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A fixed-point algorithm of e-enriched non-expansive mappings with an application

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Abstract

This work introduces a four-step iterative scheme for approximating fixed points of E-enriched nonexpansive mappings in uniformly convex Banach spaces. Theoretical analysis establishes a strong convergence under suitable assumptions on the mapping or the underlying space. To demonstrate its practical relevance, the scheme is applied to an image restoration problem formulated within a fixed-point framework. The numerical results show stable reconstructions with enhanced PSNR and SNR values, indicating superior performance in handling inverse imaging tasks. These findings underline both the theoretical strength and the computational efficiency of the proposed approach.

Subject Classification: *47H09, 47H10, 54H25.*

Keywords: *Fixed point theory, E-enriched nonexpansive mappings, Iterative algorithm, Weak convergence, Strong convergence, Uniformly convex Banach spaces, Image restoration.*

1. Introduction

A classical milestone is the contraction principle established by the authors [1] in nonlinear analysis fixed-point theory, which guarantees the existence and uniqueness of fixed points and provides a powerful framework for addressing problems arising in functional analysis, optimization, variational inequalities, and applied mathematics [2].

Later, the results were extended to include non-expansive operators defined on closed convex subsets of a uniformly convex Banach space (shortly, UCBS), obtained independently by authors [3-5]. Because many nonlinear

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operators remain outside the classical no expansive framework, Berinde [6] introduced enriched no expansive mappings as a proper generalization of nonexpansive mappings. Berinde [6] systematically investigated this class and established existence and approximation results using Krasnosel'skii-type iterative schemes in Hilbert and Banach spaces [6-7]. However, considerable effort has been devoted to designing iterative algorithms with improved convergence performance, such as the Mann iteration [8] and the Ishikawa iteration [9], Agarwal, O'Regan, and Sahu [10] and Noor [11], AK iteration [12], The other authors [13-15], Furthermore, the iteration has demonstrated superior convergence speed compared to many existing algorithms [16]. Other studies have also investigated stability issues, random iterative procedures, and convergence behavior in related frameworks with applications [17-21].

More recently, other generalizations have been proposed, including E-enriched non-expansive mappings (E-ENM) [22], to cover a wider range of nonlinear operators.

Based on these developments, we propose a new four-step iterative framework for approximating the E-ENM fixed points in a UCBS. The proposed scheme refines existing multi-step processes by incorporating additional control parameters that improve convergence behavior. The remainder of this paper is organized as follows. Section 2 presents the necessary preliminaries and basic definitions related to this study. Section 3 introduces the proposed four-step iterative scheme and establishes its strong convergence results. Section 4 discusses the application to an image restoration problem. Finally, Section 5 concludes the paper and outlines possible directions for future research.

2. Preliminaries

Throughout this paper, X is a real Banach space, $\emptyset \neq C$ is a convex subset of X, $\Lambda: C \to C$ is a mapping, and $Fix(\Lambda) :=$ The set of all fixed points of Λ. Recall some of the required mappings.

Definition 1: A self-mapping Λ on C is said to be.

1. A nonexpansive mapping if $\|\Lambda(q) - \Lambda(s)\| \leq \|q - s\|, \forall\, q, s \in C$. [5]
2. Is called a quasi-nonexpansive mapping if for all $q \in C$ and $z \in Fix(\Lambda) \neq \emptyset, \|\Lambda(q) - z\| \leq \|q - z\|$. [23]
3. Satisfies the Suzuki-type generalized nonexpansive condition if $\forall\, q, s \in C$ satisfy $\frac{1}{2}\|q - \Lambda(q)\| \leq \|q - s\| \Longrightarrow \|\Lambda(q) - \Lambda(s)\| \leq \|q - s\|$. [24]
4. An enriched nonexpansive mapping if $\exists d \geq 0$ such that $\|d(q - s) + \Lambda(q) - \Lambda(s)\| \leq (d + 1)\|q - s\|, \forall q, s \in C$. [7]
5. Suzuki-enriched nonexpansive mapping if $\exists d \in [0, \infty), \alpha \in X$ such that $\frac{1}{2}\|q - \Lambda(q)\| \leq (d + 1)\|q - s\| \Longrightarrow \|d(\alpha - s) + \Lambda(q) - \Lambda(s)\| \leq (d + 1)\|q - s\|, \forall\, q, s \in C$. [25]
6. An enriched quasi-nonexpansive mapping if $\exists d \in [0,1)$, $\forall\, q \in C$ and $z \in Fix(\Lambda)$ such that $\|d(q - z) + \Lambda(q) - z\| \leq (d + 1)\|q - z\|$. [26]

7. An E-ENM if $\exists d \geq 0, L \geq 1$ such that $\|d(q-s)+q-\Lambda(s)\| \leq L\|q-\Lambda(q)\|+(d+1)\|q-s\|, \forall\, q,s \in C$. [22]

Definition 2: [26] A mapping $\Lambda_\alpha: C \to C$ is said to be an α -Krasnosel'skii mapping associated with Λ *if* $\exists \alpha(0,1)$ such that, $\Lambda_\alpha(q) = (1-\alpha)q + \alpha\Lambda(q), \forall q \in C$.

Remark 1: The interrelations among the above classes of mappings are well documented in the literature. Detailed discussions on these implications can be found, for example, in [17-21].

Lemma 1: [5] *If X is a real Banach space and $\emptyset \neq C \subset X$ is a convex set. Suppose that $\Lambda: C \to C$ is a mapping. For a fixed parameter $\alpha(0,1)$, define the operator $\Lambda_\alpha: C \to C$ be an α -Krasnosel'skii mapping associated with Λ, then $Fix(\Lambda_\mu) = Fix(\Lambda)$.*

Lemma 2: [27] *Assume that X is a UCBS, and the constants satisfy $0 < b < c < 1$. Let $\{a_v\} \subset [b,c]$ be a sequence and let $\{q_v\}, \{s_v\} \subset X$ be two sequences such that, for some $d \geq 0$, $lim_{v\to\infty}\|a_v\, q_v + (1-a_v)\, s_v\| = d$, $\lim\limits_{v\to\infty} sup\, \|q_v\| \leq d, lim_{v\to\infty}\, sup\|s_v\| \leq d$. Then it follows that $lim_{v\to\infty}\|q_v - s_v\| = 0$.*

3. Main Results

The iterative algorithm under study $\{q_v\}$ states that

$$q_{v+1} = \Lambda_\mu^2 p_v\ ,\ p_v = \Lambda_\mu(\alpha_v \Lambda_\mu b_v + (1-\alpha_v)b_v)$$
$$b_v = \Lambda_\mu(\eta_v \Lambda_\mu a_v + (1-\eta_v)a_v)$$
$$a_v = \Lambda_\mu(\omega_v \Lambda_\mu q_v + (1-\omega_v)q_v) \quad \text{(I)}$$

where the sequence $\{\alpha_v\}, \{\eta_v\}, \{\omega_v\} \subseteq (0,1)$ and $\mu = \frac{1}{D+1} \in (0,1)$.

Theorem 1: *Assume that X be a UCBS X. If $\emptyset \neq C \subset X, C$ is a closed convex set, and $\Lambda: C \to C$ is E-ENM with $Fix(\Lambda) \neq \emptyset$. If $\{q_v\}$ is given by (I), then $lim_{v\to\infty}\|q_v - z\|$ exists.*

Proof: Since Λ is E-ENM with a fixed point $z \in Fix(\Lambda)$, then by [Theorem 2, [21]], there exists Λ_μ mapping satisfying as follows $\|q - \Lambda_\mu(s)\| \leq L\|q - \Lambda_\mu(s)\| + \|q-s\|$, where $\mu = \frac{1}{D+1} \in (0,1)$, and by Lemma 1, $Fix(\Lambda_\mu) = Fix(\Lambda) \neq \emptyset$.

Now $$\|\Lambda_\mu(q) - \Lambda_\mu(s)\| \leq (1+L)\|q - \Lambda_\mu(q)\| + \|q-s\| \quad (1)$$

From iteration (I), obtain inequalities.

$$\|a_v - z\| \leq \|q_v - z\| + (1+L)[(2+L) + \omega_v]\|\Lambda_\mu q_v - q_v\| \quad (2)$$

$$\|b_v - z\| \leq \|a_v - z\| + (1+L)[(2+L) + \eta_v]\|\Lambda_\mu a_v - a_v\| \quad (3)$$

$$\|p_v - z\| \leq \|b_v - z\| + (1+N)[(2+N)+\alpha_v]\|\Lambda_\mu b_v - b_v\| \quad (4)$$

$$\|q_{v+1} - z\| \leq \|p_v - z\| + (1+L)(3+L)\|p_v - \Lambda_\mu(p_v)\| \quad (5)$$

Using (2), (3), (4) in (5), then we have $\|q_{v+1} - z\| \leq \|q_v - z\| + \epsilon_v$, where

$$\epsilon_v = (1+L)\big([(2+L)+\omega_v]\|\Lambda_\mu q_v - q_v\| + [(2+L)+\eta_v]\|\Lambda_\mu a_v - a_v\| + [(2+L)+\alpha_v]\|\Lambda_\mu b_v - b_v\| + (3+L)\|p_v - \Lambda_\mu(p_v)\|\big).$$

By the boundedness of the iteration $\{q_v\}$, and $\{\alpha_v\},\{\eta_v\},\{\omega_v\} \subseteq (0,1)$ claim that $\sum_{v=0}^{\infty}\|\Lambda_\mu q_v - q_v\| < \infty$, $\sum_{v=0}^{\infty}\|\Lambda_\mu a_v - a_v\| < \infty, \sum_{v=0}^{\infty}\|\Lambda_\mu b_v - b_v\| < \infty, \sum_{v=0}^{\infty}\|\Lambda_\mu p_v - p_v\| < \infty$, then $\sum_{v=0}^{\infty}\epsilon_v < \infty$ implies that $\epsilon_v \to 0$.

From this, $\|q_{v+1} - z\| \leq \|q_v - z\|$. Hence, the sequence $\{\|q_v - z\|\}$ is a bounded and decreasing function; therefore, its limit exists.

Theorem 2: *Assume that X be a UCBS X. If $\emptyset \neq C \subset X, C$ is a closed convex set, $\Lambda: C \to C$ is E-ENM. Then $Fix(\Lambda) \neq \emptyset$ if and only if the sequence $\{q_v\}$ defined by (I) is bounded in C and satisfied $lim_{n\to\infty}\|\Lambda_\mu q_v - q_v\| = 0$, where $\mu = \frac{1}{d+1}$.*

Proof: Let $z \in Fix(\Lambda_\mu)$. Then, by Theorem 1, we obtain that $\lim_{n\to\infty}\|q_v - z\|$ exists and $\{q_v\}$ is bounded. Let $\lim_{n\to\infty}\|q_v - z\| = \ell$. From Theorem 1, we obtained $\|a_v - z\| \leq \|q_v - z\|$. This implies.

$$\limsup_{n\to\infty} \|a_v - z\| \leq \limsup_{n\to\infty}\|q_v - z\| = \ell. \quad (6)$$

Utilizing inequality (1), which further implies

$$\|\Lambda_\mu q_v - z\| \leq (1+L)\|q_v - \Lambda_\mu q_v\| + \|q_v - z\| \quad (7)$$

Then, taking the limit superior and by (6)

$$\limsup_{n\to\infty}(\|\Lambda_\mu q_v - z\| - \|q_v - z\|) \leq (1+L)\limsup_{n\to\infty}\|q_v - \Lambda_\mu q_v\| \quad (8)$$

From Lemma 2, $lim_{n\to\infty}\|\Lambda_\mu q_v - q_v\| = 0$. Conversely, assume that $\{q_v\}$ be a bounded sequence in C and $lim_{n\to\infty}\|\Lambda_\mu q_v - q_v\| = 0$. Since $\{q_v\} \subset C$ is bounded, suppose $\emptyset \neq C \subset X$ closed convex. Let $A_c(C,\{q_v\}) = \{z \in C: r(z,\{q_v\}) = inf_{a\in C} r(a,\{q_v\})\}$ asymptotic center where $r(z,\{q_v\}) = lim_{v\to\infty} sup\|z - q_v\|$ is a functional of a bounded sequence $\{q_v\}$ (see [28], [29], [30]). By (1) and the assumption $lim_{n\to\infty}\|\Lambda_\mu q_v - q_v\| = 0$, we have

$$r(\Lambda_\mu z,\{q_v\}) \leq (1+L)\|z - \Lambda_\mu z\| + r(z,\{q_v\}) \quad (9)$$

If $z \neq \Lambda_\mu z$, then by (9) implies that $r(\Lambda_\mu z,\{q_v\}) > r(z,\{q_v\})$. Leads to a contradiction. Since, by definition of the asymptotic center $A_c(C,\{q_v\})$, the point z is the unique element in C that satisfies $r(z,\{q_v\}) = inf_{a\in C} r(a,\{q_v\})$. Therefore, we must have $z = \Lambda_\mu z$ and Λ_μ has a fixed point.

Theorem 3: *Suppose that X be a UCBS, $\emptyset \neq C \subset X$ be a closed convex set, and $\Lambda: C \to C$ is an E-ENM with $Fix(\Lambda) \neq \emptyset$. If C is compact, then $q_v \to z \in Fix(\Lambda_\mu)$, where $\{q_v\}$ as in (I).*

Proof: Since C is compact and $\{q_v\} \subset C$ is bounded, there exists a subsequence $\{q_{v_n}\}$ of $\{q_v\}$ converging to some $z \in C$ such that

$$lim_{v\to\infty}\|q_{v_n} - z\| = 0 \tag{10}$$

$$\|q_{v_n} - \Lambda_\mu z\| \leq (2 + L)\|q_{v_n} - \Lambda_\mu q_{v_n}\| + \|q_{v_n} - z\| \tag{11}$$

By Theorem (2), $lim_{v\to\infty}\|q_{v_n} - \Lambda_\mu q_{v_n}\| = 0$ and by (11) $lim_{v\to\infty}\|q_{v_n} - z\| = 0$, then $\lim_{v\to\infty}\|q_{v_n} - \Lambda_\mu z\| = 0$ and $z \in Fix(\Lambda_\mu)$. Again, from Theorem (1) $\lim_{v\to\infty}\|q_v - z\|$ exists. Therefore, $\{q_v\}$ converge strongly to the fixed point $z \in Fix(\Lambda_\mu)$.

Remark 2: The next corollary extends Theorem 3 under a weaker condition.

Corollary 1: *Let X be a UCBS, $\emptyset \neq C \subset X$ be a closed convex set, $\Lambda_\mu: C \to C$ is an E-ENM with$F_{\Lambda_\mu} \neq \emptyset$. Suppose $\{q_v\}$ be defined by (I), if $lim_{v\to\infty} d\left(q_v, F_{\Lambda_\mu}\right) = 0$, where $d\left(q_v, F_{\Lambda_\mu}\right) = inf_{z\in F_{\Lambda_\mu}}\|q_v - z\|$, then the sequence $\{q_v\}$ converge strongly to the fixed point of Λ_μ.*

Proof: This result follows directly from Theorem 3 under the assumption that implies the sequence $\{q_v\}$ have a convergence condition in Theorem 3.

Assumption: This condition will be used in the next Theorem. Let f is a function that satisfies $f(r) = 0$ if $r = 0, f(r) > 0$ if $r > 0$, and $d\left(r, F_{\Lambda_\mu}\right) = inf_{z\in F_{\Lambda_\mu}}\|r - z\|$, then.

$$\|r - \Lambda_\mu r\| \geq f\left(d\left(r, F_{\Lambda_\mu}\right)\right), \text{ for all } r \in C \tag{12}$$

Theorem 4: *Let $\emptyset \neq C \subset X$, a closed convex set of UCBS X, $\Lambda_\mu: C \to C$ is an E-ENM having at least fixed point z. Suppose $\{q_v\}$ be sequence defied by (I), if Λ_μ satisfied condition (12) then the sequence $\{q_v\}$ converge strongly to the fixed point of Λ_μ.*

Proof: Since Λ_μ satisfied condition (12), then we get

$\|q_v - \Lambda_\mu q_v\| \geq f(d\left(q_v, F_{\Lambda_\mu}\right))$ and by Theorem 2, $lim_{v\to\infty}\|\Lambda_\mu q_v - q_v\| = 0$.

Which is implies that $lim_{v\to\infty}\, inf\, f(\, d\left(q_v, F_{\Lambda_\mu}\right)) = 0$.

Now, by assumption $f(r) = 0$, when $r = 0$, we obtain.

$lim_{v\to\infty}\, inf\ (\, d\left(q_v, F_{\Lambda_\mu}\right)) = 0$. And by Corollary 1, the sequence $\{q_v\}$ converge strongly to the fixed point of Λ_μ.

4. Application

The iterative algorithm (I) is applied to an image restoration problem where a color image is degraded by Gaussian blur and additive Gaussian noise. The image is resized to 256 × 256 and normalized to [0,1], so that the unknown image is represented as a vector in $\mathbb{R}^N$, with N = 256 × 256 × 3. The degradation model is defined as follows. A Gaussian blurring operator A with blur strength $\sigma_{blur} = 2.0$ is applied to the original image x_true, producing $b_{blur} = A(x_{true})$. The operator A is implemented using a Gaussian filter of size $=2[2\sigma_{blur}] + 12$. Zero-mean Gaussian noise with standard deviation $\sigma_{noise} = \frac{8}{255}$ is then added $b = b_{blur} + \sigma_{noise}\xi$, where ξ is a standard normal array. The degraded image is clipped to [0,1] and used as the initial point $x_0 = b$. For restoration, we consider the projected gradient-type mapping $\Psi(X) = P_{[0,1]}(X - \tau A^*(A(X) - b))$, where A^* denotes the adjoint of A, $P_{[0,1]}$ is the projection onto [0,1], and the step size is fixed at $\tau = 0.15$. The total number of iterations is set to $v = 10$. The control parameters are chosen as $\alpha_v = \frac{2v}{10+v^2}, \eta_v = \frac{1}{v+5}, w_v = \frac{v+20}{v+21}, v \geq 1$. The performance of Algorithm (I) is compared with the Thakur, Ali, and D** algorithms under the same conditions. Image quality is evaluated using PSNR and SNR. Since images are normalized to [0,1], PSNR and SNR are defined in the standard way. Numerical results are presented via visual comparisons, quantitative tables, and convergence plots, demonstrating the effectiveness of Algorithm (I). Using the same initial point and the same operator Ψ, the Thakur algorithm, Ali algorithm, D**-algorithm, and algorithm (I) are important under identical condition. The restoration quality is evaluated using the peak signal-to-noise ratio PSNR and the signal-to-noise ratio SNR. Since the images are normalized to [0,1], PSNR is defined as $PSNR(x_{true}, X) = 10\log_{10}\left(\frac{1}{MSE(x_{true},X)}\right)$, where $MSE(x_{true}, X) = \frac{1}{v}\sum_{i=1}^{v}\left(x_{true,i}, X_i\right)^2$.Moreover,$SNR$ is defined by $SNR(x_{true}, X) = 20\log_{10}\left(\frac{\|x_{true}\|_2}{\|x_{true}-X\|_2}\right)$.

The numerical results are presented through visual comparisons of the original, degraded, and restored images, as illustrated in Figure 1. In addition, quantitative evaluation is performed using the PSNR and SNR measures to assess the quality of the restored images. Both the visual inspection and the numerical values confirm the effectiveness of Algorithm (I) in restoring images degraded by Gaussian blur and Gaussian noise.

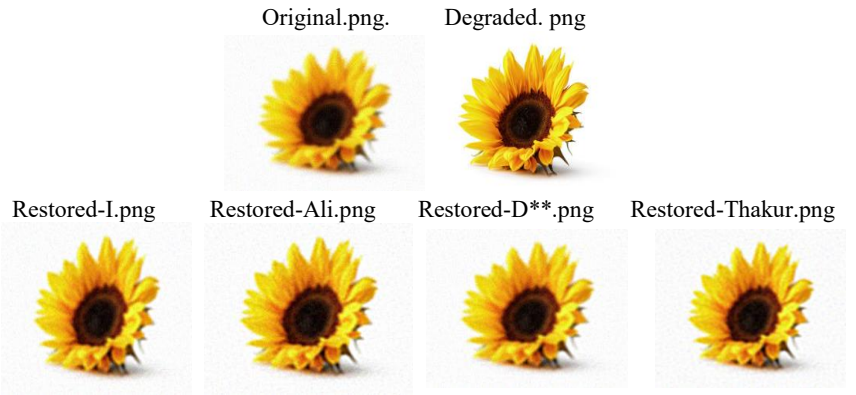

Figure 1
The first row shows the original and degraded images. The subsequent rows display the restored images obtained using Algorithm (I), the Ali algorithm, the D algorithm, and the Thakur algorithm, respectively**

4. Conclusion

A strong convergence of the generated sequence is established under appropriate assumptions on the operator or the underlying space. To demonstrate the practical relevance of the theoretical results, we consider an image restoration model formulated within a fixed-point framework governed by an E-ENM. The numerical experiments indicate that the proposed method produces stable reconstructions with improved convergence characteristics. In the future, we look forward to working in Hilbert space, which enables the development of convergence studies of algorithms on α-strong inverse monotone operators of type. The work may require combining a specific algorithm with the shrinking projection method and the viscosity algorithm.

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