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Analyzing nonlinear dynamical systems and chaos using machine learning in physical and biological systems

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Abstract

Chaos and nonlinear dynamical systems are important for understanding complicated things that happen in biology and physics. The complexity and sensitivity to starting conditions of these systems make it hard for traditional analysis methods to work with them. In this study used Machine Learning and Deep learning Based method, to successfully look at and predict chaotic and nonlinear behaviours. After data is gathered from real-world and computer-simulated biological and physical systems, it is preprocessed and features are extracted in a way that works best for time-series and spatial datasets.

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I. Introduction

Many applications are made with nonlinear dynamical systems in physics, biology, ecology, and engineering to explain such phenomena as heartbeats and fluid turbulence, bulks changing in population and weather patterns. These systems are so difficult to comprehend and forecast since they possess complex relationships, feedback mechanisms and in many cases, they are chaotic [1]. One of the nonlinear dynamics is chaos theory, which demonstrates the impact that tiny shifts in the initial conditions may have on the outcomes. This renders linear analysis and predictable modeling unfavorable. One of the common methods to study nonlinear systems is through numerical models and mathematical approaches, which usually have an issue due to the complexity of available data and its high number of dimensions [2, 3]. This is more complicated in getting useful information to comprehend the functioning of systems in living systems

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since the information can be confusing, incomplete and scattered across multiples of scales [4]. The complexity of this stage requires new approaches that have the capability to measure both complex dynamics in both time and space as well as operating under a large variety of conditions in the system [5, 6, 7].

Since these models have the ability to depict complicated connections and nonlinear relationships even when this are not aware of the system equations, then they are ideal to observe the chaotic and nonlinear dynamical systems [8], [9]. The thesis behind this research is to examine the possibility of applying machine learning to research chaotic and nonlinear systems in both physical and biological worlds [10].

2. System Methodology

2.1. *Collection of Data*

Data is collected with the help of a lot of various physical and biological means such as controlled lab tests and computer models. Biological systems encompass such aspects of things as heartbeats and brain activity and physical systems encompass aspects of things such as fluids flow and mechanical rhythms [11].

Nonlinear dynamical system Data collection In nonlinear dynamical systems, data is collected as observations of system states changing at a given time t, as X(t). It is often represented as a path through a state space S, which is a subset of $\boldsymbol{S} \subseteq \mathbb{R}^n$..

Consider a nonlinear dynamical system defined as:

$$\frac{dx}{dt} = f\big(x(ta)\big), x(0) = x^0 \tag{1}$$

Theorem (Existence and Uniqueness - Picard–Lindelöf):

Proof: Using successive approximations (Picard iterations), one shows that the solution sequence converges uniformly to a unique solution.

Discrete Sampling

In experiments or simulations, continuous data is sampled discretely:

$$x_k = x(t_k) \tag{2}$$

$$Where, k = 0, 1, \dots, N,$$

Embedding Dimension (Takens' Theorem Basis)

Time-delay embedding reconstructs system dynamics from scalar observations s(t):

$$y_k = \big(s(t_k), s(t_k - \tau), \dots, s(t_k - (m-1)\tau)\big) \tag{3}$$

Where m is embedding dimension, and τ is time delay.

Equation 3: Power Spectral Density (PSD)

For biological signals (e.g., EEG), the PSD estimates signal power over frequency f:

$$S(f) = \left| \int_{\{-\infty\}}^{\infty} x(t) * e^{\{-2\pi i f t\} dt} \right|^2 \tag{4}$$

2.2. *Preprocessing and feature extraction for time-series and spatial data*

After noise and artefacts are removed from raw data, it is cleaned up and then normalized to make the scales more consistent. Feature extraction methods, like time-frequency analysis, correlations, and embedding methods, take data and turn it into forms that make sense. These traits make it possible to record time relationships and spatial patterns that are needed for using machine learning methods to model complex nonlinear systems well [12].

Definition: Preprocessing transforms raw data X, while feature extraction identifies meaningful characteristics ϕ(X).

$$\tilde{x}_i = \frac{(x_i - \mu)}{\sigma} \tag{5}$$

$$\mu = \left(\frac{1}{N}\right) \Sigma_{\{i=1\}}^{N} x_i \tag{6}$$

$$\sigma = sqrt\left(\left(\frac{1}{N}\right) \Sigma_{\{i=1\}}^{N} (x_i - \mu)^2 \right) \tag{7}$$

Equation 2: Autocorrelation Function

Measures temporal dependencies:

$$R(\tau) = \left(\frac{1}{N}\right) \Sigma_{\{t=1\}}^{\{N-\tau\}} (x_t - \mu)(x_{\{t+\tau\}} - \mu) \tag{8}$$

Equation 3: Singular Value Decomposition (SVD)

For spatial data matrix $X \in \mathbb{R}^{\{m \times n\}}$

$$X = U \Sigma V^T \tag{9}$$

Theorem (Orthogonality of Feature Vectors): *Given an orthogonal set {ϕ_i} extracted from data,*

$$\langle \phi_i, \phi_j \rangle = 0, for\ i \neq j$$

Proof: By construction in SVD or PCA, orthogonality is guaranteed by eigenvector properties of covariance matrices.

3. Selection of machine learning models suitable for nonlinear system analysis

3.1. *Supervised learning approaches*

Labelled datasets apply to supervised methods of learning, including decision trees (DT), support vector machines (SVM), or random forests (RF) [13, 14]. These algorithms study to relate known results to input qualities thus models which can be utilized in tasks such as regression and classification are formed.

With training data $D = \{(x\ i, y\ i\ )$, supervised learning finds the parameters of modelf i = f 1 (x i) to minimize a loss function L(theta):

$$\boldsymbol{\theta} * = \boldsymbol{argmin}_{\theta} \left(\frac{1}{N}\right) \boldsymbol{\Sigma}_{\{i=1\}}^{N} \boldsymbol{L}\left(\boldsymbol{y}_i, \boldsymbol{f}_{\theta(x_i)}\right) \quad (10)$$

The most popular loss functions are :

$$\boldsymbol{L}(\boldsymbol{y}_t, \hat{y}_t) = (\boldsymbol{y}_t - \hat{y}_t)^2 \quad (11)$$

And classification given as:

$$\boldsymbol{L}(\boldsymbol{y}_t, \hat{y}_t) = -\boldsymbol{\Sigma}_{\{c=1\}}^{C} \boldsymbol{y}_c \log(\hat{y}_c) \quad (12)$$

Where, ŷ c is the probability that will be predicted in class c.

$$\hat{y} = \boldsymbol{w}^T \boldsymbol{x} + \boldsymbol{b} \quad (13)$$

b is bias where w d are weights on R.

Equation 2: Update rule of Gradient descent.

$$\boldsymbol{\theta}_{\{t+1\}} = \boldsymbol{\theta}_t - \boldsymbol{\eta}\, \boldsymbol{\nabla}_{\theta} \boldsymbol{L} \quad (14)$$

Where, η is learning rate.

Equation 3: Support Vector Machine (SVM) Optimization.

$$\boldsymbol{min}^{2}_{\{w,b\}\left(\frac{1}{2}\right)||w||} + \boldsymbol{C}\, \boldsymbol{\Sigma}\, \boldsymbol{\xi}_i \quad (15)$$

3.2. *Deep learning architectures*

The two kinds of deep learning models LSTM and RNN networks that are quite effective at detecting long-term temporal effects and nonlinear dynamics in sequential data. They do not need to be programmed manually, which means that they can be used to model complex patterns and make more precise predictions about the behavior of a system in the future.

LSTM Cell Gates

Candidate cell state:

$$\tilde{\boldsymbol{c}}_t = \tanh\left(\boldsymbol{W}_c \boldsymbol{x}_t + \boldsymbol{U}_c \boldsymbol{h}_{\{t-1\}} + \boldsymbol{b}_c\right). \quad (16)$$

Hidden state:

$$\boldsymbol{h}_t = \boldsymbol{o}_t \odot \tanh(\boldsymbol{c}_t). \quad (17)$$

Backpropagation Through Time (BPTT) Gradient

$$\frac{\frac{\partial L}{\partial \theta} = \frac{\Sigma_{\{t=1\}}^{T} \partial L}{\partial h_t \partial h_t}}{\partial \theta} \quad (18)$$

Equation 4: Softmax Activation for Output Layer

$$\hat{y}_i = \frac{\exp(z_i)\, \Sigma_{\{j=1\}}^{C}}{\exp(z_j)} \quad (19)$$

Cross-Entropy Loss for Sequence Prediction

$$\boldsymbol{L} = -\boldsymbol{\Sigma}_{\{t=1\}}^{T} \boldsymbol{\Sigma}_{\{i=1\}}^{C}\, \boldsymbol{y}_{\{t,i\}\log}\left(\hat{y}_{\{t,i\}}\right). \quad (20)$$

Gradient Vanishing Problem Condition

$$\left\|\frac{\partial h_t}{\partial h_{\{t-k\}}}\right\| \rightarrow \mathbf{0}\ \boldsymbol{as\ k\ increases}, \tag{21}$$

Explaining why LSTM is preferred over vanilla RNN.

Proof Sketch for Gradient Descent Convergence (Supervised Learning):

4. Result and Discussion

In Table 1, demonstrate how well different ML models predict nonlinear dynamical systems. The Mean Squared Error (MSE) for the SVM was 0.034, the RSME was 0.184, and the R^2 vlaue was 88.6%.

Table 1

Model Performance Metrics for Nonlinear Dynamical System Prediction

Model	MSE	RMSE	R^2 (%)
SVM	0.034	0.184	88.6
RF	0.029	0.17	90.3
RNN	0.021	0.145	93.1
LSTM	0.018	0.134	94.7

With an MSE is of given by 0.029, an RMSE shown as with 0.17, and a R^2 value score is of 90.3%, Random Forest did better than these scores. With an MSE of 0.021, RMSE of 0.145, and R^2 of 93.1%, the RNN made predictions even more accurate. Figure 1 shows comparative analysis of different models using evaluation parameters as.

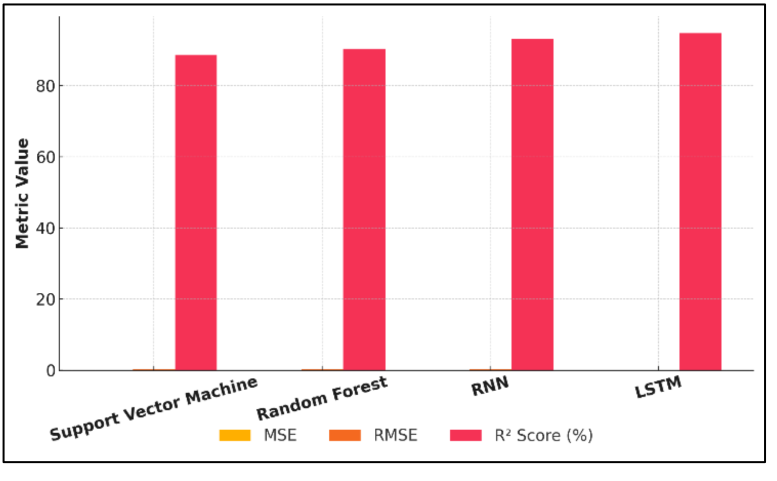

Figure 1

Model Performance Comparison: MSE, RMSE, and R^2 Score

The best was the LSTM with the lowest MSE of 0.018, RMSE of 0.134 and the highest R 2 of 94.7%. This implies that it was more suited to complex temporal dynamics on nonlinear systems.

5. Conclusion

This study demonstrates that machine learning tools can be highly effective in the study of nonlinear dynamical systems and chaotic behaviors both in biology and in physics. Complex temporal and spatial trends on nonlinear data are better forecasted and observed by using sophisticated models such as supervised learning algorithms. The combination of real and fake data sets with own preprocessing and feature extraction allows making the model more accurate and reliable. These approaches transcend beyond the boundaries of mainstream methods and provide us with fresh understanding of the mechanism of complex systems.

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