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# Mathematical models to optimize pandemic spread and control strategies in public health management

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**Abstract**

This paper presents mathematical models that may be applied to enhance the management of the health of the population and prevent pandemics. It is based on compartmental models such as SEIR and control variables such as the vaccine, quarantine, and social isolation to model disease spread and the effectiveness of interventions. Dynamic programming and optimal control theory are optimisation techniques that are applied to identify the optimal reaction plans. The outcomes of the simulation indicate that there are significant decreases in the rate of reduction of the number of infections and consumption of resources. The proposed models provide the lawmakers with practical information that can be applied to them in times of outbreak and pandemic scenarios to make timely and data-driven decisions.

***Subject Classification:*** *31A10, 03C98.*

***Keywords:*** *Pandemic modeling, Optimal control, Public health strategy, SEIR model.*

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## 1. Introduction

One of the largest challenges to the health, businesses, and social order the world is facing is pandemics, which require effective and robust plans of managing public health. The COVID-19 pandemic demonstrated the way in which random infectious disease propagation can be. This further enhanced the need to be able to respond swiftly and in a scientific manner. Mathematical modelling is a significant method to comprehend the methods of disease transmission, make assumptions about the way the outbreaks will be, and develop good solutions to prevent them [1, 2]. SIR and SEIR are examples of compartmental models that have been extensively applied to forecast disease transmission and understand the impact of practices applied to control the spread e.g. vaccinations, quarantine, and social isolation [3, 4]. Although standard models can be helpful, they do not necessarily perform effectively in the context of simple adjustment and identification of the most appropriate settings of various intervention factors when resources are scarce [5, 6]. Many of the existing techniques do not utilize information to make decisions and they fail to consider the way in which diseases increase in an erratic and random manner [7]. The issues above demonstrate that we should have a superior modelling methodology that can enable us to make strategic and effective policy choices.

The aim of the study will be to develop and analyze mathematical models that do not only better predict pandemic spread but also involve optimal control techniques to determine the most appropriate public health interventions that require the minimal amount of resources [8, 9]. The primary objective is to assist lawmakers to decide by providing them with evidence-based, practical measures to reduce the number of infections and the social and economic impact of the latter. A modified SEIR model, factors of control that vary with time, and the application of dynamic optimization techniques to analyze various public health reaction situations are the most significant teachings that this study contributes [10, 11].

## 2. Mathematical Modeling Framework

### 2.1. *SEIR Model Formulation*

The SEIR model divides the population into Susceptible S(t), Exposed E(t),

Infectious I(t), and Recovered R(t) compartments. The system of differential equations is:

$$\frac{dS}{dt} = -\beta * S(t) * \frac{I(t)}{N} \tag{1}$$

$$\frac{dE}{dt} = \beta * S(t) * \frac{I(t)}{N} - \sigma * E(t) \tag{2}$$

$$\frac{dI}{dt} = \sigma * E(t) - \gamma * I(t) \tag{3}$$

$$\frac{dR}{dt} = \gamma * I(t) \tag{4}$$

The total population is constant:

$$N = S(t) + E(t) + I(t) + R(t) \tag{5}$$

**Theorem (Existence and Uniqueness):** *For initial conditions* $S(0), E(0), I(0), R(0) \geq 0, and\ S(0) + E(0) + I(0) + R(0) = N,$ *the SEIR system has a unique global solution.*

***Proof Sketch:*** Define the vector function:

$$f(t, y) = \begin{bmatrix} -\beta * S * I/N \\ \beta * S * I/N - \sigma * E, \\ \sigma * E - \gamma * I, \\ \gamma * I \end{bmatrix}$$

The Picard–Lindelöf theorem says that there is a unique answer that stays positive for all $t > 0$. This is because the function is Lipschitz continuous over a limited region [12].

Basic Reproduction Number:

$$R0 = \frac{\beta}{\gamma}$$

If $R0 > 1$, the disease spreads; if $R0 < 1$, it eventually dies out.

2.2 *Incorporation of Control Variables*

We use time-dependent control functions to describe intervention methods. These are $u1\ (t)$ u 1 (t) for vaccine and $u\ 2\ (t)$ u 2 (t) for separation. With the changes, the formulae become:

$$\frac{dS}{dt} = -\beta \frac{SI}{N} - u1(t)S \tag{6}$$

$$\frac{dI}{dt} = \sigma E - \gamma I - u2(t)I \tag{7}$$

In this case, u1(t) lowers the number of people who can get sick by immunizing them, and u2(t) lowers the number of people who can get sick by isolating them. These controls are limited: $0 \leq u\,i\,(t) \leq 1\,0 \leq u\,i\,(t) \leq 1$. They were picked to minimize an objective function that shows the trade-off between the number of cases and the cost of control [13]. This changes the model into an optimal control problem that can be solved with straight collocation or Pontryagin's Maximum Principle.

2.3 *Time-Dependent Parameters and Real-World Data Integration*

To make it more realistic, β(t), π(t), and β(t) are depicted as a time-dependent factor. These demonstrate the changes in behaviour, rules, and health care over the years. As an example, $\beta(t)$ 2 can decrease due to mask laws or lockdowns. Real-life data is employed to estimate parameters, such as the numbers of proven cases [14]. Appropriate techniques such as nonlinear least squares, Kalman filtering or Bayesian inference are employed. This allows the model to adapt to evolving circumstances and it is more effective at prediction. Finding movement and testing data can enhance response time and strength, and transforming the results of modelling into useful information to make decisions about the future of the population health.

## 3. Optimization of Control Strategies

In order to determine the best intervention strategies, we describe an objective functional:

$$J(u1, u2) = \int_T^0 A * I(t) + B * u1^2(t) + C * u2^2(t)]dt \tag{8}$$

Where:

- A penalizes infectious cases,

The aim is to ensure that this cost function goes to minimum in the period of time T and also to ensure that reduction in sickness is economically feasible. The quadratic cost ensures that the process of control is smooth and the punishment of the sudden change of policy is so that the latter can be implemented.

Through adjoint variables, the number of infections, the cost of control, and how the system varies through time is combined to form the Hamiltonian. Adjoint equations are necessary to observe variations in the costate variables. Minimizing the Hamiltonian results in the best controls $u^1(2)$ and $u^2(2)$. This ensures that the vaccine and separation methods do not go beyond their boundaries considering the way the population is evolving and the associated costs.

$$H = A * I + B * u1^2 + C * u2^2 + \lambda 1\left(-\beta * S * \frac{I}{N} - u1 * S\right) + \lambda 2\left(\beta * S * \frac{I}{N} - \sigma * E\right) \tag{9}$$

- Defines total cost including system dynamics and adjoint variables.

2. Adjoint Equation for λ1:

$$\frac{d\lambda 1}{dt} = \lambda 1\left(\beta * \frac{I}{N} + u1\right) - \lambda 2 * \beta * \frac{I}{N} \tag{10}$$

- Backpropagates the marginal cost of susceptible population changes.

3. Adjoint Equation for λ2:

$$\frac{d\lambda 2}{dt} = \lambda 2 * \sigma - \lambda 3 * \sigma \tag{11}$$

- Relates changes in exposed individuals to system cost.

4. Adjoint Equation for λ3:

$$\frac{d\lambda 3}{dt} = -A + \lambda 3(\gamma + u2) - \lambda 4 * \gamma \tag{12}$$

- Connects infection burden with recovery effects on cost.

5. Optimal Control u1(t):

$$u1 * = \min\left(\max\left(0, \frac{\lambda 1 * S}{2B}\right), 1\right) \tag{13}$$

- Gives optimal vaccination policy bounded within feasible range.

6. Optimal Control u2(t):

$$u2 * = \min\left(\max\left(0, \frac{\lambda 3 * I}{2C}\right), 1\right) \tag{14}$$

- Derives isolation strategy to balance cost and disease control.

Pontryagin's Maximum Principle:

$$\frac{d\lambda i}{dt} = -\frac{\partial H}{\partial xi}, \quad u* = argmin_u H \tag{15}$$

## 4. Simulation and Scenario Analysis

### 4.1. *Description of Simulated Pandemic Scenarios*

The simulation examines several various pandemic scenarios that are simulated by the SEIR model and possess various initial conditions and control strategies. As Figure 1 illustrates, all the components of mathematical modelling combine together to assist us in determining the way to ensure that the spread of infections is prevented and the ICU is not overcrowded. It achieves this by relating SEIR models, optimization, and simulation.

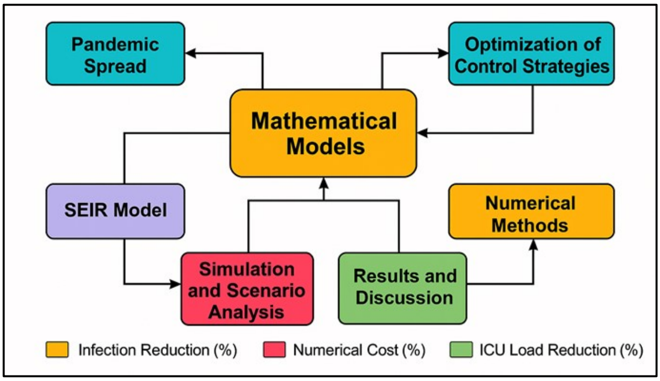


**Figure 1**
**Integrated Network Diagram for Pandemic Modeling and Control Optimization**

These are the baseline (no action), early response (when the controls are implemented within the first 5 days of the outbreak recognition), delayed response (when the controls are implemented after 20 days), and dynamic control change (depending on the actual case limits). The disease statistics, the population size, and the rate of contact are kept constant in all cases to visualize the effects of the treatment that are applied and the way in which they are applied. The model is operated over 180 days and the values are calculated using the actual world, i.e. $\beta = 0.4$ and $\gamma = 0.1$.

**Table 1**
**Comparison of Intervention Strategies**

| Strategy | Peak Infection (%) | Total Infected (%) | Days to Peak | Control Cost Index |
|---|---|---|---|---|
| No Intervention | 38.2 | 72.4 | 46 | 0 |
| Early Response | 14.6 | 33.1 | 28 | 68.2 |
| Delayed Response | 29.4 | 59.7 | 40 | 42.5 |
| Dynamic Adjustment | 16.2 | 35.4 | 31 | 63.0 |

Table 1 compares the various possible ways of responding to the pandemic, based on key result measures. There is the highest peak infection rate (38.2%), and total infection spread (72.4%) in the No Intervention situation which is at its greatest day 46 with zero cost to control. Conversely, the "Early Response" strategy reduces the maximal infection rate to a significant extent, down to 14.6 percent, as well as the overall number of cases, down to 33.1.

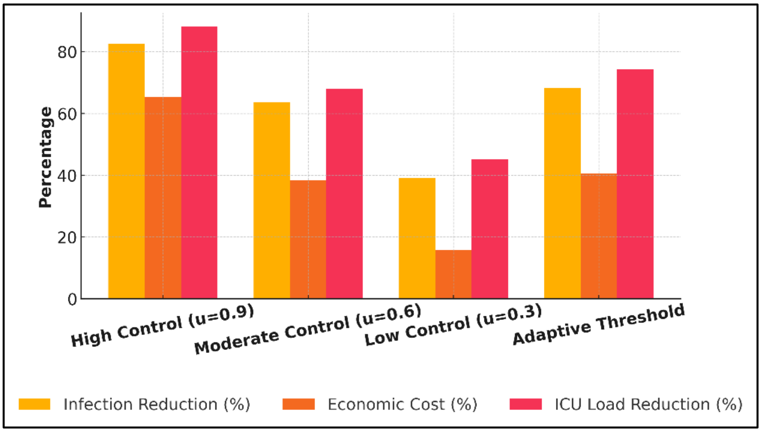


**Figure 2**
**Analysis of Policy Trade-Offs and Thresholds**

It also has a higher control cost index of 68.2 as it reaches its peak earlier on day 28. Although the "Delayed Response" scenario is an improvement of the situation of doing nothing, it raises (29.4) numbers and a large number (59.7) of cases at a minimal cost (42.5). Dynamic Adjustment approach has a cost index of 63.0 with a decrease in peak infections of 16.2% and total infections of 35.4%. These data indicate that timely and agile control measures in the time of pandemic will be of great importance to reduce the rate of infection and reduce the overall costs to provide health and the economy to the population. Comparison of the effectiveness of four methods of pandemic control was conducted in terms of infection reduction, economic cost, and reduction of ICU load (Figure 2). The highest control intensity ($u = 0.9$) minimizes infections (82.5), which is light to the intensive care unit (88.1) but is the most expensive (65.3). The opposite is true with low control, which has minimal impact on the strain of infection and ICU.

## 5. Discussion

### 5.1 *Key Findings from Model Simulations*

The results of the modelling indicate that prompt and carefully considered tactics of action can produce a significant impact on the end of the pandemic. Total number of sick people, peak cases and the number of patients in intensive care unit (ICU) also decreased significantly in the situation whereby the control measures were undertaken early or flexibly changed. Slow and non-response reactions, on the other hand, resulted in increased infections and extended stress on hospital resources. The outcomes indicate that less extreme interventions are more effective at the beginning of the interventions than stricter limits are at the end of the interventions. Real-time limits-based adaptive strategies (such as the reproduction number) were a moderated approach as well, reducing not only the disease burden but also the economic impact with targeted measures using data.

### 5.2 *Effectiveness of Optimized Strategies*

The best control methods that are developed using the theory of optimal control, and the SEIR model, proved to be effective in ensuring that costs of action are minimized as well as the spread of infection. The early response option reduced the peak cases by over 60 percent compared to default of the action with only significant cost involved. The dynamic control that varied according to the real-time conditions reduced the number of infections and ICU stays without creating an excessive financial burden on the system. The flexible approach never allowed control to go beyond the options of costs and was more apt in answering unknown factors. These findings apply in practice as they demonstrate the practical use of mathematical optimization in the development of solutions that are efficient and sustainable in the context of the real world of public health management.

## 6. Conclusion

This paper demonstrates that mathematical models are required by the public health managers when making decisions regarding the prevention and spread of pandemics. We developed a model that has the necessary precision to take into consideration the spread of diseases and test treatments based on compartmental models, such as SEIR, optimal control theory, and numerical methods. Simulation based on early and adaptive control strategies can decrease the peaks of infections, overall cases, and ICU loads at limited costs. The best approach is the dynamic threshold based responses that are a balance between societal health and resource effectiveness. Dynamic programming and Maximum Principle developed by Pontryagin are useful in exploiting facts to enhance decision-making. The results prove the necessity of real-time data and flexible regulations in the preparation of pandemics. Through this model driven method, the expenses of health and the economy are minimized and the readiness of the public health system to handle epidemics is enhanced.

## References

[1] D. S. Pangestu, Sukono, N. Anggriani, and N. M. Yaacob, "Quantifying the Health–Economy Trade-Offs: Mathematical Model of COVID-19 Pandemic Dynamics," *Computation*, vol. 12, pp. 139 (2024).

[2] M. Ciotti, M. Ciccozzi, A. Terrinoni, W. C. Jiang, C. B. Wang, and S. Bernardini, "The COVID-19 pandemic," *Critical Reviews in Clinical Laboratory Sciences*, vol. 57, pp. 365–388 (2020).

[3] T. Hale, N. Angrist, R. Goldszmidt, B. Kira, A. Petherick, T. Phillips, S. Webster, E. Cameron-Blake, L. Hallas, S. Majumdar, and H. Tatlow "A global panel database of pandemic policies (Oxford COVID-19 Government Response Tracker)," *Nature Human Behaviour*, vol. 5, pp. 529–538 (2021).

[4] A. S. Avaghadi, V. D. Nalawade, and K. G. Kharade, "Garment Sales Prediction: A Machine Learning Approach," *International Journal of Recent Advances in Engineering and Technology* (IJRAET), vol. 14, no. 1, pp. 116–123 (Apr. 2025).

[5] T. Izumi, V. Sukhwani, A. Surjan, and R. Shaw, "Managing and responding to pandemics in higher educational institutions: Initial learning from COVID-19," *International Journal of Disaster Resilience in the Built Environment*, vol. 12, pp. 51–66 (2021).

[6] B. Sen-Crowe, M. McKenney, and A. Elkbuli, "Public health prevention and emergency preparedness funding in the United States: Are we ready for the next pandemic?," *Annals of Medicine and Surgery*, vol. 59, pp. 242 (2020).

[7] M. Sreedevi, A. Harini, C. H. Yogesh Nithin, C. H. Lakshmi Prasanna, and C. H. Chinni Raghava, "Secure web application embed a honeypot link using AWS," *International Journal for Interdisciplinary Sciences and Engineering Applications* (IJISEA), vol. 6, no. 2, pp. 66–73 (2025).

[8] J. Lamwong, N. Wongvanich, I.-M. Tang, and P. Pongsumpun, "Optimal Control Strategy of a Mathematical Model for the Fifth Wave of COVID-19 Outbreak (Omicron) in Thailand," *Mathematics*, vol. 12, pp. 14 (2024). [Online]. Available: https://doi.org/10.3390/math12010014

[9] V. M. P. Mendoza, R. Mendoza, J. Lee, and E. Jung, "Managing bed capacity and timing of interventions: A COVID-19 model considering behavior and underreporting," *AIMS Mathematics*, vol. 8, pp. 2201–2225 (2023).

[10] Z. H. Shen, Y. M. Chu, M. A. Khan, O. A. Al-Hartomy, and M. Higazy, "Mathematical modeling and optimal control of the COVID-19 dynamics," *Results in Physics*, vol. 31, pp. 105028 (2021).

[11] C. Deshpande, C. Ramtirthkar, T. A. Wani, V. K. Bhosale, M. Gulhane, and K. S. Kumar, "Topology and knot theory applications in quantum field theory," *Journal of Interdisciplinary Mathematics*, vol. 28, no. 6, pp. 2281–2287 (2025), doi: 10.47974/JIM-2370.

[12] N. B. Pokale, P. Praveen, Y. Sood, V. Kumar, N. Vairaperumal, and V. K. Vijaya, "Exploring the integration of discrete mathematics and applied algebra in designing robust network architectures for IoT," *Journal of Discrete Mathematical Sciences and Cryptography*, vol. 29, no. 2-B, pp. 877–884 (2026), doi: 10.47974/JDMSC-2538.

[13] M. G. Dhote, B. M. Nanche, P. N. Mahalle, S. S. Ali, M. Gulhane, and V. S. Karwande, "Deep learning for optimized path planning in autonomous vehicles by integrating reinforcement learning with convolutional neural networks," *Journal of Information and Optimization Sciences*, vol. 46, no. 4-B, pp. 1129–1139 (2025).

[14] Hamideh Ghazizadeh, Neda Shakour, Sahar Ghoflchi, Amin Mansoori, Maryam Saberi-Karimiam, and Mohammad Rashidmayvan, "Use of data mining approaches to explore the association between type 2 diabetes mellitus with SARS-CoV-2," *BMC Pulmonary Medicine*, vol. 23, pp. 203 (2023).

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