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Coefficients estimate of holomorphic functions defined by neutrosophic q-poisson distribution in conic domain

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Abstract

In This work, a neutrosophic q-Poisson's distributions are use to estimate coefficients of holomorphic functions associated with conic domains. Through the application for Neutrosophic q-Poisson's series, two novel subclasses for the function have been introduced, and upper bound of coefficient $|a_2|$, $|a_3|$ have been determined within these subclasses. These findings provide significant insights into the use of Neutrosophic distribution tools and contribute to the study of Holomorphic functions linked to conic domains.

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1. Introduction

Let Ŧ denotes to family for every holomorphic function Υ define in open unit disks $\breve{U} = \{\text{ʓ} \in C: |\text{ʓ}| < 1\}$, normalize by condition $\Upsilon(0) = 0$ & $\Upsilon'(0) - 1 = 0$. so, $\forall \Upsilon \in$ Ŧ has Taylor series expansions for a forms:

$$\Upsilon(\text{ʓ}) = \text{ʓ} + \sum_{n=2}^{\infty} \varkappa_n \text{ʓ}^n, \quad \text{ʓ} \in \breve{U}. \tag{1}$$

According to this, each functions $\Upsilon \in \mathcal{S}$ have inverse maps Υ^{-1} that satisfies the following conditions:

$$\Upsilon^{-1}(\Upsilon(\text{ʓ})) = \text{ʓ}, \quad \text{ʓ} \in \breve{U},$$

and

$$\Upsilon(\Upsilon^{-1}(\varpi)) = \varpi, \quad |\varpi| < r_0(\Upsilon), \quad r_0(\Upsilon) \geq \frac{1}{4}.$$

In fact, an inverse functions be

$$\Upsilon^{-1}(\varpi) = \varpi - \varkappa_2\varpi^2 + (2\varkappa_2^2 - \varkappa_3)\varpi^3 - (5\varkappa_2^3 - 5\varkappa_2\varkappa_3 + \varkappa_4)\varpi^4 + \cdots. \tag{2}$$

Let C be the complex plane. A subsets $\mathbb{G}$ for complex plane C named domain if it is open and any two point in $\mathbb{G}$ is connect by broke line segments in $\mathbb{G}$. The domain $\mathbb{G}$ is said to be simply connected if it has no holes.

Furthermore, we use the notation $\mathcal{S} \subset$ Ŧ to represent a family for each function which is univalent in $\breve{U}$. For functions Υ, $g \in$ Ŧ, the Υ named subordinated to g in $\breve{U}$, and this relation is denoted by

$$\Upsilon(\text{ʓ}) \prec g(\text{ʓ}),$$

where there is Schwarz functions $\varpi(\text{ʓ})$, holomorphic in $\breve{U}$, where $\varpi(0) = 0$ & $|\varpi(\text{ʓ})| < 1 \ \forall \text{ʓ} \in \breve{U}$, s.t

$$\Upsilon(\text{ʓ}) = g(\varpi(\text{ʓ})) \quad \text{for all } \text{ʓ} \in \breve{U}.$$

Where a functions g be the univalent in the $\breve{U}$, so we get a equivalences:

$$\Upsilon(\text{ʓ}) \prec g(\text{ʓ}) \Leftrightarrow \Upsilon(0) = g(0) \ \& \ \Upsilon(\breve{U}) \subset g(\breve{U}). \tag{3}$$

A function $\Upsilon \in$ Ŧ is called strongly starlike of order $\xi (0 < \xi < 1)$ if and only if

$$\frac{z\Upsilon'(z)}{\Upsilon(z)} \prec \left(\frac{1+z}{1-z}\right)^{\xi}. \tag{4}$$

One can find that ([1])

$$\frac{z\Upsilon'(z)}{\Upsilon(z)} \prec \left(\frac{1+z}{1-z}\right)^{\xi} \Leftrightarrow \left(\frac{z\Upsilon'(z)}{\Upsilon(z)}\right)^{1/\xi} = \frac{1+z}{1-z}, \tag{5}$$

equivalently, this requirement may be expressed in the following form:

$$\left|\arg\left(\frac{z\Upsilon'(z)}{\Upsilon(z)}\right)\right| < \frac{\xi\pi}{2}.$$

The authors [2, 3] provide definitions for the this operators, while Exton [4] was first to use q-differences operators $\mathfrak{D}_q$ to extend classes for S^* in quantum calculus. Additionally, (see [5, 6, 7]) were pioneer in applying fundamentals (or q-) hyper - geometric function within Geometric Function Theory's. Since then, numerous mathematicians have conducted extensive research in this area, significantly contributing to its development and influencing several branches of mathematical analysis and geometric function theory.

For example, [2] explored new applications for a this operators in relations to q-Raina functions, [6] develop and explain the family for the star - shaped function associate with symmetric booth lemniscates. For further details on the theory of the q-calculus operator in, one can refer to works [8, 9, 10, 11]. This research builds upon earlier studies in a fields of that employed q-calculus.

[12] introduced the q-derivatives operators $\mathfrak{D}_q$ for functions Υ given as equation (1) as follows:

$$\mathfrak{D}_q\Upsilon(z) = \begin{cases} \frac{\Upsilon(z)-\Upsilon(qz)}{(1-q)z}, if\ z \neq 0 \\ \Upsilon'(z), if\ z = 0 \end{cases}, \tag{6}$$

for $q \in (0,1)$. It is clear that

$$\lim_{q\to 1^-} \mathfrak{D}_q\,\Upsilon(z) = \Upsilon'(z),$$

for Υ given by equation (1).

The formula for q-derivatives for functions z^n as follows:

$$\mathfrak{D}_q z^n = [n]_q z^{n-1}, \quad n \in N, \tag{7}$$

where

$$[n]_q = \sum_{k=1}^{n} q^{k-1}, \quad [0]_q = 0, \tag{8}$$

be q-analogue for natural numbers (named a basic number n). It follows from (8) that

$$[n]_q = \frac{1-q^n}{1-q}, \quad \lim_{q\to 1^-}[n]_q = n, \quad q \in (0,1).$$

The following properties of the q-derivative operator $\mathfrak{D}_q$ hold:

$$\mathfrak{D}_q[\Upsilon(z)g(z)] = \Upsilon(qz)\mathfrak{D}_q g(z) + g(z)\mathfrak{D}_q\Upsilon(z), \tag{9}$$

$$\mathfrak{D}_q[a\Upsilon(z) + bg(z)] = a\mathfrak{D}_q\Upsilon(z) + b\mathfrak{D}_q g(z), \quad a, b \in C, (5)$$

$$\mathfrak{D}_q\left[\frac{\Upsilon(z)}{g(z)}\right] = \frac{g(z)\mathfrak{D}_q\Upsilon(z) - \Upsilon(z)\mathfrak{D}_q g(z)}{g(qz)g(z)}, \quad g(qz)g(z) \neq 0. \tag{11}$$

The general Leibniz rule for the n-th q-derivative of a product is

$$\mathfrak{D}_q^{(n)}(\Upsilon g)(\wp) = \sum_{k=0}^{n}\binom{n}{k}_q \mathfrak{D}_q^{(k)}\Upsilon(q^{n-k}\wp)\mathfrak{D}_q^{(n-k)}g(q^k\wp), \tag{12}$$

where the q-binomials coefficient are

$$\binom{n}{k}_q = \frac{[n]_q!}{[k]_q![n-k]_q!},$$

with $[n]_q! = \prod_{k=1}^{n}[k]_q$ and $[0]_q! = 1$. Here, $[n]_q$ is the q-factorial function, and it satisfies

$$\lim_{q\to 1^-}[n]_q\,! = n!.$$

Similarly,

$$\lim_{q\to 1^-}\binom{n}{k}_q = \binom{n}{k}.$$

We refer to [12, 13, 14] for additional properties of the difference operator $\mathfrak{D}_q$. From (12), it follows that

$$\mathfrak{D}_q^{(2)}\Upsilon(\wp) = \mathfrak{D}_q\left(\mathfrak{D}_q\Upsilon(\wp)\right), \quad \mathfrak{D}_q^{(n)}\Upsilon(\wp) = \mathfrak{D}_q\left(\mathfrak{D}_q^{(n-1)}\Upsilon(\wp)\right), \quad n \geq 3. \tag{13}$$

Using (6) and (1), we have

$$\mathfrak{D}_q\Upsilon(\wp) = 1 + \sum_{n=2}^{\infty}[n]_q a_n \wp^{n-1}, \tag{14}$$

and

$$\mathfrak{D}_q\left[\wp\mathfrak{D}_q\Upsilon(\wp)\right] = \mathfrak{D}_q\Upsilon(\wp) + \wp\mathfrak{D}_q^{(2)}\Upsilon(\wp). \tag{15}$$

A family for q-starlike function was introduce in [15] as follow:

Definition 1.1: The functions $\Upsilon \in \mathcal{F}$ named belong to a class S_q^* where

$$S_q^* = \left\{\Upsilon \in \mathcal{F}: \mathrm{R}\left(\frac{\wp\mathfrak{D}_q\Upsilon(\wp)}{\Upsilon(\wp)}\right) > 0,\ q \in (0,1),\ \wp \in \breve{U}\right\}. \tag{16}$$

In the limiting case as $q \to 1^-$, a family S_q^* reduces to traditionals classes S^*.

The discrete random variables 6 named have a q-Poisson distributions when it take values $0, 1, 2, 3, \ldots$ with corresponding probabilities.

$$e_q^{-\delta}, \quad \frac{\delta e_q^{-\delta}}{[1]_q!}, \quad \frac{\delta^2 e_q^{-\delta}}{[2]_q!}, \quad \frac{\delta^3 e_q^{-\delta}}{[3]_q!}, \quad \ldots,$$

where δ is parameters, e_q^{6} be q-exponential functions define by

$$e_q^{6} = 1 + \frac{6}{[1]_q!} + \frac{6^2}{[2]_q!} + \frac{6^3}{[3]_q!} + \cdots = \sum_{m=0}^{\infty}\frac{6^m}{[m]_q!},$$

and $[m]_q! = [m]_q[m-1]_q \cdots [2]_q[1]_q$ is the q-factorial function see [16, 17, 18].

For the q-Poisson distributions, a probabilities mass functions express by

$$P_q(6 = m) = \frac{\delta^m e_q^{-\delta}}{[m]_q!}, \quad m = 0,1,2, \ldots.$$

The corresponding q-Poisson distributions series define as

$$\mathcal{P}_q(ʅ) = ʅ + \sum_{n=2}^{\infty} \frac{\mu^{n-1} e_q^{-\mu}}{[n-1]_q!} ʅ^n, \quad ʅ \in \breve{\mathrm{U}}.$$

The ratio test shows that this series has an infinite radius of convergence. Building on this idea, neutrosophic theory's offers the framework of dealing with imprecise parameter. As an example, neutrosophic q-Poisson distributions extends a classical case by allowing an intervals - value parameters $m_{\aleph}$ (see [18, 19, 20]). Its probability mass function is

$$\aleph P_q(6 = k) = \frac{(m_{\aleph})^k e_q^{-m_{\aleph}}}{[k]_q!}, \quad k = 0,1,2, \ldots,$$

for $\aleph = d + I$ represent the neutrosophic statistically numbers&both expectations & variance be $m_{\aleph}$.

A neutrosophic q-Poisson distributions be

$$M_q(m_{\aleph}, ʅ) = ʅ + \sum_{n=2}^{\infty} \frac{(m_{\aleph})^{n-1} e_q^{-m_{\aleph}}}{[n-1]_q!} ʅ^n, \quad ʅ \in \breve{\mathrm{U}}.$$

[18] The linear operator $\mathcal{B}_{m_{\aleph}}(\Upsilon(ʅ)): Ŧ \to Ŧ$, define via convolutions (Hadamard products), be the

$$\mathcal{B}_{m_{\aleph}}(\Upsilon(ʅ)) = M_q(m_{\aleph}, ʅ) * \Upsilon(ʅ) = ʅ + \sum_{n=2}^{\infty} \frac{(m_{\aleph})^{n-1} e_q^{-m_{\aleph}}}{[n-1]_q!} a_n ʅ^n, \quad ʅ \in \breve{\mathrm{U}}.$$

Historically, the conic domain $\psi_{\hbar}, \hbar \geq 0$, was introduced by Alsoboh et al. [18], Kanas [21] as

$$\psi_{\hbar} = \left\{u + iv: u > \hbar\sqrt{(u-1)^2 + v^2}\right\}. \tag{17}$$

For fixed $\hbar$, these domains represent a right half - plane $\hbar = 0$, a parabola $\hbar = 1$, right branch's for the hyperbola $0 < \hbar < 1$, and an ellipse $\hbar > 1$ (see also [22, 23, 24, 25]).

An extremal function of these conic region be:

$$p_{\hbar}(ƶ) = \begin{cases} \frac{1+ƶ}{1-ƶ}, \hbar = 0, \\ \frac{1}{1-\hbar^2} \cosh\left\{\left(\frac{2}{\pi}\cos^{-1}\hbar\right) \log\frac{1+\sqrt{ƶ}}{1-\sqrt{ƶ}}\right\} - \frac{\hbar^2}{1-\hbar^2}, 0 \leq \hbar < 1, \\ 1 + \frac{2}{\pi^2}\left(\log\frac{1+\sqrt{ƶ}}{1-\sqrt{ƶ}}\right)^2, \hbar = 1, \\ \frac{1}{\hbar^2-1} \sin\left(\frac{\pi}{2K(\hbar)} \int_0^{\frac{\mathrm{u}(ƶ)}{\sqrt{\hbar}}} \frac{dt}{\sqrt{1-t^2}\sqrt{1-\hbar^2 t^2}}\right) + \frac{\hbar^2}{\hbar^2-1}, \hbar > 1, \end{cases} \tag{18}$$

where

$$u(ʅ) = \frac{ʅ-\sqrt{\hbar}}{1-\sqrt{\hbar}ʅ}, \quad ʅ \in \breve{\mathrm{U}},$$

and $\hbar \in (0,1)$ is chosen such that $\hbar = \cosh\left(\frac{\pi K'(\hbar)}{4K(\hbar)}\right)$. so, $K(\hbar)$ the complete elliptic integrals for first type, & $K'(\hbar) = K\left(\sqrt{1-\hbar^2}\right)$ is its complementary integral (see [26, 27]).

If $p_{\hbar}(\gimel) = 1 + \sigma_{\hbar}\gimel + \cdots$, it is shown in [25] that

$$\sigma_{\hbar} = \begin{cases} \frac{8\left(\cos^{-1}\hbar\right)^2}{\pi(1-\hbar^2)}, 0 \leq \hbar < 1 \\ \frac{8}{\pi^2}, \hbar = 1, \\ \frac{\pi^2}{4\hbar^2(1+\hbar)\sqrt{\hbar}K^2(\hbar)}, \hbar > 1. \end{cases} \tag{19}$$

In recent years, several subclasses of holomorphic and bi-univalent functions have attracted considerable attention. Inspired by these works, we define two novel subclass for class Σ and obtain coefficients bound for $|a_2|$, $|a_3|$ for function belong to these subclasses.

2. The Sets for Lemmas

To establish primary findings, we will need to use next lemma.

Lemma 2.1: [1] *Let the function* $s(\gimel)$ *be given by*

$$s(\gimel) = 1 + s_1\gimel + s_2\gimel^2 + s_3\gimel^3 + \cdots, (8)$$

where $s(\gimel)$ belongs to the class $\mathcal{P}$ of all functions holomorphic in Ŭ and $\mathrm{R}\{s(\gimel)\} > 0$ for all $\gimel \in$ Ŭ. Then

$$|s_n| \leq 2, \quad n \in N = \{1,2,3,\dots\}. \tag{21}$$

This inequality is sharp.

Lemma 2.2: *Let the function* $s(\gimel)$ *be given by*

$$s(\gimel) = 1 + s_1\gimel + s_2\gimel^2 + s_3\gimel^3 + \cdots,$$

and suppose it is subordinate to functions $t(\gimel)$ *are*

$$t(\gimel) = 1 + t_1\gimel + t_2\gimel^2 + t_3\gimel^3 + \cdots.$$

If the function $t(\gimel)$ be univalent in Ŭ & $t(\text{Ŭ})$ the convex, so

$$|s_n| \leq |t_1|, \quad n \in N. \tag{22}$$

For a deeper review of more recent works, the reader is referred to [16, 17], and [18].

3. Coefficient Bound for the Function Class $\mathfrak{T}_q^{\Sigma}(\alpha, \lambda, m_{\aleph})$

This part, investigate a coefficients bounds of function belonging to classes $\mathfrak{T}_q^{\Sigma}(\alpha, \lambda, m_{\aleph})$. We aim to established explicit estimate for a initials Taylor coefficient and explore their relationships under the defined q-differential operators and associated conditions.

Definition 3.1: A function $\Upsilon \in Ŧ$ for a forms specified in (1) be included in the function class $\mathfrak{T}_{q}^{\Sigma}(\alpha, \zeta, m_{\aleph})$ if the following conditions are met: $\Upsilon \in \Sigma$,

$$\mathfrak{D}_{q}\mathcal{B}_{m_{\aleph}}\big(\Upsilon(z)\big) + z\zeta\mathfrak{D}_{q}^{(2)}\mathcal{B}_{m_{\aleph}}\big(\Upsilon(z)\big) \prec \big[p_{\mathfrak{h}}(z)\big]^{\alpha} \text{ or}$$

$$\Big(\mathfrak{D}_{q}\mathcal{B}_{m_{\aleph}}\big(\Upsilon(z)\big) + z\zeta\mathfrak{D}_{q}^{(2)}\mathcal{B}_{m_{\aleph}}\big(\Upsilon(z)\big)\Big)^{1/\alpha} \prec p_{\mathfrak{h}}(z), \tag{23}$$

and

$$\mathfrak{D}_{q}\mathcal{B}_{m_{\aleph}}\big(g(\varpi)\big) + \varpi\zeta\mathfrak{D}_{q}^{(2)}\mathcal{B}_{m_{\aleph}}\big(g(\varpi)\big)$$

$$\prec \big[p_{\mathfrak{h}}(\varpi)\big]^{\alpha} \text{ or } \Big(\mathfrak{D}_{q}\mathcal{B}_{m_{\aleph}}\big(g(\varpi)\big) + \varpi\zeta\mathfrak{D}_{q}^{(2)}\mathcal{B}_{m_{\aleph}}\big(g(\varpi)\big)\Big)^{1/\alpha} \prec p_{\mathfrak{h}}(\varpi). \tag{24}$$

where $z, \varpi \in \breve{U}$ and g is given by (2).

Theorem 3.2: *Put the functions $\Upsilon \in Ŧ$ of a forms describe in the (1) that belong to functions class $\mathfrak{T}_{q}^{\Sigma}(\alpha, \zeta, m_{\aleph})$. Then:*

$$|a_2| \leq \alpha\sqrt{\frac{2[2]_q!\sigma_{\mathfrak{h}}}{2[3]_q\alpha(m_{\aleph})^2 e_q^{-m_{\aleph}}(1+[2]_q\zeta)-(\alpha-1)\big([2]_q m_{\aleph} e_q^{-m_{\aleph}}\big)^2(1+\zeta)^2}}, \tag{25}$$

and

$$|a_3| \leq \frac{(\alpha\sigma_{\mathfrak{h}})^2}{\big([2]_q m_{\aleph} e_q^{-m_{\aleph}}\big)^2(1+\zeta)^2} + \frac{\alpha\sigma_{\mathfrak{h}}[2]_q!}{[3]_q(m_{\aleph})^2 e_q^{-m_{\aleph}}(1+[2]_q\zeta)}. \tag{26}$$

Proof: Firstly, it is evident from conditions (23) and (24) that

$$\mathfrak{D}_{q}\mathcal{B}_{m_{\aleph}}\big(\Upsilon(z)\big) + z\zeta\mathfrak{D}_{q}^{(2)}\mathcal{B}_{m_{\aleph}}\big(\Upsilon(z)\big) = [s(z)]^{\alpha}, \tag{27}$$

where

$$s(z) \prec p_{\mathfrak{h}}(z),$$

$$1 + s_1 z + s_2 z^2 + s_3 z^3 + \cdots \prec 1 + \sigma_{\Upsilon} z + \cdots. \tag{28}$$

and

$$\mathfrak{D}_{q}\mathcal{B}_{m_{\aleph}}\big(g(\varpi)\big) + \varpi\zeta\mathfrak{D}_{q}^{(2)}\mathcal{B}_{m_{\aleph}}\big(g(\varpi)\big) \prec \big[p_{\mathfrak{h}}(\varpi)\big]^{\alpha} \tag{29}$$

Where

$$\wp(\varpi) \prec p_{\mathfrak{h}}(\varpi),$$

$$1 + \wp_1\varpi + \wp_2\varpi^2 + \wp_3\varpi^3 + \cdots \prec 1 + \sigma_{\Upsilon}\varpi + \cdots. \tag{30}$$

By virtue of (28), (30), and Lemma 1.2, it follows that

$$|s_n| < |\sigma_{\mathfrak{h}}| \quad \text{and} \quad |\wp_n| < |\sigma_{\mathfrak{h}}|. \tag{31}$$

By equating the coefficients in (27) and (29), we obtain

$$[2]_q m_{\aleph} e_q^{-m_{\aleph}}(1+\zeta)a_2 = \alpha s_1, \tag{32}$$

$$\frac{[3]_q}{[2]_q!}(m_{\aleph})^2 e_q^{-m_{\aleph}}\big(1+[2]_q\zeta\big)a_3 = \alpha s_2 + \frac{\alpha(\alpha-1)}{2}s_1^2, \tag{33}$$

$$-[2]_q m_{\aleph} e_q^{-m_{\aleph}}(1+\zeta)a_2 = \alpha\wp_1, \tag{34}$$

and

$$\frac{[3]_q}{[2]_q!}(m_\aleph)^2 e_q^{-m_\aleph}\left(1+[2]_q\zeta\right)(2a_2^2-a_3)=\alpha\wp_2+\frac{\alpha(\alpha-1)}{2}\wp_1^2. \tag{35}$$

Based on (32) and (34), we find

$$s_1=-\wp_1, \tag{36}$$

$$2\left([2]_q m_\aleph e_q^{-m_\aleph}\right)^2(1+\zeta)^2 a_2^2=\alpha^2(s_1^2+\wp_1^2). \tag{37}$$

Subsequently, by combining equations (33), (35), and (35), we obtain

$$\frac{2[3]_q}{[2]_q!}(m_\aleph)^2 e_q^{-m_\aleph}\left(1+[2]_q\zeta\right)a_2^2=\alpha(s_2+\wp_2)+\frac{\alpha(\alpha-1)}{2}\frac{2\left([2]_q m_\aleph e_q^{-m_\aleph}\right)^2(1+\zeta)^2 a_2^2}{\alpha^2}.$$

$$2[3]_q\alpha(m_\aleph)^2 e_q^{-m_\aleph}\left(1+[2]_q\zeta\right)a_2^2=\alpha^2[2]_q!\,(s_2+\wp_2)$$
$$+(\alpha-1)\left([2]_q m_\aleph e_q^{-m_\aleph}\right)^2(1+\zeta)^2 a_2^2. \tag{38}$$

Thus, we conclude

$$a_2^2=\frac{\alpha^2[2]_q!(s_2+\wp_2)}{2[3]_q\alpha(m_\aleph)^2 e_q^{-m_\aleph}(1+[2]_q\zeta)-(\alpha-1)\left([2]_q m_\aleph e_q^{-m_\aleph}\right)^2(1+\zeta)^2}. \tag{39}$$

Finally, applying (31) together with (39), we derive the desired estimate for the coefficient $|a_2|$ as mentioned in (25).

By subtracting (35) from (35), we aim to prove (26). Consequently, we observe that

$$\frac{2[3]_q}{[2]_q!}(m_\aleph)^2 e_q^{-m_\aleph}\left(1+[2]_q\zeta\right)a_3-\frac{2[3]_q}{[2]_q!}(m_\aleph)^2 e_q^{-m_\aleph}\left(1+[2]_q\zeta\right)a_2^2$$
$$=\alpha(s_2-\wp_2)+\frac{\alpha(\alpha-1)}{2}(s_1^2-\wp_1^2), \tag{40}$$

As a consequence of (36), (37), and (39), we deduce that

$$a_3=\frac{\alpha[2]_q!(s_2-\wp_2)}{2[3]_q(m_\aleph)^2 e_q^{-m_\aleph}(1+[2]_q\zeta)}+\frac{\alpha^2(s_1^2+\wp_1^2)}{2\left([2]_q m_\aleph e_q^{-m_\aleph}\right)^2(1+\zeta)^2}, \tag{41}$$

In conclusion, by using (31) along with (41), we achieve the required estimate for the coefficient $|a_3|$ as specified in (26).

From Theorem 3.2, we can note that by setting $\zeta=1$ and $\zeta=0$, we respectively derive the following corollaries:

Corollary 3.3: *Assume $\Upsilon\in Ŧ$ is a function of a forms specified in (1) which falls under functions class $\mathfrak{T}_q^{\Sigma}(\alpha,1,m_\aleph)$. Then:*

$$|a_2|\le\alpha\sqrt{\frac{2[2]_q!\sigma_\hbar}{2[3]_q\alpha(m_\aleph)^2 e_q^{-m_\aleph}(1+[2]_q)-4(\alpha-1)\left([2]_q m_\aleph e_q^{-m_\aleph}\right)^2}},$$

and

$$|a_3|\le\frac{(\alpha\sigma_\hbar)^2}{4\left([2]_q m_\aleph e_q^{-m_\aleph}\right)^2}+\frac{\alpha\sigma_\hbar[2]_q!}{[3]_q(m_\aleph)^2 e_q^{-m_\aleph}(1+[2]_q)}.$$

Corollary 3.4: *Assume* $\Upsilon \in Ŧ$ *is a function of forms specified in (1) which falls under functions classes* $\mathfrak{T}_{\mathrm{q}}^{\Sigma}(\alpha, 0, m_{\aleph})$. *Then:*

$$|a_2| \leq \alpha \sqrt{\frac{2[2]_q!\sigma_{ħ}}{2[3]_q\alpha(m_{\aleph})^2 e_q^{-m_{\aleph}} - (\alpha-1)\left([2]_q m_{\aleph} e_q^{-m_{\aleph}}\right)^2}},$$

and

$$|a_3| \leq \frac{(\alpha\sigma_{ħ})^2}{\left([2]_q m_{\aleph} e_q^{-m_{\aleph}}\right)^2} + \frac{\alpha\sigma_{ħ}[2]_q!}{[3]_q(m_{\aleph})^2 e_q^{-m_{\aleph}}}.$$

5. Discussion and Conclusion

This study employed a Neutrosophic q-Poisson distributions to investigated coefficient problems for holomorphic functions related to conic domains. By means for Neutrosophic q-Poisson series, two novel subclasses for the holomorphic functions were defined, and explicit upper bound of coefficient $|a_2|$ *and* $|a_3|$ were obtained. The results highlight the effectiveness of Neutrosophic distribution techniques in geometric function theory and offer a useful framework for further research on holomorphic functions associated with conic domains.

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