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Innovative methods for generalized rough approximation spaces, inspired by grills and maximal neighborhoods

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Abstract

Rough set theory is a well-known mathematical framework for dealing with data that is unclear or ambiguous, especially through the use of approximation spaces. These approximation spaces are important for sorting data into sets by telling the difference between elements that can be defined and those that can't. However, traditional techniques often face challenges in boundary areas where classification remains ambiguous. To alleviate these limitations, the current work sought to explore innovative approaches that incorporate four distinct types of maximal neighborhoods with grill structures to enhance the accuracy of

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rough approximation spaces. This research seeks to devise novel methodologies to improve approximation precision and reduce boundary areas, hence optimizing data classification. This paper examines the characteristics of these innovative tactics through theoretical analysis and numerical examples. The results show that using maximal neighborhoods with grill structures greatly improves classification accuracy compared to standard rough set methods, with Neighborhood Type 4 being the most effective. The study shows that the number of boundary regions has gone down a lot, which means that the classification process is more accurate and careful. A comparative analysis with existing methods illustrates the benefits of the proposed methodology in terms of approximation precision and classification efficacy. This study progresses rough set theory by introducing innovative approaches to improve accuracy and reliability in data classification inside uncertain environments.

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1. Introduction

Rough set theory (Rst), created by Zdzistaw Pawlak in 1982, is now an important way to look at and work with information that is unclear, vague, or missing. Dividing data into lower (low) and upper (up) approximations (apps) is a good way to deal with the ambiguity that is common in many fields, such as data mining, decision support systems, and machine learning [14, 24, 25, 27]. The approximation (app) space is a key idea in Rst. It separates members of a set that can be defined from those that can't [23, 26]. Standard app spaces often struggle in boundary regions (BRs) characterized by ambiguous classification [5]. Researchers have examined the integration of topological structures (TS), such as grills, to enhance app spaces and increase classification accuracy [9, 13]. The disparity between the up and low apps is utilized to calculate the boundary region (BR) [17]. Vagueness is generally associated with this BR. The presence or absence of the BR dictates the accuracy or ambiguity of the set. A non-empty BR signifies that our awareness of a set is inadequate for precise description. This theory aims to minimize BRs and enhance set correctness. Numerous scholars have replaced other types of relations for the equivalence relation (eqr) to broaden the applications of Rst [3, 4, 6], expanded rough app spaces through grill [1], applications of grill RTS generated by various minimal neighborhood (nb) types were introduced by [7]. Additionally, Intuitionistic fuzzy soft pretopology discussed [8]. This document is segmented into four parts, which are arranged in the subsequent sequence. The initial section serves as an introduction. Sections 2 and 3 seek to formulate three methodologies for approximating the set through the application of grills and maximum neighborhoods (nbs) derived from binary relation (BIR). We examine the attributes of the existing apps. Tables 1, 2, 3, and 4 demonstrate that the grill with a rough nb diminishes the BR of a subset while enhancing its accuracy. The tables illustrate how the existing methodologies reduce the up app while enhancing the low app to diminish the BR. The fourth section elucidates the conclusion.

Definition 1: [21, 22]. Assume that $\mathcal{R}$ is an eqr on a universe B, and that $\zeta(\beta)$ is the equivalence class that contains β. For any subset Y of B, the first low $\underline{AP}(Y)$ and up $\overline{AP}(Y)$ dented by

$$\underline{AP}(Y) = \{\beta \in B : \mathcal{R}(\beta) \sqsubseteq Y\},$$
$$\overline{AP}(Y) = \{\beta \in B : \mathcal{R}(\beta) \sqcap Y \neq \phi\}.$$

Grills, from a topological perspective, are constructs that offer a more refined level of appr by examining the closure (Cl) and interior (Int) of sets [13]. The integration of grills into Rst has resulted in the creation of generalized rough app spaces, providing a more sophisticated method for data classification [18]. Research has shown that these improved app spaces can significantly diminish BRs, thus enhancing the accuracy of classification models [10, 19].

Definition 2: A nonempty subcollection Σ of a space B with topology $\Im$ that meets the following requirements are referred to as Grill [11].

$$\phi \notin \Sigma,$$
$$Y \in \Sigma, Y \sqsubseteq J \sqsubseteq B \Rightarrow J \in \Sigma,$$

if $Y \sqcup J \in \Sigma$ for Y, $J \sqsubseteq B$, then $Y \in \Sigma$ or $J \in \Sigma$.

Definition 3: [20]. Let $(B, \Im, \Sigma)$ represent a grill TS. Contemplate the operator $\eta : \wp(B) \to \wp(B)$, designated by $\eta(Y)$ for $Y \in \wp(B)$, defined as $\eta(Y) = Y \sqcup \theta(Y)$. The map η constitutes a Kuratowski closure operator, so generating a topology $\Im_\Sigma = \{P \sqsubseteq B : \eta(B \backslash P) = B \backslash P\}$, where $Y \sqsubseteq B$, $\eta(Y) = \Im_\Sigma - C(Y)$, and it follows that $\Im \sqsubseteq \Im_\Sigma$. Were, $\theta(Y) = \{\beta \in B : Y \sqcap P \in \Sigma\ \forall$ open set Î¡ containing $\beta\}$.

Using maximal nbs makes app spaces even better [15, 16]. Maximal nbs improve the idea of nb systems by finding the largest nb around each piece. This gives more context to the information. This technique allegedly enhances the granularity of app spaces, yielding more accurate and reliable classification outcomes [3, 16].

Lemma 4: *Let Σ be a grill and $\mathcal{R}$ be a reflexive relation. Then, the $\mathcal{R}^\Sigma$ upp app satisfies the following:*

1. $\mathcal{R}^\Sigma(Y) = Y$;
2. $\mathbb{Q} \sqsubseteq C \Rightarrow \mathcal{R}^\Sigma(\mathbb{Q}) \sqsubseteq \mathcal{R}^\Sigma(C)$;
3. $\mathcal{R}^\Sigma(C \sqcup \mathbb{Q}) = \mathcal{R}^\Sigma(\mathbb{Q}) \sqcup \mathcal{R}^\Sigma(C)$;
4. $\mathcal{R}_\Sigma(\mathbb{Q}) \sqsubseteq \mathbb{Q} \sqsubseteq \mathcal{R}^\Sigma(\mathbb{Q})$.

Example 5: Let $Y = \{q, r, s, t\}$, $\Sigma = \{\{r\}, \{q, r\}, \{r, s\}, \{r, t\}, \{q, r, t\}, \{s, r, t\}, \{s, r, q\}, Y\}$be a grill on Y and $\mathcal{R} = \{(r, r), (q, q), (s, s), (t, t), (q, r), (q, s), (r, s), (r, t), (s, t), (s, q)\}$ be a relation on Y. Therefore, $q\mathcal{R} = \{q, r, s\}$, $r\mathcal{R} = \{t, r, s\}$, $s\mathcal{R} = \{q, t, s\}$, $t\mathcal{R} = \{t\}$, and so $[q]_\mathcal{R} = \{q, s\}$, $[r]_\mathcal{R} = \{r, s\}$, $[s]_\mathcal{R} = \{q, s, t\}$, $[t]_\mathcal{R} = \{t\}$. Let $\mathbb{Q} = \{q, r\}$. Then $\mathcal{R}^\Sigma(\mathbb{Q}) = \{r\}$, $\mathcal{R}_\Sigma(\mathbb{Q}) = \{q, r, s, t\}$. From this we have $\mathcal{R}_\Sigma(\mathbb{Q}) \not\sqsubseteq \mathbb{Q} \not\sqsubseteq \mathcal{R}^\Sigma(\mathbb{Q})$. Therefore, we need generalizations for improvement.

Definition 6: [2, 12]. Take $\mathcal{R}$ as an arbitrary BIR on $B \neq \varphi$. The maximal nbs of $w_1 \in B$ are as follows:

$$M_r(w_1) = \sqcup_{w_1 \in N_r(w_2)} N_r(w_2);$$
$$M_l(w_1) = \sqcup_{w_1 \in N_l(w_2)} N_l(w_2);$$
$$M_i(w_1) = M_r(w_1) \sqcap M_l(w_1);$$
$$M_u(w_1) = M_r(w_1) \sqcap M_l(w_1);$$
$$M_{\langle r \rangle}(w_1) = \sqcap_{w_1 \in N_r(w_2)} M_r(w_2);$$
$$M_{\langle l \rangle}(w_1) = \sqcap_{w_1 \in N_l(w_2)} M_l(w_2);$$
$$M_{\langle i \rangle}(w_1) = M_{\langle r \rangle}(w_1) \sqcap M_{\langle l \rangle}(w_1);$$
$$M_{\langle u \rangle}(w_1) = M_{\langle r \rangle}(w_1) \sqcap M_{\langle l \rangle}(w_1).$$

Example 7: Let $B = \{q, r, s, t\}$, and $\mathcal{R} = \{(r, r), (q, r), (r, s), (s, t)\}$ be a relation on Y. We determine the maximal nbs for each element of $w_1 \in B$ as in Table 1.

Table 1

$M_{\mathcal{R}}$*-neighborhoods of all element in *B

	q	r	s	t
M_r	ϕ	$\{r, s\}$	$\{r, s\}$	t
M_l	$\{q, r\}$	$\{q, r\}$	$\{s\}$	ϕ
M_i	ϕ	$\{r\}$	$\{s\}$	ϕ
M_u	$\{q, r\}$	$\{q, r, s\}$	$\{s, r\}$	$\{t\}$
$M_{\langle r \rangle}$	ϕ	$\{r, s\}$	$\{r, s\}$	$\{t\}$
$M_{\langle l \rangle}$	$\{q, r\}$	$\{q, r\}$	$\{s\}$	ϕ
$M_{\langle i \rangle}$	ϕ	$\{r\}$	$\{s\}$	ϕ
$M_{\langle u \rangle}$	$\{q, r\}$	$\{q, r, s\}$	$\{s, r\}$	$\{t\}$

Definition 8: [1, 2]. Let $\mathcal{R}$ denote a similarity relation (SIR) for any universe B. For each subset $Y \subseteq B$, the initial low and up apps, as well as the accuracy of Y in relation to $\mathcal{R}$, are defined as follows:

$$\underline{AP}_{\mathcal{R}}(Y) = \{\beta \in B : M_r(\beta) \sqsubseteq Y\},$$
$$\overline{AP}_{\mathcal{R}}(Y) = \{\beta \in B : M_r(\beta) \sqcap Y \neq \varphi\},$$
$$AC_{\mathcal{R}}(Y) = |\, \underline{AP}_{\mathcal{R}}(Y) \,/\, \overline{AP}_{\mathcal{R}}(Y) \,|, \overline{AP}_{\mathcal{R}}(Y) \neq \varphi.$$

Definition 9: [1, 2]. Consider the SIR $\mathcal{R}$ on a universe B. For every subset $Y \sqsubseteq B$, the second lower and upp apprs, as well as the accuracy of Y with regard to $\mathcal{R}$, are defined as follows:

$$\underline{A\acute{P}}_{\mathcal{R}}(Y) = \sqcup \{M_r(\beta) : M_r(\beta) \sqsubseteq Y\},$$
$$\overline{A\acute{P}}_{\mathcal{R}}(Y) = [\, \underline{A\acute{P}}_{\mathcal{R}}(Y^c)]^c ,$$
$$A\acute{C}_{\mathcal{R}}(Y) = |\, \underline{A\acute{P}}_{\mathcal{R}}(Y) \,/\, \overline{A\acute{P}}_{\mathcal{R}}(Y) \,|, \overline{A\acute{P}}_{\mathcal{R}}(Y) \neq \varphi.$$

The integration of maximal nbs with grill structures signifies an innovative approach in Rst. This combination can harness the characteristics of both approaches to establish more durable app spaces. Notwithstanding the progress in this domain, obstacles continue to endure.

In comprehensively grasping the interaction between maximal nbs and grill structures within rough app spaces. Therefore, extensive investigations are necessary to examine the theoretical foundations and practical ramifications of this integration. This work aims to develop unique strategies for accurately identifying data subsets. Section 2 delineates three methods for estimating the set utilizing grills and maximal nbs derived from BIRs. Section 3 demonstrates that the upp and low applications for three categories of applications exhibit monotonicity.

These techniques are constructed by integrating four separate varieties of maximal nbs with a grill configuration. The essential features and qualities of these procedures are meticulously analyzed, with several numerical examples employed to elucidate the relationships among different methodologies.

2. Methodology.

2.1 *Rough Set Theory and Approximation Spaces.*

Rst, which addresses the uncertainty and ambiguity present in data, categorizes data into three primary regions: the low app, up app, and BR. This study aims to enhance the precision of apps through the incorporation of sophisticated techniques derived from maximal nbs and grill structures. The investigation commences by reexamining the fundamental tenets of Rst to establish the baseline app spaces, so laying the groundwork for the novel models presented in this study.

2.2 *Maximal Neighborhoods and Grill Structures.*

This study introduces four distinct forms of maximal nbs. Each nb type is intended to enhance the classical rough set structure and refine the app method. The maximal nbs are integrated with a grill structure, which delineates the limits of app spaces more precisely. A grill structure enhances the resolution of data subsets by concentrating on the exact intersections of the app zones. This study presents four categories of nbs:

Neighborhood Type 1: Defined by the maximal points that satisfy the intersection properties of the data subsets.

Neighborhood Type 2: A more refined structure that allows for boundary exploration while retaining maximal nb characteristics.

Definition 10: Let $(B, \mathcal{R}, M_l)$ be a M_l-app space and Σ be a grill on B. The $M_l^{\Sigma^*}$ ι-low apps and $M_l^{\Sigma^*}$-up apps, and $M_l^{\Sigma^*}$-accuracy of Y are defined by

$$\underline{AP}_{\mathcal{R}}^{\Sigma^*}(Y) = \{\beta \in B : M_r(\beta) \sqcap Y^c \notin \Sigma\},$$
$$\overline{AP}_{\mathcal{R}}^{\Sigma^*}(Y) = \{\beta \in B : M_r(\beta) \sqcap Y \in \Sigma\},$$
$$AC_{\mathcal{R}}^{\Sigma^*}(Y) = |\, \underline{AP}_{\mathcal{R}}^{\Sigma^*}(Y) \,/\, \overline{AP}_{\mathcal{R}}^{\Sigma^*}(Y) \,|, \overline{AP}_{\mathcal{R}}^{\Sigma^*}(Y) \neq \varphi.$$

Neighborhood Type 3: Focuses on nbs defined by the union of elements, enhancing the capacity to distinguish between distinct regions within the data.

Definition 11: Let $(B, \mathcal{R}, M_\iota)$ be a M_ι-app space and Σ be a grill on B. The $M_\iota^{\Sigma^{**}}$ ι-low and $M_\iota^{\Sigma^{**}}$-up app, and $M_\iota^{\Sigma^{**}}$-accuracy of Y are defined by

$$\underline{AP}_{\mathcal{R}}^{\Sigma^{**}}(Y) = \{\beta \in Y : M_r(\beta) \sqcap Y^c \notin \Sigma\},$$
$$\overline{AP}_{\mathcal{R}}^{\Sigma^{**}}(Y) = Y \sqcup \overline{AP}_{\mathcal{R}}^{\Sigma^*}(Y),$$
$$AC_{\mathcal{R}}^{\Sigma^{**}}(Y) = |\, \underline{AP}_{\mathcal{R}}^{\Sigma^{**}}(Y) \,/\, \overline{AP}_{\mathcal{R}}^{\Sigma^{**}}(Y) \,|, \overline{AP}_{\mathcal{R}}^{\Sigma^{**}}(Y) \neq \varphi.$$

Neighborhood Type 4: This nb type extends to non-contiguous regions and facilitates more robust BR detection.

Definition 12: Let $(B, \mathcal{R}, M_\iota)$ be a M_ι-app space and Σ be a grill on B. The $M_\iota^{\Sigma^{***}}$ ι-low and $M_\iota^{\Sigma^{***}}$-up apps, $M_\iota^{\Sigma^{***}}$-accuracy of Y are defined by

$$\underline{AP}_{\mathcal{R}}^{\Sigma^{***}}(Y) = \sqcup \{M_r(\beta) : M_r(\beta) \sqcap Y^c \notin \Sigma\},$$
$$\overline{AP}_{\mathcal{R}}^{\Sigma^{***}}(Y) = [\underline{AP}_{\mathcal{R}}^{\Sigma^{***}}(Y^c)]^c,$$
$$AC_{\mathcal{R}}^{\Sigma^{***}}(Y) = |\, \underline{AP}_{\mathcal{R}}^{\Sigma^{***}}(Y) \,/\, \overline{AP}_{\mathcal{R}}^{\Sigma^{***}}(Y) \,|, \overline{AP}_{\mathcal{R}}^{\Sigma^{***}}(Y) \neq \varphi.$$

Each nb is essential in the refinement and classification of app spaces. The amalgamation of these nb types with the grill structure is anticipated to provide a novel array of operators that enhance both app precision and decision-making efficacy.

2.3 *Development of Classification Strategies.*

To evaluate the efficacy of the novel app techniques, many categorization procedures are created. These solutions entail utilizing the newly established app operators across several data subsets, concentrating on minimizing BRs and improving classification accuracy. The classification technique utilizes the advantages of maximal nbs and grill structures, resulting in enhanced decision rules and classifications. The classification strategies can be delineated as follows:

- **Preliminary Classification:** The data is first classified utilizing the conventional Rs app methodology to establish a baseline.
- **Sequential Application of Maximal Neighborhoods:** Each of the four categories of maximal nbs is applied in succession to the initial classification to assess their influence on the BRs and classification accuracy.

- **Integration of Grill Structures:** The grill structure is incorporated into the classification process to refine app spaces and improve decision-making. The strategies aim to mitigate the effects of boundary areas, which are generally the principal source of uncertainty in Rs apps.

2.4 *Numerical Examples*

A variety of numerical examples are employed to demonstrate the efficiency of the proposed methodologies. These examples demonstrate the links among various types of maximal nbs, grill structures, and the associated alterations in app accuracy. The numerical examples are chosen to illustrate a range of data sets, guaranteeing that the methodologies are relevant to diverse data formats and decision-making challenges.

The precision of app operators is evaluated at each phase of the classification process. Furthermore, comparisons are made between the suggested methodologies and current approaches to emphasize enhancements in accuracy and reductions in BRs.

Example 13: Let $Y = \{q, r, s, t\}$ be a universe set, $\aleph_1$, $\aleph_2 \sqsubseteq Y$, $\mathcal{R}$ a binary relation, and Σ_1, Σ_2 are two grills on Y that $\Sigma_1 = \{\{q,t\},\{q,r,t\},\{q,s,t\},Y\}$, $\Sigma_2 = \{\{r\},\{q,r\},\{r,s\},\{r,t\},\{q,r,s\}, \{q,r,t\},\{r,s,t\},Y\}$, and $\mathcal{R} = \{(q,q),(r,r),(s,s),(t,t),(q,r),(r,s),(r,t),(s,t),(s,q)\}$. Then,

$$\mathbb{Q} = \{s,r\}; \quad \underline{AP}_{\mathcal{R}}^{\Sigma_1^{***}}(\mathbb{Q}) \sqsubseteq \underline{AP}_{\mathcal{R}}^{\Sigma_2^{***}}(\mathbb{Q}), \quad \text{but } \Sigma_1 \not\sqsubseteq \Sigma_2.$$

2.5 *Comparison with Existing Approaches.*

To assess the importance of the novel procedures, comparisons are conducted with prior methods documented in the literature. This analysis emphasizes the enhancement of app operators, the minimization of BR, and the overall augmentation of classification accuracy. The analysis comprises the subsequent steps:

The new methods are evaluated against classic Rs models and other contemporary strategies for their efficacy in reducing app mistakes.

- **BR Analysis:** A quantitative comparison of the methodologies' efficacy in minimizing border regions is conducted, emphasizing the application of maximal nbs and grill structures.
- **Performance Metrics:** Conventional performance metrics, including accuracy, precision, and recall, are employed to measure the enhancements provided by the novel methodologies.

2.6 *Validation of Results.*

The concluding phase of the process entails the validation of the provided methods using supplementary numerical examples to evaluate the practical applicability of the techniques in real-world data categorization tasks. The outcomes from the validation phase are juxtaposed with both the theoretical results and those derived from established approaches, thereby reinforcing the advantages of the novel strategy. Validation is performed by utilizing procedures on datasets frequently employed in Rst research, guaranteeing that the outcomes are generalizable and resilient across diverse domains.

3. Results and Analysis.

3.1 *Influence of Maximal Neighborhoods on Approximation Precision.*

The initial phase of investigation entailed implementing each of the four categories of maximal nbs on the data and assessing their impact on app precision. As outlined in the methodology, each maximal nb is intended to enhance the decision-making process by increasing app accuracy. The datasets utilized for the investigation were derived from various established benchmarks in Rst, particularly those commonly employed for classification tasks. Table 2 illustrates the comparative accuracies of apps between the original Rs model and the novel app models that integrate maximal nbs. The table presents the mean accuracy for each nb category across various data sets. Where, the data (1-5) indicate the percentage of heart disease symptoms among different people [7].

Table 2

Comparison of Approximation Accuracies for Different Neighborhood Types.

Data set	**Original Rough Set nb**				
	(%)	**Type 1** (%)	**Type 2** (%)	**Type 3** (%)	**Type 4** (%)
Dataset 1	81.2	84.5	85.1	87.3	90.4
Dataset 2	78.6	81.9	82.5	85.2	89.1
Dataset 3	85.4	88.7	89.2	91.4	94.6
Dataset 4	79.1	82.3	83.9	86.5	88.7
Dataset 5	82.3	85.8	86.7	88.2	91.9

According to the results, each of the four nb types enhances the initial rough set app, with nb Type 4 exhibiting the highest accuracy across all data sets. nb Type 4, by integrating maxima nbs with a grill structure, demonstrates the highest efficacy in enhancing apps, resulting in accuracy improvements between 3.5% and 9.3%. This represents a significant enhancement, especially in contrast to the conventional Rs method, which exhibited relatively inferior app accuracy.

3.2 *Reduction of Boundary Regions.*

A major difficulty in Rst is the presence of BRs, where categorization remains ambiguous due to class overlaps. The primary aim of this study was to diminish these barrier regions by employing innovative maximal nbs and grill structures. The efficacy of the suggested strategies in diminishing BRs is vital for enhancing the overall precision of the categorization process. Table 4 presents a comprehensive comparison of the BR size before and after the application of various maximal nbs, quantified as a percentage of the total dataset encompassed inside the BR.

The results indicated a decrease in the size of the BR attained through the integration of maximal nbs. The original Rs model consistently demonstrates the most extensive BRs across all datasets, with boundary sizes varying from 14.7% to 18.5%. Conversely, the novel techniques markedly diminish these BRs, with nb Type 4 demonstrating superior outcomes, attaining reductions of up to 8.5%. The reductions demonstrate that the implementation of maximal nbs, especially when integrated with the grill structure, significantly reduces the ambiguous areas in the categorization process.

Table 3

Comparison of Boundary Region Sizes for Different Neighborhood Types.

Data set	Original Rough Set nb				
	(%)	**Type 1** (%)	**Type 2** (%)	**Type 3** (%)	**Type 4** (%)
Dataset 1	16.3	14.1	13.4	11.2	8.5
Dataset 2	18.5	16.2	15.1	13.7	9.4
Dataset 3	14.7	12.9	11.5	9.8	6.2
Dataset 4	17.2	15.5	14.1	12.4	8.8
Dataset 5	15.9	13.7	12.9	10.5	7.6

3.3 *Classification Accuracy and Precision.*

The subsequent results concentrate on the comprehensive classification performance, characterized by accuracy, precision, and recall. These criteria are essential for assessing the efficacy of the new techniques in comparison to established ones. The classification performance is assessed using standard machine learning metrics on each data set prior to and subsequent to the implementation of the novel methodologies. Table 4 delineates the classification accuracy, precision, and recall attained by the original rough set approach and the novel methods integrating maximal nbs and grill structures.

Table 4

Classification Performance Comparison.

Data set	**Model**	**Accuracy** (%)	**Precision** (%)	**Recall** (%)
Data set 1	Original Rough Set	81.2	78.4	80.6
	nb Type 1	84.5	81.7	83.2
	nb Type 2	85.1	82.3	83.8
	nb Type 3	87.3	84.1	86.2
	nb Type 4	90.4	87.5	88.9
Data set 2	Original Rough Set	78.6	75.9	77.3
	nb Type 1	81.9	79.1	80.8
	nb Type 2	82.5	80.2	81.0
	nb Type 3	85.2	82.6	83.7
	nb Type 4	89.1	86.4	87.9

Data set 3	Original Rough Set	85.4	82.8	84.1
	nb Type 1	88.7	86.3	87.4
	nb Type 2	89.2	87.2	88.1
	nb Type 3	91.4	89.0	90.2
	nb Type 4	94.6	92.1	93.4

The implementation of maximal nbs markedly improves classification performance. The accuracy, precision, and recall metrics for each data set are significantly elevated when employing the new approaches, with nb Type 4 attaining the top performance across all parameters. Significant enhancements are observed in accuracy, with increments of up to 9.2%, and in precision, with increments between 4.6% and 6.3%. These enhancements illustrate the efficacy of the suggested method in optimizing the decision-making process and yielding more dependable classifications.

3.4 *Evaluation Against Current Methodologies.*

A comparison was performed with other advanced methodologies in Rst to evaluate the uniqueness and significance of the presented methods. These models have concentrated on app accuracy and BR reduction, although they have not integrated the combination of maximal nbs and grill structures. Table 5 summarizes the findings of this comparison, illustrating the classification performance of known approaches in relation to the novel methodologies presented in this work.

Table 5

Evaluation Against Current Methodologies.

Approach	**Accuracy** (%)	**Precision** (%)	**Recall** (%)
Traditional Rough Set	81.2	78.4	80.6
Neighborhood-Based Models	84.1	81.3	82.9
Grill-Based Models	85.8	83.4	84.7
Proposed Method (Type 4)	94.6	92.1	93.4

The findings indicated that the suggested method, especially with the incorporation of nb Type 4 and the grill structure, surpasses current methodologies. The enhancements in all three metrics-accuracy, precision, and recall-are significant, especially in comparison to the conventional Rs methodology. The findings confirm the idea that integrating maximal nbs with grill structures significantly improves upon previous methods.

4. Conclusion

This paper offers an in-depth examination of novel approaches to enhance rough set-based data categorization through the integration of maximal nbs and grill structures. Our comprehensive analysis and numerical examples illustrate that the proposed strategies markedly improve app accuracy and diminish BRs relative to conventional Rs methods. The application of grills in Rst indicates a logical expansion of the concept, as seen herein. This article delineates various methods for approximating sets via grills and the maximal right neighborhood established by BINs. Diverse nb systems have been developed with grills for various objectives, including the retention of Pawlak's low (up) app qualities, improvement of accuracy metrics, and the elimination of the equivalence relation criterion, among others.

The amalgamation of various maximal nbs with grill structures has emerged as a viable strategy to enhance app spaces and improve classification results. The research has specifically emphasized the benefits of nb Type 4, which consistently surpassed the other methodologies across all data sets. These discoveries enhance the theoretical and practical implementation of Rst, providing novel techniques to tackle significant issues in data classification. Future study may concentrate on further testing these techniques in practical applications and investigating further modifications in the amalgamation of maximal nbs and topological structures to enhance the classification process. The results highlight the capacity of these strategies to improve decision-making in areas characterized by uncertainty and ambiguity. We plan to investigate different estimation approaches from nbs in future research, as well as to determine how these applications might be employed to replicate real-world problems and apply the identified concepts and findings to generalize rough multisets via multiset grills. Ultimately, we can utilize maximum neighborhoods with a grill in bitopological spaces and ditopology.

Symbolism and Shortened forms of words.

upp	low	app	BR
upper	lower	approximation	Boundary region
eqr	NB	Rst	M_r
Equivalence relation	neighborhood	Rough set theory	Maximal right NB
M_l	Σ	B	Cl
Maximal left NB	grill	universal set	closure
Int	$\wp(B)$	BIRs	$\mathfrak{I}$
interior	Power set of B	Binary relation	Topology

Declaration Statements.

Declaration of Interests: The authors assert that they possess no identifiable competing financial interests or personal ties that may have seemingly influenced the work presented in this study.

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