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Solving porous medium equation using SEE transform homotopy perturbation method

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Abstract

This work innovatively solves the porous medium equation for the first time by using what is now called the "SEE transform." This newly devised integral transform is employed in tandem with the homotopy perturbation method. The nonlinear part of the porous medium equations is skillfully treated by the homotopy perturbation approach, and solution to these equations are constructed with several examples to demonstrate the efficiency and dependability of combining the homotopy perturbation method with the SEE transform. Further outcome underscores the efficiency and practical appropriateness of combining the SEE transform with the homotopy perturbation technique in addressing these challenges.

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Keywords: *SEE transform, Homotopy perturbation method, Non-linear partial differential equation, Porous medium equation.*

1. Introduction

Partial differential equations govern many physical processes and phenomena in various scientific and engineering fields, as explained by Polyanin and Zaitsev [1]. As a nonlinear diffusion model, the porous medium equation is used to capture heat transfer processes in various physical contexts. A well-known example of a nonlinear partial differential equation is the porous medium equation, according to Vázquez [2].

$$\frac{\partial z}{\partial \psi} = \frac{\partial}{\partial \xi}\left[z^q \frac{\partial z}{\partial \xi}\right] \tag{1}$$

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The Boussinesq equation emerges when q=2, serving as a basic illustration of a parabolic-type nonlinear evolution equation. Its theoretical framework and properties are significantly distinct from those of the traditional heat equation. This equation is applicable to various natural phenomena, as elaborated by Gupta and Gupta [3]. It frequently appears in physical applications, particularly concerning fluid dynamics, heat transfer, or diffusion processe. Boussinesq's 1903 study on groundwater infiltration contains an earlier instance. Around 1950, Zel'dovich and his colleagues conducted a noteworthy study on heat radiation in plasmas, which served as the foundation for the theory's rigorous mathematical development.

Numerous methods, such as the Adomian decomposition method, variational iteration method, Laplace decomposition method, and weighted finite difference method, have been advanced to deal nonlinear equations. However, these techniques often involve complex procedures, including the calculation of Lagrange multipliers and Adomian polynomials, which may lead to computational challenges and inconsistent results. These limitations have driven the search for more reliable solution methods. In light of recent advancements, this work introduces the Homotopy Perturbation Transform Method (HPTM). The HPTM provides solutions in the form of rapidly convergent series, potentially resulting in closed-form expressions. The method combines two powerful analytical approaches, incorporating He’s polynomials for nonlinear terms as originally suggested by Ghorbani [4]. Notably, the HPTM operates without round-off errors, restrictive assumptions, or discretization, making it appealing to both scientists and mathematicians. In during the past decade, Considerable research efforts have been devoted to developing methods for solving nonlinear differential equations. These include the variational iteration method, the homotopy perturbation transform method, the homotopy decomposition method introduced by Hashim et al. [5], the homotopy perturbation method (see Sweilam and Khader [6], Sharma and Methi [7], Kuffi, Mansour, and Issa [8]), and various combinations of these methods. Recently, Mishra, Pradhan and Mehta [9] and Aboodh [10] applied the homotopy perturbation method to porous medium equations. However, it is not suitable for nonlinear equations due to the complexities associated with nonlinear variables. Utilizing the homotopy perturbation technique, the nonlinear component is decomposed into a sequence of linear subproblems, leading to an iterative series representation of the solution. The operational properties of generalized fractional calculus operators associated with special functions have been extensively studied in recent literature [11], [12].

As a consequence, both the SEE transform and homotopy perturbation techniques can be hired to tackle diversity of nonlinear challenges. The effectiveness and precision of the SEE transform homotopy perturbation method in solving nonlinear porous medium equations are examined in this study. The resulting solutions can be represented in closed form and are obtained as fast-converging series. A notable benefit of this proposed method over decomposition methods is its capability to address nonlinear problems without the necessity for Adomian polynomials.

2. SEE Transform Homotopy Perturbation Method

With the following initial conditions, A general nonlinear non-homogenous partial differential equation has been the focus of scientific investigation:

$$Dz(\xi,\psi)+Rz(\xi,\psi)+Nz(\xi,\psi)=g(\xi,\psi) \tag{2}$$

$$z(\xi,0)=\mathfrak{H}(\xi),z_{\psi}(\xi,0)=\mathfrak{I}(\xi)$$

In the above equation, $D$is a linear differential operator of order two, $R$is a linear differential operator of lower order, $g(\xi,\psi)$denotes the source function, and Nrepresents a general nonlinear operator. The application of the SEE transform to both sides of equation (2) results in:

$$S[Dz(\xi,\psi)]+S[Rz(\xi,\psi)]+S[Nz(\xi,\psi)]=S[g(\xi,\psi)] \tag{3}$$

With the aforementioned initial conditions and the differentiation property of SEE transforms, we obtain:

$$S[z(\xi,\psi)]=\frac{1}{v^2}S[g(\xi,\psi)]+\frac{1}{v^{n+1}}\mathfrak{H}(\xi)+\frac{1}{v^{n+2}}\mathfrak{I}(\xi)$$

$$-\frac{1}{v^2}S\{[Rz(\xi,\psi)]+[\,Nz(\xi,\psi)]\} \tag{4}$$

After both sides of equation (4) have undergone the inverse SEE transform,

$$z(\xi,\psi)=G(\xi,\psi)-S^{-1}\left[\frac{1}{v^2}S\{[Rz(\xi,\psi)]+[\,Nz(\xi,\psi)]\}\right] \tag{5}$$

The quantity $G(\xi,\psi)$encapsulates the influence of both the source term and the given initial conditions.

In order for,

$$G(\xi,\psi)=\frac{1}{v^2}S[g(\xi,\psi)]+\frac{1}{v^{n+1}}\mathfrak{H}(\xi)+\frac{1}{v^{n+2}}\mathfrak{I}(\xi) \tag{6}$$

$$z(\xi,\psi)=\sum_{\eta=0}^{\infty}P^{\eta}\,z_{\eta}(\xi,\psi) \tag{7}$$

Furthermore, the non-linear term can be broken down into

$$Nz(\xi,\psi)=\sum_{\eta=0}^{\infty}P^{\eta}\,H_{\eta}(z) \tag{8}$$

Regarding certain He's polynomials H_{η} which are provided by

$$H_{\eta}(z_0,z_1,\dots\dots..,z_{\eta})=\frac{1}{\eta!}\frac{\partial^{\eta}}{\partial P^{\eta}}[N(\textstyle\sum_{\iota=0}^{\infty}P^{\iota}z)]_{P=0}\,,\eta=1,2,3,\dots\dots$$

Substituting the equation (8) and (7) in (6), we get

$$\sum_{\eta=0}^{\infty}P^{\eta}\,z_{\eta}(\xi,\psi)=G(\xi,\psi)$$

$$-P\left[S^{-1}\left[\frac{1}{v^2}S\left\{\left[R\sum_{\eta=0}^{\infty}P^{\eta}\,z_{\eta}(\xi,\psi)\right]+\sum_{\eta=0}^{\infty}P^{\eta}\,H_{\eta}(z)\right]\right\}\right]\right] \tag{9}$$

The coefficients corresponding to identical powers of Pare matched through the SEE transform coupled with the homotopy perturbation technique, resulting in the following projected solutions.

$$P^0: z_0(\xi,\psi) = -\frac{1}{v^2} S[Rz_0(\xi,\psi) + H_0(z)]$$
$$P^1: z_0(\xi,\psi) = -\frac{1}{v^2} S[Rz_1(\xi,\psi) + H_0(z)]$$
$$P^2: z_0(\xi,\psi) = -\frac{1}{v^2} S[Rz_1(\xi,\psi) + H_1(z)]$$
$$P^3: z_0(\xi,\psi) = -\frac{1}{v^2} S[Rz_2(\xi,\psi) + H_2(z)]$$

The most accurate estimates for the answers are

$$z(\xi,\psi) = \lim_{P\to 1} z_\eta(\xi,\psi) = z_0(\xi,\psi) + z_1(\xi,\psi) + z_2(\xi,\psi) + \cdots \quad (11)$$

3. Applications

This section is devoted to exploring the use of the SEE transform homotopy-perturbation technique for determining reliable analytical and approximate solutions to the porous medium equations.

Example 3.1: By substituting $q = -1$into equation (1), we obtain

$$\frac{\partial z}{\partial \psi} = \frac{\partial}{\partial \xi}\left((z)^{-1}\frac{\partial z}{\partial \xi}\right) \quad (12)$$

With conditions $z(\xi,0) = \frac{1}{\xi}$

When the SEE transform is applied to equation (12) on both sides, the outcome is

$$-\frac{1}{v^n} z(\xi,0) + vS[z(\xi,\psi)] = S\left[(z^{-1})\frac{\partial^2}{\partial \xi^2}\right] - S\left[(z^{-2})\left(\frac{\partial z}{\partial \xi}\right)^2\right] \quad (13)$$

$$[z(\xi,\psi)] = \left(\frac{1}{v^{n+1}}\right)\left(\frac{1}{\xi}\right) + S^{-1}\left[\frac{1}{v}S\left[(z^{-1})\frac{\partial^2}{\partial \xi^2}\right] - \left[(z^{-2})\left(\frac{\partial z}{\partial \xi}\right)^2\right]\right] \quad (14)$$

Now we apply the homotropy perturbation method

$$z(\xi,\psi) = \sum_{\eta=0}^{\infty} P^\eta z_\eta(\xi,\psi) \quad (15)$$

Furthermore, the non-linear term can be broken down into

$$Nz(\xi,\psi) = \sum_{\eta=0}^{\infty} P^\eta H_\eta(z) \ (16)$$

Using (15) and (16) in (14) we get,

$$\sum_{\eta=0}^{\infty} P^\eta z_\eta(\xi,\psi) = \left(\frac{1}{\xi}\right) + PS^{-1}\left\{\frac{1}{v}S\left[\sum_{\eta=0}^{\infty} P^\eta H_\eta(z)\right]\right\} \quad (17)$$

Regarding certain He's polynomials H_η which are provided by

$$H_0(z) = ({z_0}^{-1})\frac{\partial^2 z_0}{\partial \xi^2} - ({z_0}^{-2})\left(\frac{\partial z_0}{\partial \xi}\right)^2$$

$$H_1(z) = ({z_0}^{-1})\left[-\frac{z_1}{z_0}\frac{\partial^2 z_0}{\partial \xi^2} + \frac{\partial^2 z_1}{\partial \xi^2}\right] - ({z_0}^{-2})\left[-2\frac{z_1}{z_0}\left(\frac{\partial z_0}{\partial \xi}\right)^2 + 2\frac{\partial z_0}{\partial \xi}\frac{\partial z_1}{\partial \xi}\right]$$

The coefficients of corresponding powers of P can be compared to get

$$P^0 = z_0(\xi,\psi) = \frac{1}{\xi}$$

$P^1 = z_1(\xi,\psi)$

$$= S^{-1}\left[\frac{1}{v}S\left[(z_0{}^{-1})\left[-\frac{z_1}{z_0}\frac{\partial^2 z_0}{\partial\xi^2}+\frac{\partial^2 z_1}{\partial\xi^2}\right]\right]-S^{-1}\left[\frac{1}{v}S\left[(z_0{}^{-2})\left[-2\frac{z_1}{z_0}\left(\frac{\partial z_0}{\partial\xi}\right)^2+2\frac{\partial z_0}{\partial\xi}\frac{\partial z_1}{\partial\xi}\right]\right]\right]\right] \tag{18}$$

By following a similar process, we can gain further values. For instance, by substituting the previously mentioned values in equation (11), we obtain a series-based solution.

$$z(\xi,\psi) = \frac{1}{\xi}+\frac{\psi}{\xi^2}+\frac{\psi^2}{\xi^3}+\cdots\ldots = \frac{1}{\xi-\psi}$$

which is the precise closed-form solution to equation (12).

Example 3.2: Let us take q=1 in equation (1), we get

$$\frac{\partial z}{\partial\psi} = \frac{\partial}{\partial\xi}\left(z\frac{\partial z}{\partial\xi}\right) \tag{19}$$

With $z(\xi,0) = \xi$

Taking the SEE transform to equation (19) on both sides results in

$$-\frac{1}{v^n}z(\xi,0)+vS[z(\xi,\psi)] = S\left[z\frac{\partial^2 z}{\partial\xi^2}+\left(\frac{\partial z}{\partial\xi}\right)^2\right] \tag{20}$$

On applying specific initial conditions on above equation,

$$[z(\xi,\psi)] = \left(\frac{1}{v^{n+1}}\right)(\xi)+ S\left[z\frac{\partial^2 z}{\partial\xi^2}+\left(\frac{\partial z}{\partial\xi}\right)^2\right] \tag{21}$$

Taking inverse SEE transform on both side of equation (21)

$$[z(\xi,\psi)] = \xi + PS^{-1}\left[\frac{1}{v}S\left[z\frac{\partial^2 z}{\partial\xi^2}+\left(\frac{\partial z}{\partial\xi}\right)^2\right]\right] \tag{22}$$

Now we apply the homotropy perturbation method

$$z(\xi,\psi) = \sum_{\eta=0}^{\infty} P^{\eta}\, z_{\eta}(\xi,\psi) \tag{23}$$

Furthermore, the non-linear term can be broken down into

$$Nz(\xi,\psi) = \sum_{\eta=0}^{\infty} P^{\eta}\, H_{\eta}(z) \tag{24}$$

Using (23) and (24) in (22) we get,

$$\sum_{\eta=0}^{\infty} P^{\eta}\, z_{\eta}(\xi,\psi) = \xi + PS^{-1}\left\{\frac{1}{v}S\left[\sum_{\eta=0}^{\infty} P^{\eta}\, H_{\eta}(z)\right]\right\} \tag{25}$$

Regarding certain He's polynomials H_{η} which are provided by

$$H_0(z) = z_0\frac{\partial^2 z_0}{\partial\xi^2}+\left(\frac{\partial z_0}{\partial\xi}\right)^2$$

$$H_1(z) = \left[z\frac{\partial^2 z_0}{\partial\xi^2}+z_0\frac{\partial^2 z_1}{\partial\xi^2}\right]+2\frac{\partial z_0}{\partial\xi}\frac{\partial z_1}{\partial\xi} \tag{26}$$

The coefficients of corresponding powers of P can be compared to get

$$P^0 = z_0(\xi,\psi) = \xi$$

$$P^1 = z_1(\xi,\psi) = S^{-1}\left[\frac{1}{v}S\left[z\frac{\partial^2 z}{\partial\xi^2}+\left(\frac{\partial z}{\partial\xi}\right)^2\right]\right] = \psi \tag{27}$$

By following a similar process, we can gain further values and found that $z(\xi,\psi) = 0\ for\ n \geq 2$. By inserting the obtained values into equation (11), a series solution is obtained.

$$z(\xi,t) = \frac{1}{\xi} + \frac{\psi}{\xi^2} + \frac{\psi^2}{\xi^3} + \cdots \ldots = \frac{1}{\xi-\psi} \quad (28)$$

which is the precise closed-form solution to equation (19).

Conclusion

To retain an analytical solution for the porous medium problem, the proposed findings aims to investigate the adoption of homotopy perturbation technique in combination with the novel SEE transform. When combined, these techniques yield very accurate and trustworthy results. A major superiority of this algorithm is its extent to deliver an analytical approximation, frequently resulting in an exact solution, through a sequence that converges rapidly with precisely calculated terms. Its quick convergence and lower computational requirements compared to other numerical techniques highlight the method's dependability and offer a significant improvement over existing methods for solving partial differential equations. Ultimately, we conclude that HPTM is a substantial advancement over current numerical techniques and is expected to be widely applied in various scientific fields.

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