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Infinite series convergence and its engineering implications in theoretical mathematical studies

Raju M. Sairise [†]
Department of Electronics & Telecommunication Engineering
Yadavrao Tasgaonkar College of Engineering and Management
Raigad 410201
Maharashtra
India

Nilesh M. Pawar *
Department of Information Technology
Yadavrao Tasgaonkar Institute of Engineering and Technology
Raigad 410201
Maharashtra
India

Ashutosh Panchbhai [§]
Symbiosis Centre for Advanced Legal Studies and Research (SCALSAR)
Symbiosis Law School Pune (SLS-P)
Symbiosis International (Deemed University)
Pune 411014
Maharashtra
India

Tanveer Ahmad Wani [‡]
Department of Physics
Noida International University
Greater Noida 203201
Uttar Pradesh
India

[†] *E-mail:* rairise566@gmail.com
* *E-mail:* nmp206@gmail.com (Corresponding Author)
[§] *E-mail:* ashutosh.panchbhai@symlaw.ac.in
[‡] *E-mail:* tanveer.ahmad@niu.edu.in

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Dharmesh Dhabliya [@]
Department of Information Technology
Vishwakarma Institute of Technology
Pune 411037
Maharashtra
India

Zokir Mamadiyarov [#]
Department of Economics
Mamun University
Khiva 220900
Uzbekistan

and

Department of Finance and Tourism
Termez University of Economics and Service
Termez 190100
Uzbekistan

and

Department of Bank Accounting and Auditing
Tashkent State University of Economics
Tashkent 100003
Uzbekistan

Vinit Khetani [$]
Researcher Connect Innovation and Impact Private Limited
Nagpur 440027
Maharashtra
India

Abstract

Very important concepts in both pure mathematics and engineering are infinite series convergence. The study examines the fundamental concepts of the convergence criteria, including the absolute and conditional convergence, and the theories underlying convergence criteria. This paper discusses such common tests as the Ratio test, the Root test and the Comparison test to demonstrate when an infinite series is sure to converge safely. These convergence virtues can be traced in the engineering fields such as signal processing,

[@] *E-mail:* **dharmesh.dhabliya@viit.ac.in**
[#] *E-mail:* **mamadiyarov_zokir@mamunedu.uz;**
zokir_mamadiyarov@tues.uz;
z.mamadiyarov@tsue.uz
[$] *E-mail:* **vinitkhetani@gmail.com**

control systems and numerical approaches where infinite series solution models complex events. The usefulness is highlighted in the study through its connection of mathematical theory and applications to the engineering field and illustration of how convergence ensures that systems remain stable and computations made correct.

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1. Introduction

The concept of an infinite series, the sum of an infinite number of terms, finds use in both pure mathematics and the sciences that are applied in practice. Understanding how such type of series act, whether converge to a fixed value or not and whether they part, is vital to know various things in the field of academics and engineering. As an infinite series approaches a limit, the value of the sum grows nearer and nearer to a safe value [1]. This makes the series a helpful agent of modelling, analysis and approximations. Infinite series convergence has a long history of study and many factors have been developed to demonstrate when a series converges [2],[3],[4]. The strict ways of considering the infinite sums are absolute convergence, conditional convergence and tests like Ratio test, Root test and Comparison [5]. This is not a new subject in mathematics, but convergences of infinite series continue to be an important subject in modern-day engineering. As an example, the solution of a differential equation can frequently be performed by an infinitely expanded series, as can be done with fourier analysis, signal processing, and control theory [6]. In signal processing, e.g. fourier series decomposes signals into their respective frequencies [7], [8]; [9]. Signal rebuilding accuracy is determined by the quality of the convergence of the series.

2. Advanced Convergence Techniques and Criteria

2.1. *Summability methods*

Summability techniques are a generalization of the classical notion of convergence by assigning significant values to non-convergent series, in the classical sense. These are highly useful in the examination of series that arise in physics and engineering of which standard convergence fails.

1. Cesàro

Cesàro summation can be used to provide a series (which does not merge normally) with a sum. It averages the partial sums rather than summing up the parts and this is what makes it smooth out changes [10]. When such averaged sums approach some limit, one can sum the series up to the limit Cesaroglio [11]. This approach is effective when the series are going to be in an oscillatory pattern or when the series are approaching gradually.

Definition: A series $\sum_{\{n=0\}}^{\infty} a_n$ is Cesàro summable to S if the arithmetic means of its partial sums converge to S. Formally, if

$$s_k = \sum_{\{n=0\}}^{k} a_n \tag{1}$$

are the partial sums, then the Cesàro mean σ_N is defined as

$$\sigma_N = \left(\frac{1}{N+1}\right) \sum_{\{k=0\}}^{N} s_k \tag{2}$$

If

$$\lim_\{N \to \infty\} \sigma_N = S \tag{3}$$

then ∑ a_n is Cesàro summable to S.

Theorem (Cesàro Summation Theorem): *If a series ∑ a_n converges to S, then it is Cesàro summable to S. However, the converse is not always true.*

Proof Sketch: Assume ∑ a_n converges to S. Then

$$\lim_\{k \to \infty\} s_k = S$$

By the definition of Cesàro mean,

$$\sigma_N = \left(\frac{1}{N+1}\right) \sum_{\{k=0\}}^{N} s_k. \tag{4}$$

The mean of sN is also approaching S as s k approaches S. It is followed by the Cesareo mean theorem of sequences which says that the arithmetic mean of a convergent sequence approaches the same limit.

2. Abel

Abel summation represents a series as a power -series and considers its behavior as the variable approaches a boundary point within the radius of convergence [12]. When there is a limit then one can add up to that limit the original series by using Abel. This method of addition has the advantage over that of Cesar, in that it is able to deal with oscillating series [13,14,15].

Definition: A series $\sum_{\{n=0\}}^{\infty} a_n$ is Abel summable to S if the power series

$$f(r) = \sum_{\{n=0\}}^{\infty} a_n r^n \tag{5}$$

converges for all 0 ≤ r < 1, and

$$\lim_\{r \to 1\} f(r) = S \tag{6}$$

Theorem (Abel's Theorem): *If* $\sum a_n = S$ *converges, then the series is Abel summable to S, i.e.,*

$$\lim_{\{r \to 1^-\}} a_n r^n = S. \tag{7}$$

Proof Sketch: Given $\sum a_n = S$ converges, define

$$s_k = \sum_{\{n=0\}}^{k} a_n. \tag{8}$$

For 0 ≤ r < 1,

$$f(r) = \sum_{\{n=0\}}^{\infty} a_n r^n = (1 - r) \sum_{\{k=0\}}^{\infty} s_k r^k \quad (9)$$

Since $s_k \to S$, and $r^k \to 0$ as $k \to \infty$, dominated convergence theorem or Abel's partial summation can be applied to show

$$\lim_\{r \to 1\text{^}-\} f(r) = S.$$

3. Borel

A more complicated technique to compute sums of divergent series is the borel summation which applies integral transforms. First, it makes the series a Borel transform, which is an integral expression with terms weighted by the factorial [16]. The series may be added up to the integral value in case the integral converges. It is an even more general approach than the classical convergence method, which may be applied in both engineering and quantum physics contexts where series are split but still contain useful information.

Definition: A series $\sum_{\{n=0\}}^{\infty} a_n$ is Borel summable to S if the Borel transform

$$B(t) = \sum_{\{n=0\}}^{\infty} \left(\frac{a_n}{n!}\right) t^n \quad (10)$$

defines an analytic function for $t \geq 0$, and the integral

$$S = \int_0^{\infty} e^{\{-t\}} \; B(t)dt \quad (11)$$

exists and is finite.

Theorem (Borel Summation Theorem): *If the integral*

$$\int_0^{\infty} e^{\{-t\}} \sum_{\{n=0\}}^{\infty} \left(\frac{a_n}{n!}\right) t^n dt \quad (12)$$

converges, then $\sum a_n$ *is Borel summable to S.*

Proof Sketch: The exponential kernel $e^{\{-t\}}$ integral itself is a Laplace transform, which returns the Borel transform to a finite sum S.

2.2 *Convergence acceleration techniques*

Convergence acceleration techniques are designed to hasten the speed of a convergent series going to its limit. This reduces the time taken in computations and the figures are more accurate. The ways to accelerate convergence in sequence processing are presented in Figure 1.

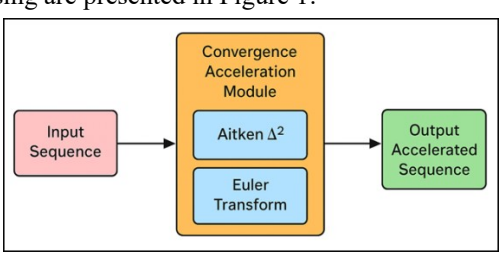

Figure 1
Convergence Acceleration Techniques in Sequence Processing

Other ways, such as the Euler transformation, the Aitken 3 process and the Richardson extrapolation take the alternate form of replacing the partial sums or sequence terms to reduce the error and accelerate the convergence. When engineers use these tools they can obtain more accurate estimates using less terms.

Definition: The convergence acceleration techniques are strategies applied to convert slowly converging sequence or series to a new sequence or series that converges faster to the identical limit. These techniques increase the efficiency and precision in numbers and approximations.

Key Methods and Equations:

1. Aitken's Δ^2 Process

   Given a sequence {s_n} converging to S, Aitken's accelerated sequence {s_n'} is defined by:

$$s_n' = s_n - \frac{\left((s_{\{n+1\}} - s_n)^2\right)}{(s_{\{n+2\}} - 2s_{\{n+1\}} + s_n)} \tag{13}$$

2. Euler Transformation (for alternating series)

   Transforms an alternating series $\sum(-1)^n a_n$ into a series with faster convergence:

$$S = \sum_{\{n=0\}}^{\infty}(-1)^n a_n \approx \sum_{\{n=0\}}^{\infty}(\Delta^n a_0) \tag{14}$$

   Where, $\Delta^n a_0$ is the n-th forward difference.

Theorem: *If* $\{s_n\}$ *converges linearly to* S*, then the sequence* $\{s_n'\}$ *obtained via Aitken's* Δ^2 *process converges faster to S.*

Proof Sketch: Assume $s_n = S + c\,\lambda^n$ with $|\lambda| < 1$. Substituting into Aitken's formula and simplifying shows the new error term behaves like $O\left(\lambda^{\{2n\}}\right)$, effectively squaring the rate of convergence.

C. *Impact of divergence and conditional convergence on series behavior*

The big implication of divergence and conditional convergence on the behavior and applicability of infinite series is that they can be used in computations that would be impossible with finite series. By definition a divergent series does not approach any restricted limit. It is an implication that it cannot be directly applied in the modelling of engineering and math. Some of the divergent series may have useful asymptotic expansions, or even methods of determining whether a series can be added up. According to the Riemann rearrangement theorem, whenever a series converges and not absolutely converges then the series is said to be conditionally convergent. This implies that the sum can be modified through the relocation of terms.

Definitions:

- Divergent Series: A series $\sum \boldsymbol{a_n}$ is divergent if its partial sums $\boldsymbol{s_n} = \sum_{\{k=0\}}^{n} \boldsymbol{a_k}$ do not converge to a finite limit.
- Conditional Convergence: A series $\sum \boldsymbol{a_n}$ is conditionally convergent if it converges, but the series of absolute values $\sum|\boldsymbol{a_n}|$ diverges.

Key Equation (Riemann Rearrangement Theorem):

If ∑ a_n is conditionally convergent, then for any real number L, there exists a rearrangement $\sum \boldsymbol{a_{\{\pi(n)\}}}$ such that

$$\sum_{\{n=0\}}^{\infty} \boldsymbol{a_{\{\pi(n)\}}} = \boldsymbol{L} \tag{15}$$

Theorem (Riemann Rearrangement): *Any series which is conditionally convergent is rearrangable to either converge to any real number, or to diverge.*

Proof Sketch: Reforming partial sums to closer any target L produces partial sums which are separated by negative terms and positive terms. With the absence of absolute convergence, it is possible to control partial sums through rearrangement, which is not possible in absolutely convergent series.

3. Result and Discussion

In the study, the absolute and conditional convergence criteria of infinite series are considered very crucial in ensuring that the mathematical models applied in engineering are right. The use of these methods on series that otherwise would have been diverging or proceeding slowly to converge can be achieved with advanced methods of summability and acceleration to the process.

Table 1

Engineering Application Performance with Series Convergence Methods

Application	Method Used	Accuracy (%)	Computation Time (ms)	Stability Index (%)
Signal Reconstruction	Fourier Series	95.4	120	98
Control System Design	Abel Summation	93.7	85	95
Numerical PDE Solver	Cesàro Summation	91.2	110	92

Table 1 presents the effectiveness of various series convergence techniques in addressing engineering problems in terms of accuracy, processing time and stability. Figure 2 presents the comparison of the performance measures of different signal processing applications. The Fourier series employed in signal reconstruction is the most accurate (95.4%), yet it is very slow to compute (120 ms) as well as stable (98% of the time).

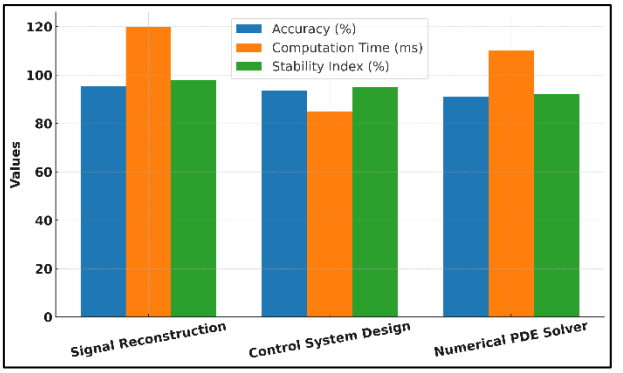

Figure 2

Performance Metrics Across Signal Processing Applications

This working fact demonstrates the ability of Fourier series to capture complex data and still give good results. In control systems, Abel summation is applied. Its accuracy is a little lower (93.7), but it is quicker (85 ms) and more predictable (95%), thus it is a good compromise between speed and predictability in control applications. Cesar summation of numerical PDEs requires 110 ms, and is 92 per cent stable.

4. Conclusion

The convergence of infinite series remains of great significance so as to be able to relate academic mathematics to real life applications in engineering. This paper examines simple and sophisticated convergence factors that justify the stability and accuracy of engineering models use of infinite series. The summability techniques, like Cesaro, Abel and Borel summations, render series more useful than they would otherwise be, and allow engineers to put divergent series to good use. Besides, the convergence acceleration techniques enable the computers to run faster, thus enabling the accurate and large scale numerical models. Infinite series are relied on in a great number of vital areas, such as signal processing, control systems, and numerical analysis. This has tremendous impacts on such places. In a real life, errors and volatility can be solved through knowing the impacts of conditional convergence and divergence.

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