [11, 12] defined and investigated the interrelationships of various fuzzy topological spaces on fuzzy digraphs. They introduced left (right) fuzzy vertex and edge topologies, based on adjacency relations of fuzzy vertices and edges in fuzzy digraphs.

Although many researchers have studied fuzzy topological spaces on various fuzzy graphs, the literature on this concept remains limited. The use of fuzzy topological structures has enabled us to study multiple aspects of fuzzy graphs, which has motivated us to introduce new fuzzy topological spaces on them.

The primary aim of this study is to delineate fuzzy topological spaces within the framework of fuzzy graphs. In our recent research, we introduced a novel category of fuzzy topological space associated with fuzzy undirected graphs, which we designate as fuzzy Non-Adjacency Vertex Topology. We shall examine the benefits of fuzzy Non-Adjacency Vertex Topology concerning essential categories of main types of fuzzy undirected graphs.

We defined our new fuzzy topology (fuzzy Non-Adjacency Vertex Topology) for fuzzy graphs and provided examples of basic fuzzy graphs. we discussed some future work and conclusions.

2. Preliminaries

This section introduces fuzzy graphs theory and fuzzy topology, basic definitions, and preliminaries. All of these fuzzy concepts are widely used and are found in references like [1-4].

Definition 2.1: [4] If S the universe sets. A map $\gamma : S \to [0,1]$ represents a fuzzy subset for the S, and any subsets for the S × S named fuzzy relations on S. For two fuzzy subsets X and Y of S, their join and meet are defined point-wise by

$(X \vee Y)(n) = max\text{Error! Bookmark not defined.}, (X \wedge Y)(n) = min\{X(n), Y(n)\}$ for every $n \epsilon S$.

Definition 2.2: [12] For X the nonempty fuzzy sets. The collection $\mathcal{T}_F \subseteq \mathcal{P}(X)$ named fuzzy topology on X if:

1. $X \in \mathcal{T}_F$.
2. Whenever $A, B \in \mathcal{T}_F$, their intersection $A \cap B$ is also in $\mathcal{T}_F$.
3. For any sub-collection $\{A_i\}_{i\epsilon I} of \mathcal{T}_F$, the union $\bigcup_{i\epsilon I} A_i$ belongs to $\mathcal{T}_F$. element for $\mathcal{T}_F$ are referred to as fuzzy open set.

Definition 2.3: [1] The fuzzy graphs $\mathbb{G}_\mathrm{f} = (V, \gamma, \varpi)$ named built from nonempty vertex sets V, a membership function $\gamma : V \rightarrow [0,1]$ which assigns all vertex a degree of belonging, and a symmetric membership function $\varpi : V \times V \rightarrow [0,1]$ that assigns a degree to every potential edge. These functions satisfy $\varpi(a, b) \leq min\{\gamma(a), \gamma(b)\}$ for all $a, b \epsilon V$. A triple $H = (U, \rho, \nu)$ with $U \subseteq V$ named partial fuzzy sub - graphs when $\rho \leq \gamma$ over U and $\nu \leq \varpi$ on $U \times U$; it becomes spanning when U equals V. the path for the length n be ordered list for the distinct vertices $a_0, a_1, \ldots, a_\mathrm{n}$ with $\varpi(a_{\{i-1\}}, a_i) > 0$ for every $i = 1, \ldots, n$. Two vertices are said to be connected if at least one such path exists between them, and the graphs is called connected when this property holds for each pairs for the vertices.

For the paths $(a_0 = a, a_1, \ldots, a_\mathrm{n} = b)$ in fuzzy graphs, its strength is least memberships values among its consecutive edges, namely $\wedge_{\{i=1\}}^{\{n\}} \varpi(a_{\{i-1\}}, a_i)$. The connectivity strength between vertices a & b, denote $\varpi^\infty(a, b)$, be supremum of these path strengths over all possible routes connecting them. A fuzzy graphs $\mathbb{G}_\mathrm{f} = (V, \gamma, \varpi)$ is deemed connected if $\varpi^\infty(a, b) > 0$ for every pair $a, b \epsilon V$.

Definition 2.4: [2]. The $fuzzy\ graph$s $\mathbb{G}_\mathrm{f} = (\mathcal{V}, \gamma, \varpi)$ be complete when each potential edge attains its maximal admissible weight, namely $\varpi(a, b) = \gamma(a) \wedge \gamma(b)\ \forall\ a, b \epsilon \mathcal{V}$.

Definition 2.5: [2]. Fuzzy graph $\mathbb{G}_\mathrm{f} = (\mathcal{V}, \gamma, \varpi)$ the complete bipartite where vertex sets are partition to two nonempty, disjoints subset $\mathcal{V}_1$ & $\mathcal{V}_2$ s.t $\varpi(a, b) = 0$ whenever a, b lie in the same subset, and $\varpi(a, b) = \gamma(a) \wedge \gamma(b)$, whenever $a\epsilon\mathcal{V}1$ and $b\epsilon\mathcal{V}2$.

Definition 2.6: [3] Consider the structure $(X, \mathcal{T}_f)$, know as a fuzzy topological space. Suppose that β be subfamilies for the $\mathcal{T}_f$, then β be the termed the base for the $\mathcal{T}_f$, where any element for the $\mathcal{T}_f$ can be represented as union for the elements selected from β. Similarly, subfamilies S within $\mathcal{T}_f$ be termed the sub - base where collection formed by finite intersection among its elements constitutes base of the $\mathcal{T}_f$. Furthermore, for two given fuzzy topologies $\mathcal{T}_{f1}$ and $\mathcal{T}_{f2}$ on a set X, we describe $\mathcal{T}_{f1}$ as being coarser than $\mathcal{T}_{f2}$ if it satisfies the inclusion $\mathcal{T}_{f1} \subseteq \mathcal{T}_{f2}$ and conversely, $\mathcal{T}_{f2}$ as finer than $\mathcal{T}_{f1}$.

Definition 2.7: [3] fuzzy topologies $\mathcal{T}_f$ named generate by the sub - family S for the fuzzy subsets within X, provided each members for the $\mathcal{T}_f$ can be represented as the union for the finite intersection for the member from the family S.

Definition 2.:8 [13] If arbitrary intersections of $open$ subsets in a topological space $(X, \mathcal{T})$ are likewise $open$, then the space is known as an Alexandroff space.

Definition 2.9: [2] If $\mathbb{G}_f = (V,\ \gamma,\ \varpi)$ the fuzzy graphs. We call $\mathbb{G}_f$ a fuzzy bipartite graph when vertex sets V can partition to two nonempty, disjoint subset V_1 & V_2 s.t $V = V_1 \cup V_2$ & $V_1 \cap V_2 = \emptyset$. The defining condition is that every edge whose endpoints lie in the same subset must have a membership value of 0. Equivalently, $\varpi(a,\ b) > 0$ only when a and b belong to different subsets; if both vertices are in V_1 or both in V_2, then $\varpi(a,\ b) = 0$.

Definition 2.10: [2] A fuzzy graphs without loops or parallel fuzzy edges will be called a simple fuzzy graph. By a loop we mean an edge incident to a vertex that it repeats on itself. Furthermore, also parallel fuzzy edges are formed when at least two distinct edges connect the same two distinct vertices.

Definition 2.11: [2] A fuzzy path P_n is defined as a fuzzy graphs with exactly n vertex & exactly $n-1$ edge. Its vertices denote sequentially as $\mathcal{v}, \mathcal{v}_2, \dots, \mathcal{v}_n$, while the edges of the fuzzy path connect consecutive vertices $\mathcal{v}_i$ and $\mathcal{v}_{i+1}$ for $i = 1, 2, 3, \dots, n-1$. Furthermore, the fuzzy path be closed if $\mathcal{v}_0 = \mathcal{v}_n$. A fuzzy trial, defined similarly, is called closed where it contains at least one edge that forms a cycle.

Definition 2.12: [2] The fuzzy graphs $\mathbb{G}_f = (\mathcal{V}, \gamma, \varpi)$is termed the $fuzzy$ forest if it includes span fuzzy sub - graphs $F = (\gamma, \nu)$, where F is a crisp forest. The key condition is that any arc (x, y) s.t ϖ(x, y) < $\nu^{\infty}(x, y)$ not part of F. This implies that for any edge excluded from F, there must exist an alternative path within F connecting x and y, whose membership value surpasses $\mu(x, y)$. Additionally, when the fuzzy forest is connected, it is specifically classified as a fuzzy tree.

3. Fuzzy non- adjacency vertex topological space generated by fuzzy graphs

Definition 3.1: Put simple fuzzy graphs $\mathbb{G}_f = (V, \gamma, \varpi)$ with vertex sets $V(\mathbb{G}_f)$. Defined binary relations R on $V(\mathbb{G}_f)$ by $((r, \gamma(r)), (s, \gamma(s))) \in R$ whenever $\varpi(r, s) = 0$, i.e., when the two vertices are non-adjacent. For a fixed vertex $(r, \gamma(r))$, set

$$R[r] = \{(s, \gamma(s)) \in V(\mathbb{G}_f) \mid \varpi(r, s)\ 0\}\ (with\ |R[r]| \leq 2).$$

The collection $\mathcal{S}_{\aleph f} = \{R[r] \mid R[r] \subseteq V(\mathbb{G}_f)\}$ constitutes a sub-basis for a topology on $V(\mathbb{G}_f)$. Let $\beta_{\aleph f}$ is sets for finite intersection for the member for

$\mathcal{S}_{\aleph f}$; $\beta_{\aleph f}$ is basis. The family $T_{\aleph f} = \{ \cup \mathcal{B} \mid \mathcal{B} \subseteq \beta_{\aleph f} \}$ defines a fuzzy topology on $V(\mathbb{G}_{\mathrm{f}})$ called the $fuzzy$ non-adjacency vertex topology generated by $\mathbb{G}_{\mathrm{f}}$. The pair $(V(\mathbb{G}_{\mathrm{f}}), T_{\aleph f})$ is termed the fuzzy non-adjacency vertex topological space associated with $\mathbb{G}_{\mathrm{f}}$. Every set in $T_{\aleph f}$ is classified as $\aleph f$-$open$, and the complement of any such set is deemed $\aleph f$-closed.

Example 3.2: If $\mathbb{G}_{\mathrm{f}}$ the fuzzy graphs with vertex sets

$$\mathcal{V}(\mathbb{G}_{\mathrm{f}}) = \{(a, 0.2), (b, 0.5), (c, 0.4), (d, 0.7)\}$$

As shown in the Figure 1 below. Then we have;

$$R[a] = \{(a, 0.2), (d, 0.7)\}, R[b] = \{(b, 0.5), (d, 0.7)\}$$
$$R[c] = \{(c, 0.4)\}, R[d] = \{(a, 0.2), (d, 0.7)\}, \{(b, 0.5), (d, 0.7)\}$$

then the subbase $S_{\aleph f} = \{\{(a, 0.2), (d, 0.7)\}, \{(b, 0.5), (d, 0.7)\}, \{(c, 0.4)\}\}$

By taking the finite intersection of members of subbase $S_{\aleph f}$, we obtain the base

$$\beta_{\aleph f} = \{\tilde{O}, \{(a, 0.2), (d, 0.7)\}, \{(b, 0.5)(d, 0.7)\}, \{(c, 0.4)\}\}.$$

Thus $T_{\aleph f} = \{0, \{(a, 0.2), (d, 0.7)\}, \{(b, 0.5), (d, 0.7)\}, \{(c, 0.4)\}, \{(d, 0.7)\}, \{(a, 0.2), (b, 0.5), (d, 0.7)\}, \{(a, 0.2), (c, 0.4), (d, 0.7)\}, \{(b, 0.5), (c, 0.4), (d, 0.7)\}, \mathcal{V}(\mathbb{G}_{\mathrm{f}})\}$

Figure 1 illustrates fuzzy graphs employed in Example 3.2, showcasing the vertices and their membership values that motivate the definition of the fuzzy non-adjacency vertex topology.

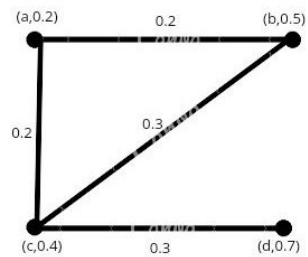

Figure 1
A fuzzy graphs employed in the example 3.2.

Example 3.3: If $\mathbb{G}_{\mathrm{f}}$ is the fuzzy graphs with vertex sets $\mathcal{V}(\mathbb{G}_f) = \{(a, 0.2), (b, 0.5), (c, 0.4)\}$ as in Figure 2 below. Then we have

$$R[a] = \{(a, 0.2), (c, 0.4)\}, \quad R[b] = \{(b, 0.5)\}, \quad R[c] = \{(a, 0.2), (c, 0.4)\},$$

and then we get $S_{\aleph f} = \{\{(a, 0.2), (c, 0.4)\}, \{(b, 0.5)\}\}$.

And $\beta_{\aleph f} = \{0, \{(a, 0.2), (c, 0.4)\}, \{(b, 0.5)\}\}$.

Thus $\mathcal{T}_{\aleph f} = \{0, \{(a, 0.2), (c, 0.4)\}, \{(b, 0.5)\}, \mathcal{V}(\mathbb{G}_f)\}$, It is clear that not discrete.

Figure 2 presents the fuzzy graphs used in Example 3.3; it demonstrates a path-like structure where the resulting fuzzy non-adjacency vertex topology is non-discrete.

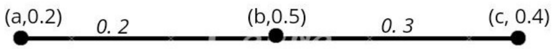

Figure 2
A fuzzy graphs employed for the example 3.3.

Remark 3.5: The fuzzy topology $\mathcal{T}_{\aleph\mathrm{f}}$ on the vertices set of the path of the fuzzy graphs P_n is not discrete if $n \leq 4$ such as in examples 3.3 above, but it is discrete if $n > 4$.

Remark 3.6: For every tree fuzzy graphs, $\mathcal{T}_{\aleph\mathrm{f}}$ satisfy a discrete fuzzy topology.

Remark 3.7: The fuzzy topology $\mathcal{T}_{\aleph\mathrm{f}}$ is not discrete on the fuzzy graph C_4, but on the fuzzy graphs C_3 and C_n when $n > 4$ is discrete.

Remark 3.8: The fuzzy topology $\mathcal{T}_{\aleph\mathrm{f}}$ on every complete fuzzy graph K_n is discrete.

Remark 3.9: The fuzzy graph topology $\mathcal{T}_{\aleph\mathrm{f}}$ on every bipartite fuzzy graph is discrete.

Proposition 3.10: If $\mathbb{G}_\mathrm{f} = (V, \gamma, \varpi)$ the fuzzy graphs. so pair $\left(V, T_{\aleph f}\right)$ be the Alexandroff topological spaces.

Proof: To prove this, it is enough to show that any arbitrary intersection of sets from $S_{\aleph f}$ remains $open$. Consider a subset $S \subseteq V$. If a vertex v belongs to $\bigcap_{\{u \epsilon S\}} R[u]$, When v lies in $R[u]$ for every u in S. Consequently, each u is contained in $R[v]$, so $S \subseteq R[v]$. Both $R[v]$ and S are finite sets. Thus, when S is infinite, the intersection $\bigcap_{\{u \epsilon S\}} R[u]$ is empty; when S is finite, intersections represents finite intersection for the $open$ set and be the therefore $open$. Furthermore, in the fuzzy graphs $\mathbb{G}_\mathrm{f} = (V, \gamma, \varpi)$, for every vertex $v \epsilon V$, the intersections for each $open$ sets includes v forms smallest $open$ neighborhood of v, denoted D_v. The family $B_{\aleph f} = \{D_v \mid v \epsilon V\}$ constitutes a minimal basis for the topology $\left(V, T_{\aleph f}\right)$.

4. Conclusions

This paper introduces the fuzzy non-adjacency vertex topological space on a fuzzy undirected graph. It shown the Alexandroff fuzzy topology as well as studies the relationship between our newly introduced fuzzy topology and several important types of fuzzy graphs, and we obtain some main results written as a remark. The paper also explores relations between the fuzzy graphs and $\mathcal{T}_{\aleph\mathrm{f}}$. In future work, this fuzzy topology can solve a variety of undirected fuzzy graphs dependent problems, like in Global Positioning System GPS. Also, this framework *opens* a various application in network vulnerability analysis, anomaly detection, and optimization of fuzzy systems such as smart grids or biological networks.

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