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Didactic study of conformable fractional derivative

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Abstract

The objective of this paper is to propose a didactic study and the possibility of teaching the results of the conformable fractional derivative given by [7] and specifying the advantages and disadvantages of this concept.

Subject Classification (2010): *26A33, 26A24, 00A35, 97Cxx, 97Dxx.*

Keywords: *Methodology of mathematics, Didactic study, Conformable fractional derivative.*

1. Introduction

The study and comprehension of mathematical concepts have been essential pillars in the foundation of numerous scientific disciplines, guiding the understanding and analysis of complex phenomena (see figure 1). In recent years, the exploration of fractional calculus has garnered significant attention due to its profound implications across various fields, including physics, engineering, and mathematics. Within this realm, the notion of conformable fractional derivatives has emerged as a promising avenue for investigating systems with non-local

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behaviors and memory effects. Conformable fractional calculus extends the traditional framework of fractional calculus by introducing a new class of fractional derivatives that exhibit advantageous properties in terms of simplicity, applicability, and mathematical tractability. Unlike conventional fractional derivatives, which often entail intricate definitions and computational complexities, conformable fractional derivatives offer a more intuitive approach, facilitating their integration into theoretical frameworks and practical applications [3, 4, 5, 7, 13, 14, 15, 16].

The study of conformable fractional derivatives encompasses an array of properties and characteristics that distinguish them as powerful tools for analyzing dynamical systems and modeling real-world phenomena. These properties include but are not limited to: linearity, compatibility with traditional calculus operations, composition rules, and the notion of conformable fractional integrals. Moreover, conformable fractional derivatives possess unique features such as being well-defined for non-differentiable functions and exhibiting algebraic closure under certain operations, further enhancing their utility in diverse contexts.

In this didactic study, we aim to explore the possibility of teaching the properties of conformable fractional derivatives to students at various educational levels. By leveraging pedagogical techniques tailored to the specific needs and backgrounds of learners, we seek to demystify the concepts of conformable fractional calculus and empower students to apply them effectively in solving real-world problems [6, 7, 8, 9, 10, 11, 12, 17].

The key motivation of this study is the importance of fractional calculus in contemporary scientific and technological endeavors. As the demand for interdisciplinary skills continues to rise, proficiency in fractional calculus has become increasingly valuable for students pursuing careers in fields such as signal processing, control theory, and data science. By introducing students to conformable fractional derivatives early in their mathematical education, we aim to equip them with the foundational knowledge and skills needed to excel in these dynamic and evolving domains. Furthermore, this study recognizes the potential benefits of incorporating historical and contextual perspectives into the teaching of conformable fractional derivatives. By tracing the historical development of fractional calculus and highlighting its connections to ancient mathematical traditions, we aim to instill a deeper appreciation for the rich heritage of mathematical knowledge that underpins contemporary advancements in the field [18]. It's an approach that enhances the understanding and fosters the sense of intellectual curiosity and engagement with mathematical concepts across different cultural and historical contexts. Through a combination of theoretical exploration, computational exercises, and real-world applications, this didactic study seeks to elucidate the fundamental principles and properties of conformable fractional derivatives in a manner that is accessible, engaging, and relevant to students' academic and professional interests. By fostering a deeper understanding of this cutting-edge mathematical framework, we aim to inspire the next generation of mathematicians, scientists, and innovators to harness the power

of conformable fractional calculus in addressing the complex challenges of the modern world.

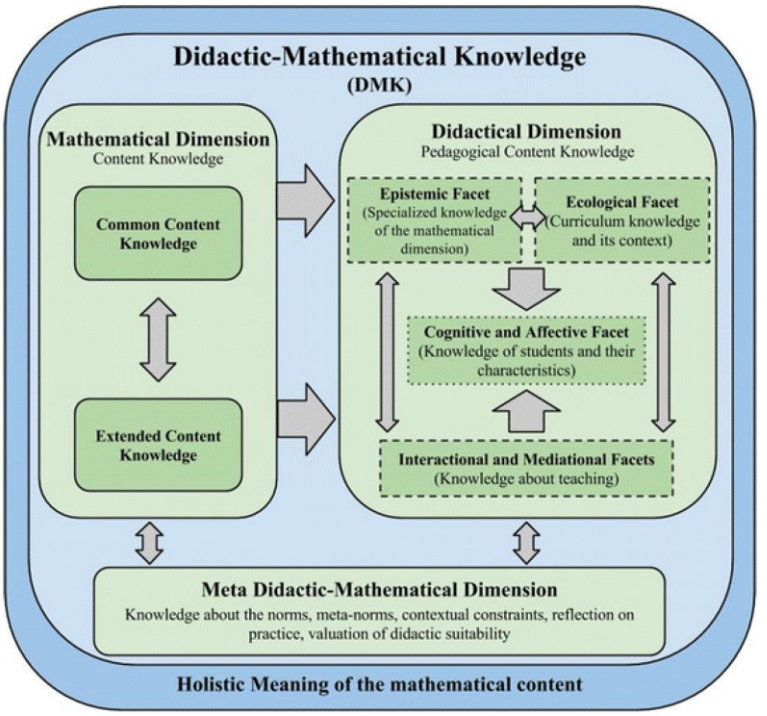


Figure 1

Dimensions and components of the DMK and its relationships with other models.

The conformable fractional derivative was first proposed by [17] in 2014 as an alternative approach to fractional calculus. The conformable fractional derivative was introduced as an attempt to address certain limitations and ambiguities associated with traditional fractional derivatives, such as the Riemann-Liouville and Caputo derivatives.

These traditional derivatives are defined using integral operators and involve non-local properties, making their interpretation and application complex in some cases.

For $\alpha \in [n-1, n[$ and $a > 0$, the well knowen α derivative of a function f is the Riemann Liouville α derivative of f given by

$$D_a^{\alpha}(f)(t) := \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(x)}{(t-x)^{\alpha-n+1}} dx,$$

The Caputo α-derivative is the variant definition :

$$D_a^{\alpha}(f)(t) := \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^{(n)}(x)}{(t-x)^{\alpha-n+1}} dx$$

Moreover, the most natural definition of fractional derivative is the conformable one defined in [7] for each $\alpha \in]0.1[$ as

$$f^{(\alpha)}(t) := \lim_{\varepsilon \to 0} \frac{f(t+\varepsilon t^{1-\alpha}) - f(t)}{\varepsilon}, \forall t > 0;$$

we define $f^{(\alpha)}(0) = \lim_{t \to 0^+} f^{(\alpha)}(t)$, when this limit exists. The conformable fractional derivative involves naturaly a low computational cost, because it does not require the evaluation of other special functions. This article aims to explore the didactic aspects of teaching conformable fractional derivatives, discussing both their advantages and disadvantages in educational contexts.

2. Notations and preliminaries

In the sequel we take $a > 0$, $f : [a, b] \to \mathbb{R}$, $\alpha \in]0,1]$. The following two theorems are our modest results recently published.

Theorem 2.1: [2] *Suppose that f is α-differentiable. Then, $\forall \lambda \in [min(f^{(\alpha)}(a), f^{(\alpha)}(b)), max(f^{(\alpha)}(a), f^{(\alpha)}(b))]$, $c \in]a, b[$: $f^{(\alpha)}(c) = \lambda$.*

Theorem 2.2: [1] *Suppose that :*

(1) $\forall x \in [a', b'] \subset [a, b], x^{\frac{1}{\alpha}} \in [a, b]$ and $f([a, b]) \subset [a', b']$;

(2) f is continuous, α-differentiable and satisfies $\max_{x \in [a,b]} |f^{\alpha}(x)| = K < \alpha$.

Then, f has an unique α-fixed point $c \in [a, b]$ and the sequence defined by its first term $(u_0)^{\frac{1}{\alpha}} \in [a, b]$ and the recurrence relation : $u_{n+1} = f((u_n)^{\frac{1}{\alpha}})$ converges to c.

The following two theorems are results given in [7]

Theorem 2.3: [7] *Suppose that f is continuous, α-differentiable and $f(a) = f(b)$. Then, $f^{(\alpha)}(c) = 0$ for at least one $c \in]a, b[$.*

Theorem 2.4: [7] *Suppose that f is continuous and α-differentiable. Then, $f^{(\alpha)}(c) = \frac{f(b) - f(a)}{\frac{1}{a} b^{\alpha} - \frac{1}{\alpha} a^{\alpha}}$ for $c \in]a, b[$.*

3. Main results

Conformable fractional calculus, introduced by Khalil in 2014 [7], extends the classical concept of derivatives to non-integer orders while maintaining the simplicity and computational advantages of integer-order calculus. Unlike traditional fractional derivatives, which involve integral operators, conformable derivatives rely on a single limit operation, making them easier to comprehend and apply in various mathematical and scientific problems.

Let's start with a simple comparison :

Let f be α-differentiable at $x_0 \in [a, b]$ and pose $\varepsilon = x - x_0$, we have :

$$f'(x_0) = \lim_{\varepsilon \to 0} \frac{f(x_0+\varepsilon)-f(x_0)}{\varepsilon} = \lim_{x \to x_0} \frac{f(x)-f(x_0)}{x-x_0},$$

and

$$f^{(\alpha)}(x_0) = \lim_{\varepsilon \to 0} \frac{f\left(x_0+\varepsilon x_0^{1-\alpha}\right)-f(x_0)}{\varepsilon} = \lim_{x \to x_0} \frac{f\left(x_0+(x-x_0)x_0^{1-\alpha}\right)-f(x_0)}{x-x_0}.$$

If f is differentiable, we have :

$$f^{(\alpha)}(x) = x^{1-\alpha} f'(x) \; \forall x \in [a,b]$$

Remarks 1: f is differentiable $\Rightarrow$ f is α-differentiable.

Reciprocal implication is false: For example, $f(x) = \sqrt{x}$ is not differentiable at 0, but it is $\frac{1}{2}$-differentiable at 0 and $f^{\left(\frac{1}{2}\right)}(0) = \frac{1}{2}$.

Example 1: The curve's allure and intuitive thinking (see figure 2)

(1) $f(x) = x^3$

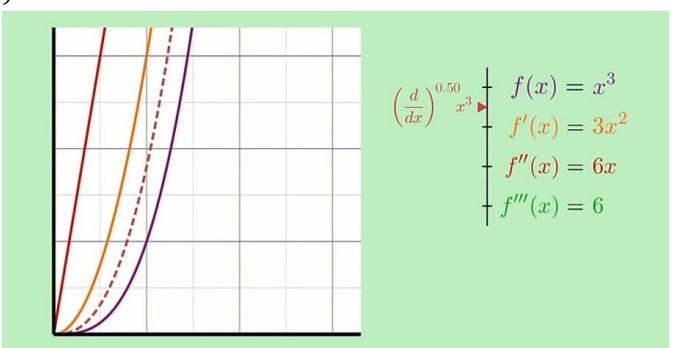
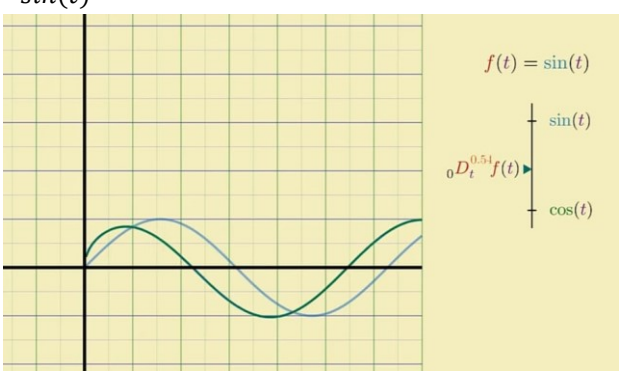


Figure 2

(2) $f(t) = sin(t)$



(3) What is happening geometrically :

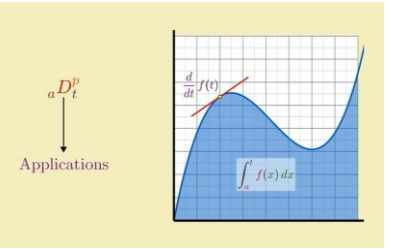


Figure 3

Fractional cord and geometric interpretation [6].

Definition 3.1: The equation fractional cord of the C_f at point $(x_0, f(x_0))$ is defined by :

$$\frac{f(x)^\alpha - f(x_0)^\alpha}{x^\alpha - x_0^\alpha} = \frac{f(x_0)^{\alpha-1}}{x_0^{\alpha-1}} f^{(\alpha)}(x_0).$$

Thus for $\alpha \in]0,1]$, the fractional cord represent deviation curve from the tangent line:

$$\lim_{\alpha\to 1} \frac{f(x)^\alpha - f(x_0)^\alpha}{x^\alpha - x_0^\alpha} = \lim_{\alpha\to 1} \frac{f(x_0)^{\alpha-1}}{x_0^{\alpha-1}} f^{(\alpha)}(x_0),$$

which means that $\frac{f(x)-f(x_0)}{x-x_0} = f'(x_0)$.

$f^{(\alpha)}(x_0)$ is then the slope of figure 3 tangent line to this fractional cord at point $(x_0, f(x_0))$ [6, Theorem 2.3].

Example 2:

(1) Let $f(x) = x^2$. The fractional cord associated with C_f at point $(2,4)$ for $\alpha = \frac{1}{2}$ is

$$\sqrt{y} - 2 = 4(\sqrt{x} - \sqrt{2}),$$

and $f^{(\frac{1}{2})}(2) = 4\sqrt{2}$ is the slope of its tangent line. See Figure 4.

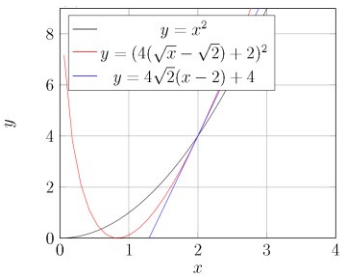


Figure 4

$$(x_0, y_0) = (2, 4), \alpha = \frac{1}{2}$$

(2) Let $f(x) = x^2$. The fractional cord associated with C_f at point (2,4) for $\alpha = \frac{1}{3}$ is

$$\sqrt[3]{y} - \sqrt[3]{4} = 4(\sqrt[3]{x} - \sqrt[3]{2}),$$

and the slope of its tangent line is $f^{(\frac{1}{3})}(2) = 4\sqrt[3]{4}$. See Figure 5.

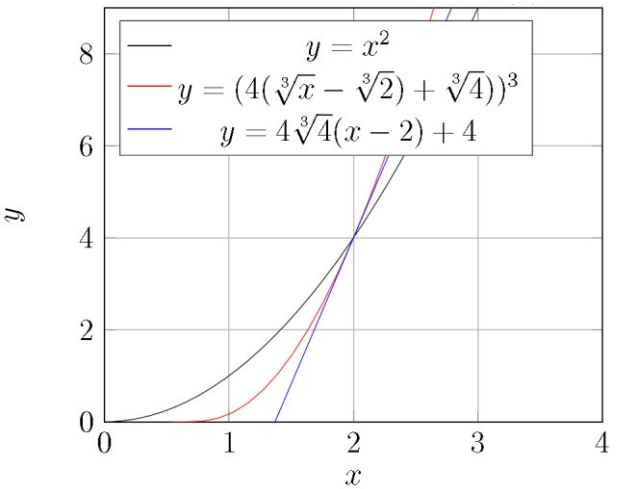


Figure 5

$(x_0, y_0) = (2, 4), \alpha = \frac{1}{3}$

Advantages of Teaching Conformable Fractional Derivative :

- Simplified Framework: Conformable fractional derivatives provide a simpler and more intuitive framework compared to other fractional calculus approaches. This simplicity facilitates easier understanding and application for students at both undergraduate and graduate levels.
- Computational Efficiency: The computational procedures involved in conformable fractional calculus are less complex, enabling faster calculations and numerical simulations. This efficiency is particularly advantageous in engineering and scientific applications where computational resources are limited.
- Seamless Integration with Classical Calculus: Conformable fractional derivatives seamlessly integrate with classical calculus concepts, allowing for a smooth transition between integer-order and fractional-order calculus. This integration enhances students' understanding of the continuity and coherence of mathematical concepts.
- Diverse Applications: Teaching conformable fractional derivatives exposes students to a wide range of real-world applications, including modeling viscoelastic materials, fractional control systems, signal processing, and

image enhancement. This interdisciplinary approach enriches students' learning experience and fosters creativity in problem-solving.

Disadvantages of Teaching Conformable Fractional Derivative: :

- Limited Historical Context: Conformable fractional calculus is a relatively new concept compared to classical calculus and traditional fractional calculus. Thus, teaching conformable derivatives may lack the historical context and depth of understanding associated with more established mathematical theories.
- Limited Educational Resources: Due to its recent development, educational resources such as textbooks, lecture notes, and exercises specific to conformable fractional calculus may be limited. This scarcity can pose challenges for educators in designing comprehensive curricula and teaching materials.
- Conceptual Complexity: While conformable fractional derivatives offer simplicity compared to other fractional calculus approaches, they still involve non-local operators and unconventional mathematical concepts. Some students may find these abstract notions challenging to grasp, requiring additional effort from educators to provide clear explanations and illustrations.

Conclusion

The didactic study of conformable fractional derivatives presents both advantages and disadvantages in educational settings. Despite its relative novelty and conceptual complexity, teaching conformable calculus offers students a simplified framework for understanding fractional calculus and its diverse applications. By addressing the challenges and leveraging the strengths of conformable fractional derivatives, educators can effectively integrate this emerging mathematical concept into their teaching methodologies, fostering a deeper understanding of non-local phenomena and enhancing students' problem-solving skills in various scientific and engineering domains.

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