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Fixed point theorems in complex public metric spaces

Mohammed Hamad Fayyadh *
Zena Hussein Maibed †
Department of Mathematics
College of Education for Pure Science (Ibn Al-Haitham)
University of Baghdad
Baghdad
Iraq

Abstract

This article introduces the complex - valued public metric spaces, with some examples, and study its properties as well as we have been proven an existence for the fixed points via weakly types compatible maps and also study ($\mathbb{CLR}_g$)) and ($\mathbb{EA}$) properties.

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1. Introduction

Generalized valued metric spaces are important in many fields, including science, engineering, and economics [1]. Numerous authors have proposed extraordinary generalizations of metric spaces, including [2]. In 1989, researchers began to expand fixed-point (F.P.) theorems for metric spaces, studying fixed-point theorems for multivalued mappings on complete metric spaces [3]. In 2007, Agarwal, Regan, and Sahu [4] presented a contraction mapping and proved that a mapping in a complete metric space has a unique fixed point.

Many studies have been conducted on the F.P. and its applications in various spaces; see [5], [6], [7], [8], [9], [10], [11], [12]. In 2011, Azam, Fisher, and Khan [13] introduced the concept of complex metric spaces and proved the existence of F.P. by contraction maps. In 2014, Mlaiki [14] introduced the concept of complex S-metric space, which is a generalization of complex metric space. In 2014, Mukheimer [15] studied F.P. Theorems in the so-called b-metric space. The authors [16] presented an interesting study in which they discovered a series of common F.P. theorems involving specific logical conditions in these

* *E-mail:* **mohammed.hamad2203@ihcoedu.uobaghdad.edu.iq (Corresponding Author)**
† *E-mail:* **zena.h.m@ihcoedu.uobaghdad.edu.iq**

complex spaces. We will present results that are considered an improvement and generalization of many studies.

In 2014, the authors [17] studied common F.P. theorem in the ***complex - value metric space*** as well as proved them via w-compatible self-maps.

we develop this work, introduced novel spaces as well as study common F.P. theorems in a complex public metric space.

if $₵$ denotes to sets for the all complex number & if $ţ_1$, $ţ_2 \in ₵$. A partials

orders $ţ_1 \precsim ţ_2$ on the $₵$ define by $ţ_1 \precsim ţ_2 \Leftrightarrow Re(ţ_1) \leq Re(ţ_2)$ & $Im(ţ_1) \leq Im(ţ_2)$.

Definition 1.1 [18]: Let $S \neq \emptyset$, and the mapping $\Delta: S \times S \to ₵$ satisfies the following:

1) $\Delta(ʂ, ƭ) \succsim 0$ for all $ʂ, ƭ \in S$ and $\Delta(ʂ, ƭ) = 0$ if and only if $ʂ = ƭ$
2) $\Delta(ʂ, ƭ) = \Delta(ƭ, ʂ)$
3) $\Delta(ʂ, ƭ) \precsim \Delta(ʂ, ţ) + \Delta(ţ, ƭ)$ for all $ʂ, ƭ, ţ \in S$

Then Δ is called CVPM on S and (S, Δ) It is called the CVPM space.

Definition 1.2 [18]: Let $S \neq \emptyset$ and $\rho: S \times S \to [1, \infty)$ be a function and $\Delta: S \times S \to ₵$ satisfies the following:

1) $\Delta(ʂ, ƭ) \succsim 0 \; \forall ʂ, ƭ \in S$ & $\Delta(ʂ, ƭ) = 0$ iff $ʂ = ƭ$
2) $\Delta(ʂ, ƭ) = \Delta(ƭ, ʂ)$
3) $\Delta(ʂ, ƭ) \precsim \rho(ʂ, ƭ)[\Delta(ʂ, ţ) + \Delta(ţ, ƭ)]$ for all $ʂ, ƭ, ţ \in S$

So (S, Δ) named the CV-extend metric spaces.

Definition 1.3 [19]: if $S \neq \emptyset$, $\rho_1, \rho_2: S \times S \to [1, \infty)$ the function & $\Delta: S \times S \times S \to ₵$ satisfies the following:

1) $\Delta(ʂ, ʂ, ƭ) \succ 0$ for all $ʂ, ƭ \in S$, $ʂ \neq ƭ$
2) If $ʂ = ƭ = ţ$, then $\Delta(ʂ, ƭ, ţ) = 0$
3) $\Delta(ʂ, ʂ, ƭ) \precsim \Delta(ʂ, ƭ, ţ)$ for all $ʂ, ƭ, ţ \in S$ and $ƭ \neq ţ$
4) $\Delta(ʂ, ƭ, ţ) = \Delta(ʂ, ţ, ƭ) = \Delta(ƭ, ʂ, ţ) = \Delta(ƭ, ţ, ʂ) = \Delta(ţ, ʂ, ƭ) = \Delta(ţ, ƭ, ʂ)$
5) $\Delta(ʂ, ƭ, ţ) \precsim \gamma[\Delta(ʂ, ƀ, ƀ) + \Delta(ƀ, ƭ, ţ)]$ for all $ʂ, ƭ, ţ, \gamma \in S$

Then (S, Δ) is called CV.G_b-metric space.

Definition 1.4 [16]: A maps $ɦ$ & $\mathcal{J}$ named weakly type compatibles (w-compatible) where are commuted at coincidences point.

Definition 1.5 [16]: A maps $ɦ$ and $\mathcal{J}$ of are satisfy $\mathbb{EA}$. property if $\exists \{ʂ_n\}$ in S s.t $lim_{n\to\infty} ɦʂ_n = lim_{n\to\infty} \mathcal{J}ʂ_n = q$, for some q in S.

Definition 1.6 [16]: A maps $ɦ$ and $\mathcal{J}$ satisfy $(\mathbb{CLR}_g)$ property if $\exists \{ʂ_n\}$ in S s.t $lim_{n\to\infty} ɦʂ_n = lim_{n\to\infty} \mathcal{J}ʂ_n = \mathcal{J}ʂ$, for some $ʂ$ in S.

2. Main Results

We defined the complex public metric space and some important theorems

Definition 2.1: Let $S \neq \emptyset$, $\rho_1, \rho_2 : S \times S \to [1, \infty)$ be a function and $\Delta : S \times S \times S \to \mathbb{C}$ satisfies the following:

1) $\Delta(\mathfrak{s}, \mathfrak{t}, \mathfrak{z}) \gtrsim 0$ for all $\mathfrak{s}, \mathfrak{t}, \mathfrak{z} \in S$
2) If $\mathfrak{s} = \mathfrak{t} = \mathfrak{z}$, then $\Delta(\mathfrak{s}, \mathfrak{t}, \mathfrak{z}) = 0$
3) $\Delta(\mathfrak{s}, \mathfrak{t}, \mathfrak{z}) = \Delta(\mathfrak{s}, \mathfrak{z}, \mathfrak{t}) = \Delta(\mathfrak{t}, \mathfrak{s}, \mathfrak{z}) = \Delta(\mathfrak{t}, \mathfrak{z}, \mathfrak{s}) = \Delta(\mathfrak{z}, \mathfrak{s}, \mathfrak{t}) = \Delta(\mathfrak{z}, \mathfrak{t}, \mathfrak{s})$
4) $\Delta(\mathfrak{s}, \mathfrak{t}, \mathfrak{z}) \lesssim \rho_1(\mathfrak{s}, \mathfrak{t})\big(\Delta(\mathfrak{s}, \mathfrak{l}, \mathfrak{l})\big) + \rho_2(\mathfrak{t}, \mathfrak{z})\big(\Delta(\mathfrak{l}, \mathfrak{t}, \mathfrak{z})\big)$

   Then Δ is called CVPM on S and (S, Δ) It is called the CVPM space.

Example 2.2: if $\Delta : S \times S \times S \to \mathbb{C}$ s.t $\Delta(\mathfrak{s}, \mathfrak{t}, \mathfrak{z}) = ki(|\mathfrak{s} - \mathfrak{t}| + |\mathfrak{t} - \mathfrak{z}| + |\mathfrak{s} - \mathfrak{z}|)$, $k \in \mathbb{N}$, $\mathfrak{s}, \mathfrak{t}, \mathfrak{z} \in S$ and $\rho_1, \rho_2 : S \times S \to [1, \infty)$, then (S, Δ) is CVPM- space.

Definition 2.3: if (S, Δ) is the CVPM- spaces, so S has

M-property if satisfy $\Delta(\mathfrak{s}, \mathfrak{t}, \mathfrak{z}) \gtrsim \Delta(\mathfrak{s}, \mathfrak{s}, \mathfrak{t}) \succ 0, \forall\, \mathfrak{s}, \mathfrak{t}, \mathfrak{z} \in S$ and $\mathfrak{s} \neq \mathfrak{t}$

Proposition 2.4: In any CVPM-space that has the M-property, then for all $\mathfrak{s}, \mathfrak{t}, \mathfrak{z} \in S$ it follows that: 1) If $\Delta(\mathfrak{s}, \mathfrak{t}, \mathfrak{z}) = 0$ then $\mathfrak{s} = \mathfrak{t} = \mathfrak{z}$

2) $\Delta(\mathfrak{s}, \mathfrak{t}, \mathfrak{t}) \lesssim [\rho_1(\mathfrak{t}, \mathfrak{s}) + \rho_2(\mathfrak{t}, \mathfrak{t})](\Delta(\mathfrak{t}, \mathfrak{s}, \mathfrak{s}))$

Theorem 2.5: In any CVPM- space (S, Δ) has M-property if

i) $\Delta(\mathfrak{h}\mathfrak{s}, \mathfrak{h}\mathfrak{t}, \mathfrak{h}\mathfrak{t}) \lesssim$

$$\varphi_1 \Delta(\mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{t}, \mathcal{J}\mathfrak{t}) + \varphi_2 \frac{\Delta(\mathfrak{h}\mathfrak{s}, \mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s})\, \Delta(\mathfrak{h}\mathfrak{t}, \mathcal{J}\mathfrak{t}, \mathcal{J}\mathfrak{t})}{1 + \Delta(\mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{t}, \mathcal{J}\mathfrak{t})} + \varphi_3 \frac{\Delta(\mathcal{J}\mathfrak{s}, \mathfrak{h}\mathfrak{t}, \mathfrak{h}\mathfrak{t})\, \Delta(\mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{t}, \mathcal{J}\mathfrak{t})}{1 + \Delta(\mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{t}, \mathcal{J}\mathfrak{t})} + \varphi_4 \frac{\Delta(\mathfrak{h}x, \mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s})\, \Delta(\mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{t}, \mathcal{J}\mathfrak{t})}{1 + \Delta(\mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{t}, \mathcal{J}\mathfrak{t})} + \varphi_5 \frac{\Delta(\mathcal{J}\mathfrak{s}, \mathfrak{h}\mathfrak{t}, \mathfrak{h}\mathfrak{t})\, \Delta(\mathfrak{h}\mathfrak{t}, \mathcal{J}\mathfrak{t}, \mathcal{J}\mathfrak{t})}{1 + \Delta(\mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{t}, \mathcal{J}\mathfrak{t})}$$

ii) $\mathfrak{h}$ and $\mathcal{J}$ have $(\mathbb{CLR}_g)$ property, iii) $\mathfrak{h}$ & $\mathcal{J}$ the w-compatibles.

So each of $\mathfrak{h}$ & $\mathcal{J}$ has common F.P.

Proof: By condition(ii), $\exists \{\mathfrak{s}_n\}$ in S s.t $lim_{n\to\infty} \mathfrak{h}\mathfrak{s}_n = lim_{n\to\infty} \mathcal{J}\mathfrak{s}_n = \mathcal{J}\mathfrak{s}$, $\mathfrak{s} \in S$. From (i),

$$\Delta(\mathfrak{h}\mathfrak{s}, \mathfrak{h}\mathfrak{s}_n, \mathfrak{h}\mathfrak{s}_n) \lesssim \varphi_1 \Delta(\mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s}_n, \mathcal{J}\mathfrak{s}_n) + \varphi_2 \frac{\Delta(\mathfrak{h}\mathfrak{s}, \mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s}) \Delta(\mathfrak{h}\mathfrak{s}_n, \mathcal{J}\mathfrak{s}_n, \mathcal{J}\mathfrak{s}_n)}{1 + \Delta(\mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s}_n, \mathcal{J}\mathfrak{s}_n)} + \varphi_3 \frac{\Delta(\mathcal{J}\mathfrak{s}, \mathfrak{h}\mathfrak{s}_n, \mathfrak{h}\mathfrak{s}_n) \Delta(\mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s}_n, \mathcal{J}\mathfrak{s}_n)}{1 + \Delta(\mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s}_n, \mathcal{J}\mathfrak{s}_n)} + \varphi_4 \frac{\Delta(\mathfrak{h}\mathfrak{s}, \mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s}) \Delta(\mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s}_n, \mathcal{J}\mathfrak{s}_n)}{1 + \Delta(\mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s}_n, \mathcal{J}\mathfrak{s}_n)} + \varphi_5 \frac{\Delta(\mathcal{J}\mathfrak{s}, \mathfrak{h}\mathfrak{s}_n, \mathfrak{h}\mathfrak{s}_n)\, \Delta(\mathfrak{h}\mathfrak{s}_n, \mathcal{J}\mathfrak{s}_n, \mathcal{J}\mathfrak{s}_n)}{1 + \Delta(\mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s}_n, \mathcal{J}\mathfrak{s}_n)}$$

$As\ n \to \infty$,we get

$$\Delta(\mathfrak{h}x, \mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s}) \lesssim \varphi_1 \Delta(\mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s}) + \varphi_2 \frac{\Delta(\mathfrak{h}\mathfrak{s}, \mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s})\, \Delta(\mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s})}{1 + \Delta(\mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s})} + \varphi_3 \frac{\Delta(\mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s})\, \Delta(\mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s})}{1 + \Delta(\mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s})} + \varphi_4 \frac{\Delta(\mathfrak{h}\mathfrak{s}, \mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s}) \Delta(\mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s})}{1 + \Delta(\mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s})} + \varphi_5 \frac{\Delta(\mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s}) \Delta(\mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s})}{1 + \Delta(\mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s})} = 0,$$

i.e. $\Delta(\mathfrak{h}x, \mathcal{J}\mathfrak{s}, \mathcal{J}\mathfrak{s}) \lesssim 0$, by (1) of proposition (2.4), we obtain $\mathfrak{h}s = \mathcal{J}s$.

Now, consider $\lambda = \mathfrak{h}\mathfrak{s} = \mathcal{J}\mathfrak{s}$ .So, $\mathfrak{h}\mathcal{J}\mathfrak{s} = \mathcal{J}\mathfrak{h}\mathfrak{s}$, implies that, $\mathfrak{h}\lambda = \mathfrak{h}\mathcal{J}\mathfrak{s} = \mathcal{J}\mathfrak{h}\mathfrak{s} = \mathcal{J}\lambda$.

Now, To prove $\mathcal{J}\lambda = \lambda$. If not .From (i), we have $\Delta(\lambda, \mathcal{J}\lambda, \mathcal{J}\lambda) = \Delta(\mathfrak{h}\mathfrak{s}, \mathfrak{h}\lambda, \mathfrak{h}\lambda)$

$$\precsim \varphi_1\Delta(\mathcal{J}\mathfrak{s},\mathcal{J}\lambda,\mathcal{J}\lambda) + \varphi_2\frac{\Delta(\hbar\mathfrak{s},\mathcal{J}\mathfrak{s},\mathcal{J}\mathfrak{s})\Delta(\hbar\lambda,\mathcal{J}\lambda,\mathcal{J}\lambda)}{1+\Delta(\mathcal{J}\mathfrak{s},\mathcal{J}\lambda,\mathcal{J}\lambda)} + \varphi_3\frac{\Delta(\mathcal{J}\mathfrak{s},\hbar\lambda,\hbar\lambda)\Delta(\mathcal{J}\mathfrak{s},\mathcal{J}\lambda,\mathcal{J}\lambda)}{1+\Delta(\mathcal{J}\mathfrak{s},\mathcal{J}\lambda,\mathcal{J}\lambda)} + \varphi_4\frac{\Delta(\hbar\mathfrak{s},\mathcal{J}\mathfrak{s},\mathcal{J}\mathfrak{s})\Delta(\mathcal{J}\mathfrak{s},\mathcal{J}\lambda,\mathcal{J}\lambda)}{1+\Delta(\mathcal{J}\mathfrak{s},\mathcal{J}\lambda,\mathcal{J}\lambda)} + \varphi_5\frac{\Delta(\mathcal{J}\mathfrak{s},\hbar\lambda,\hbar\lambda)\Delta(\hbar\lambda,\mathcal{J}\lambda,\mathcal{J}\lambda)}{1+\Delta(\mathcal{J}\mathfrak{s},\mathcal{J}\lambda,\mathcal{J}\lambda)} = \varphi_1\Delta(\lambda,\mathcal{J}\lambda,\mathcal{J}\lambda) + \varphi_3\frac{\Delta(\lambda,\mathcal{J}\lambda,\mathcal{J}\lambda)\Delta(\lambda,\mathcal{J}\lambda,\mathcal{J}\lambda)}{1+\Delta(\lambda,\mathcal{J}\lambda,\mathcal{J}\lambda)}$$

$$|\Delta(\lambda,\mathcal{J}\lambda,\mathcal{J}\lambda)| \le \varphi_1|\Delta(\lambda,\mathcal{J}\lambda,\mathcal{J}\lambda)| + \varphi_3\frac{|\Delta(\lambda,\mathcal{J}\lambda,\mathcal{J}\lambda)||\Delta(\lambda,\mathcal{J}\lambda,\mathcal{J}\lambda)|}{|1+\Delta(\lambda,\mathcal{J}\lambda,\mathcal{J}\lambda)|}$$

Since $|1 + \Delta(\lambda,\mathcal{J}\lambda,\mathcal{J}\lambda)| > |\Delta(\lambda,\mathcal{J}\lambda,\mathcal{J}\lambda)|$, then $|\Delta(\lambda,\mathcal{J}\lambda,\mathcal{J}\lambda)| \le (\varphi_1+\varphi_3)|\Delta(\lambda,\mathcal{J}\lambda,\mathcal{J}\lambda)|$,

So, $\varphi_1+\varphi_3 \ge 1$, which is a contradiction. i.e., λ is the common (F.P) of $\hbar$ and $\mathcal{J}$.

Theorem 2.6: *In Any CVPM- space (S,Δ) has M-property if the condition (iii) of theorem (2.5) satisfied and the following*:

i) $\hbar S \subseteq \mathcal{J}S$,

ii) $\Delta(\hbar\mathfrak{s},\hbar\mathfrak{t},\hbar\mathfrak{t}) \precsim$
$$\varphi_1\Delta(\mathcal{J}\mathfrak{s},\mathcal{J}\mathfrak{t},\mathcal{J}\mathfrak{t}) + \varphi_2\frac{\Delta(\hbar\mathfrak{s},\mathcal{J}\mathfrak{s},\mathcal{J}\mathfrak{s})\,\Delta(\hbar\mathfrak{t},\mathcal{J}\mathfrak{t},\mathcal{J}\mathfrak{t})}{1+\Delta(\mathcal{J}\mathfrak{s},\mathcal{J}\mathfrak{t},\mathcal{J}\mathfrak{t})} + \varphi_3\frac{\Delta(\mathcal{J}\mathfrak{s},\hbar\mathfrak{t},\hbar\mathfrak{t})\,\Delta(\mathcal{J}\mathfrak{s},\mathcal{J}\mathfrak{t},\mathcal{J}\mathfrak{t})}{1+\Delta(\mathcal{J}\mathfrak{s},\mathcal{J}\mathfrak{t},\mathcal{J}\mathfrak{t})} + \varphi_4\frac{\Delta(\hbar\mathfrak{s},\mathcal{J}\mathfrak{s},\mathcal{J}\mathfrak{s})\,\Delta(\mathcal{J}\mathfrak{s},\mathcal{J}\mathfrak{t},\mathcal{J}\mathfrak{t})}{1+\Delta(\mathcal{J}\mathfrak{s},\mathcal{J}\mathfrak{t},\mathcal{J}\mathfrak{t})} + \varphi_5\frac{\Delta(\mathcal{J}\mathfrak{s},\hbar\mathfrak{t},\hbar\mathfrak{t})\,\Delta(\hbar\mathfrak{t},\mathcal{J}\mathfrak{t},\mathcal{J}\mathfrak{t})}{1+\Delta(\mathcal{J}\mathfrak{s},\mathcal{J}\mathfrak{t},\mathcal{J}\mathfrak{t})}$$

iii) $\hbar$ and $\mathcal{J}$ satisfy ($\mathbb{EA}$) property, iv) $\mathcal{J}S$ be the closed subset for the S.

so $\hbar$ & $\mathcal{J}$ has the common (F.P).

Proof: By condition (iii), $\exists\ \{\mathfrak{s}_n\}$ in S such that

$lim_{n\to\infty}\hbar\mathfrak{s}_n = lim_{n\to\infty}\mathcal{J}\mathfrak{s}_n = q,\ q \in S$. (2.3)

But, $\mathcal{J}S$ is closed in S. So, $lim_{n\to\infty}\mathcal{J}\mathfrak{s}_n = \mathcal{J}\flat$, for some $\flat \in S$.

we have $lim_{n\to\infty}\hbar\mathfrak{s}_n = \mathcal{J}\flat$.To prove $\hbar\flat = \mathcal{J}\flat$. From (ii), we have

$$\Delta(\hbar\flat,\hbar\mathfrak{s}_n,\hbar\mathfrak{s}_n) \precsim \varphi_1\Delta(\mathcal{J}\flat,\mathcal{J}\mathfrak{s}_n,\mathcal{J}\mathfrak{s}_n) + \varphi_2\frac{\Delta(\hbar\flat,\mathcal{J}\flat,\mathcal{J}\flat)\Delta(\hbar\mathfrak{s}_n,\mathcal{J}\mathfrak{s}_n,\mathcal{J}\mathfrak{s}_n)}{1+\Delta(\mathcal{J}\flat,\mathcal{J}\mathfrak{s}_n,\mathcal{J}\mathfrak{s}_n)} + \varphi_3\frac{\Delta(\mathcal{J}\flat,\hbar\mathfrak{s}_n,\hbar\mathfrak{s}_n)\Delta(\mathcal{J}\flat,\mathcal{J}\mathfrak{s}_n,\mathcal{J}\mathfrak{s}_n)}{1+\Delta(\mathcal{J}\flat,\mathcal{J}\mathfrak{s}_n,\mathcal{J}\mathfrak{s}_n)} + \varphi_4\frac{\Delta(\hbar\flat,\mathcal{J}\flat,\mathcal{J}\flat)\Delta(\mathcal{J}\flat,\mathcal{J}\mathfrak{s}_n,\mathcal{J}x_n)}{1+\Delta(\mathcal{J}\flat,\mathcal{J}\mathfrak{s}_n,\mathcal{J}\mathfrak{s}_n)} + \varphi_5\frac{\Delta(\mathcal{J}\flat,\hbar\mathfrak{s}_n,\hbar\mathfrak{s}_n)\Delta(\hbar\mathfrak{s}_n,\mathcal{J}\mathfrak{s}_n,\mathcal{J}\mathfrak{s}_n)}{1+\Delta(\mathcal{J}\flat,\mathcal{J}\mathfrak{s}_n,\mathcal{J}\mathfrak{s}_n)}$$

As $n \to \infty$, we have

$$\Delta(\hbar\flat,\mathcal{J}\flat,\mathcal{J}\flat) \precsim \varphi_1\Delta(\mathcal{J}\flat,\mathcal{J}\flat,\mathcal{J}\flat) + \varphi_2\frac{\Delta(\hbar\flat,\mathcal{J}\flat,\mathcal{J}\flat)\Delta(\mathcal{J}\flat,\mathcal{J}\flat,\mathcal{J}\flat)}{1+\Delta(\mathcal{J}\flat,\mathcal{J}\flat,\mathcal{J}\flat)} + \varphi_3\frac{\Delta(\mathcal{J}\flat,\mathcal{J}\flat,\mathcal{J}\flat)\Delta(\mathcal{J}\flat,\mathcal{J}\flat,\mathcal{J}\flat)}{1+\Delta(\mathcal{J}\flat,\mathcal{J}\flat,\mathcal{J}\flat)} + \varphi_4\frac{\Delta(\hbar\flat,\mathcal{J}\flat,\mathcal{J}\flat)\Delta(\mathcal{J}\flat,\mathcal{J}\flat,\mathcal{J}\flat)}{1+\Delta(\mathcal{J}\flat,\mathcal{J}\flat,\mathcal{J}\flat)} + \varphi_5\frac{\Delta(\mathcal{J}\flat,\mathcal{J}\flat,\mathcal{J}\flat)\Delta(\mathcal{J}\flat,\mathcal{J}\flat,\mathcal{J}\flat)}{1+\Delta(\mathcal{J}\flat,\mathcal{J}\flat,\mathcal{J}\flat)} = 0,$$ So, $\Delta(\hbar\flat,\mathcal{J}\flat,\mathcal{J}\flat) \precsim 0$.

From (ii), we get

$$\Delta(\hbar\hbar\flat,\hbar\flat,\hbar\flat) \precsim \varphi_1\Delta(\mathcal{J}\hbar\flat,\mathcal{J}\flat,\mathcal{J}\flat) + \varphi_2\frac{\Delta(\hbar\hbar\flat,\mathcal{J}\hbar\flat,\mathcal{J}\hbar\flat)\Delta(\hbar\flat,\mathcal{J}\flat,\mathcal{J}\flat)}{1+\Delta(\mathcal{J}\hbar\flat,\mathcal{J}\flat,\mathcal{J}\flat)} + \varphi_3\frac{\Delta(\mathcal{J}\hbar\flat,\hbar\flat,\hbar\flat)\Delta(\mathcal{J}\hbar\flat,\mathcal{J}\flat,\mathcal{J}\flat)}{1+\Delta(\mathcal{J}\hbar\flat,\mathcal{J}\flat,\mathcal{J}\flat)} + \varphi_4\frac{\Delta(\hbar\hbar\flat,\mathcal{J}\hbar\flat,\mathcal{J}\hbar\flat)\Delta(\mathcal{J}\hbar\flat,\mathcal{J}\flat,\mathcal{J}\flat)}{1+\Delta(\mathcal{J}\hbar\flat,\mathcal{J}\flat,\mathcal{J}\flat)} + \varphi_5\frac{\Delta(\mathcal{J}\hbar\flat,\hbar\flat,\hbar\flat)\Delta(\hbar\flat,\mathcal{J}\flat,\mathcal{J}\flat)}{1+\Delta(\mathcal{J}\hbar\flat,\mathcal{J}\flat,\mathcal{J}\flat)}$$

$$= \varphi_1\Delta(\hbar\hbar\flat,\hbar\flat,\hbar\flat) + \varphi_3\frac{\Delta(\hbar\hbar\flat,\hbar\flat,\hbar\flat)\Delta(\hbar\hbar\flat,\mathcal{J}\flat,\mathcal{J}\flat)}{1+\Delta(\hbar\hbar\flat,\mathcal{J}\flat,\mathcal{J}\flat)},$$

$$|\Delta(\hbar\hbar\flat,\hbar\flat,\hbar\flat)| \le \varphi_1|\Delta(\hbar\hbar\flat,\hbar\flat,\hbar\flat)| + \varphi_3\frac{|\Delta(\hbar\hbar\flat,\hbar\flat,\hbar\flat)||\Delta(\hbar\hbar\flat,\hbar\flat,\hbar\flat)|}{|1+\Delta(\hbar\hbar\flat,\hbar\flat,\hbar\flat)|}$$

Since $|1+\Delta(\hbar\hbar\flat,\hbar\flat,\hbar\flat)| > |\Delta(\hbar\hbar\flat,\hbar\flat,\hbar\flat)|$, So, $|\Delta(\text{ɦɦȴ}, \text{ɦȴ}, \text{ɦȴ})| \leq \varphi_1 |\Delta(\text{ɦɦȴ}, \text{ɦȴ}, \text{ɦȴ})| + \varphi_3 \frac{|\Delta(\text{ɦɦȴ},\text{ɦȴ},\text{ɦȴ})||\Delta(\text{ɦɦȴ},\text{ɦȴ},\text{ɦȴ})|}{|\Delta(\text{ɦɦȴ},\text{ɦȴ},\text{ɦȴ})|}$ implies that, $\varphi_1 + \varphi_3 \geq 1$. Hence $\text{ɦɦȴ} = \text{ɦȴ} = \mathcal{J}\text{ɦȴ}$.

3. Discussion and Conclusion

We obtained some results in our study of common F.P. theorem in the complex-value public metric space, when the unique common (F.P) point was obtained for w-compatible mappings, and the two properties ($\mathbb{CLR}_g$) and ($\mathbb{EA}$) were used.

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