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An analytical innovation from foundations to modern applications of advancing measure theory via approach structure

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Abstract

In this research, we define and prove various features of a new measure based on the idea of approach structure, known as the approach measure. We then discuss the relationship between the new and traditional measures. We introduce another new notion, the semi-approach measure, and investigate the relation between these two concepts.

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1. Introduction

Measure theory is one of the fundamental pillars of modern mathematics and extends to many different branches and applications in the natural and engineering sciences Measure theory is an important topic in mathematical

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analysis. In 1966, Taylor [1] gave introduction to measure and integration. In 1990 Dalang and Willinger [2] made clear no-arbitrage in stochastic securities markets models and equivalent martingale measures. In 1994 Baekeland and Lowen [3] explained separability and Lindelof in approach spaces. In 2007 Hussein [4] presented equivalent measure under the process is martingale. then in 2020 Hussein [5] introduced equivalent measure in ordered Banach algebra with locally Martingale of the deflator process. In 2021 and 2022, Abbas and Hussein [6-7] presented results on normed approach space and Banach space. In 2022 [8] gave results about topological approach vector space. In 2022 Hussein and Abd [9] studied normed approach space. In 2022 Hussein and Wshayeh [10] defined space of measurable studied symmetric approach t $^{w-}$Banach algebra with some results. In 2022 Neamah and Hussein [11-14] presented results of compAssume thation t^w normed approach space [15-21].

2. Preliminary

Definition 2.1 [22, 23]: Assume that ℵ be a non-empty set and F be a collection of sub set of ℵ then F is called σ-field iff:

1) ℵ $\in$ F.
2) If ß $\in$ F then $ß^c$ $\in$ F.
3) If $A_1, A_2, \ldots \in$ F, we have $\bigcup_{n=1}^{\infty} A_n \in$ F.

A measurable space is a pair (ℵ, F).

Definition 2.2 [22, 23]: If (Ω, F) measurable space. A set function μ: F →$[0, \infty]$ called **measure on** $(\Omega, \mathbf{F})$, if

1- $E(\emptyset) = 0$
2- $\mu(\bigcup_{n=1}^{\infty} A_n) = \sum_{n=1}^{\infty} \mu(A_n)$, where $\{A_n\}$ sequence of disjoint sets in F.

Definition 2.3 [22, 23]: If (Ω,F) is measurable space and Ă is collection of subsets of Ω, we call F generated by Ă if F is the smallest σ-filed containing Ă.

Definition 2.4 [22, 23]: Assume that ℵ be a non-empty set and Ğ be a collection of sub set of ℵ then Ğ is called d-system (Dynkin class) iff:

1) $\emptyset \in$ Ğ.
2) If ß $\in$ Ğ then $ß^c$ $\in$ Ğ.
3) if $\{A_n\}$ sequence of disjoint sets in Ğ then $\bigcup_{n=1}^{\infty} A_n \in$ Ğ

The collection $\check{K}$ of subsets of ℵ is $\pi -$ system on ℵ if for each K, ß $\in$ $\check{K}$ then K $\cap$ ß $\in$ $\check{K}$.

Theorem 2.5 [22, 23]: *If ℵ is a non-empty set and Đ is $\pi - system$ of sub sets of ℵ then the σ-field generated by (Đ) coincides with d-system generated by (Đ).*

3. Approach Measure Space

Definition 3.1: If $\aleph$ is non-empty set, and F σ-filed. A function E_δ: $F \times 2^{\aleph} \rightarrow [0, \infty]$ is called approach measure on $\aleph$ if the following conditions are met

1- $E_\delta(\emptyset, ß) = 0, ß \in 2^{\aleph}$

2- $E_\delta(K, ß) = \infty$if $K = ß^c, K \in F, ß \in 2^X, K \neq \emptyset$

3- $E_\delta(\bigcup_{n=1}^{\infty} K_n, ß) = \sum_{n=1}^{\infty} E_\delta(K_n, ß)$ where $\{A_n\}$ is sequence of disjoint sets in F. The triple ($\aleph$,F, E_δ) said to be approach measure space and denoted by AMS.

Definition 3.2: Assume that $\aleph$ be a non-empty set and F σ-filed. The function μ_δ: $F \times 2^{\aleph} \rightarrow [0, \infty]$ is called semi approach measure on $\aleph$ if the following conditions are met:

1) $\mu_\delta(\emptyset, ß) = 0$, $ß \in 2^{\aleph}$

2) $\mu_\delta(K, ß) = \infty$ if $K \subseteq B^c, K \in F, ß \in 2^X, K \neq \emptyset$

3) $\mu_\delta(\bigcup_{n=1}^{\infty} K_n, ß) = \sum_{n=1}^{\infty} E_S(K_n, ß)$ for each sequence of disjoint sets $\{K_n\}_{n=1}^{\infty}$ in $F, ß \in 2^{\aleph}$. Then the triple $(\aleph, F, \mu_\delta)$ said to be **semi approach measure space** and denoted by SAMS. Note that every semi approach measure space is app-measure space.

Proposition 3.3: Assume that $(\aleph, F, E_\delta)$ be AMS then if $K \subseteq B^c$ K, $ß \in F$, $K \neq \emptyset$ then $E_\delta(K \cup ß, ß^c) = \infty$

Proof: Since $K \subseteq ß^c$ then $K \cap ß = \emptyset$,

$E_\delta(K \cup ß, ß^c) = E_\delta(K\, ß^c) + E_\delta(ß, ß^c)$

$E_\delta(A \cup ß, ß^c) = E_\delta(K, ß^c) + \infty$. Then $E_\delta(K \cup ß, ß^c) = \infty$

Proposition 3.4: Assume that$(\aleph, F, E_\delta)$ be AMS, if $K \subseteq ß$ then $E_\delta(K, C) \leq E_\delta(ß, C)$ where $K, ß \in F, C \in 2^{\aleph}$.

Proof: Assume that $A \in F$, that is $K^c \in F$, and so $K^c \cap ß \in F$ (since F σ -filed). Since $K^c \cap ß = B - A$, then $ß - K \in F$ and $K \cup (ß - K) \in F$. Then $E_\delta(ß, C) = E_\delta(K \cup (ß - K),) = E_\delta(K, C) + E_\delta(ß - K, C)$. Therefore $E_\delta(K, C) \leq E_\delta(ß, C)$.

Proposition 3.5: Assume that $(\aleph, F, E_\delta)$ be AMS. Then

$E_\delta(K \cup ß, C) = E_\delta(K, C) + E_\delta(ß, C) - E_\delta(K \cap ß, C)$ where $K, ß \in F, C \in 2^{\aleph}$.

Proof: It is clear $K \cup ß = K \cup (K^c \cap ß)$. Since $A \cap (K^c \cap ß) = (K \cap K^c) \cap ß$, that is $K \cap (K^c \cap ß) = \emptyset \cap ß = \emptyset$. Then $E_\delta(K \cup ß, C) = E_\delta(K \cup (A^c \cap ß), C)$. Therefore $E_\delta(K \cup ß, C) = E_\delta(K, C) + E_\delta(K^c \cap ß, C)$. Since $ß = (K \cap ß) \cup (K^c \cap ß)$ and $(K \cap ß) \cap (K^c \cap ß) = \emptyset$. That is $E_\delta(ß, C) = E_\delta(K \cap ß, C) + E_\delta(K^c \cap ß, C)$. $E_\delta(K^c \cap ß, C) = E_\delta(ß, C) - E_\delta(K \cap ß, C)$. Thus $E_\delta(K \cup ß, C) = E_\delta(K, C) + E_\delta(ß, C) - E_\delta(K \cap ß, C)$.

Theorem 3.6: *From any measure space it is possible construct Approach measure space.*

Proof: Assume that (X,F,E) be a measure space. We define E_δ: F× $2^X \to [0,\infty]$ by:

$$E_\delta\,(A,B)=\begin{cases}E(A)\;A=\emptyset\\ \infty\;A\neq\emptyset\end{cases}$$

We must proof E_δ is approach measure:

1) E_δ (∅,ß)=$E(\emptyset)=0$. Then E_δ (∅,ß)=0
2) Assume that K, ß $\in 2^{\aleph}$ where $K\neq\emptyset$ such that $K=ß^c$
   By definition of (E_δ) we get E_δ (K, ß)=∞
3) If $\{K_n\}_{n=1}^{\infty}$ a sequence of disjoint sets in F. If $\bigcup_{n=1}^{\infty}K_n=\emptyset$

Then, E_δ ($\bigcup_{n=1}^{\infty}K_n$, ß)=$E(\bigcup_{n=1}^{\infty}K_n)$ E_δ ($\bigcup_{n=1}^{\infty}K_n$, ß)=$E(K_1)+E(K_2)+E(K_3)+\ldots$ Since $\bigcup_{n=1}^{\infty}K_n=\emptyset$, then $K_n=\emptyset$for n=1,2,…

Since $E_\delta\,(K_n,ß)=E(K_n)$ forn=1,2,… That is $\sum_{n=1}^{\infty}E_\delta$ $(K_n,ß)$=$\sum_{n=1}^{\infty}E$ (K_n), and so $\sum_{n=1}^{\infty}E_\delta$ $(K_n,ß)$=$E(K_1)+E(K_2)+E(K_3)+\ldots$ Then, E_δ ($\bigcup_{n=1}^{\infty}K_n$,ß)=E $(K_n,ß)$. If $\bigcup_{n=1}^{\infty}K_n\neq\emptyset$ that is E_δ ($\bigcup_{n=1}^{\infty}K_n$,ß)=∞, Since $\bigcup_{n=1}^{\infty}K_n\neq\emptyset$, then there exist at least one set say $K_t\in\{K_n\}_{n=1}^{\infty}$ such that $K_t\neq\emptyset$ $(1\le t<\infty)$ so that E_δ $(K_t,ß)$=∞ and we have $\sum_{n=1}^{\infty}E_\delta$ $(K_n$, ß)=$E_\delta\,(K_1,ß)+E_\delta\,(K_2,ß)+\ldots+E_\delta\,(K_t,ß)+\ldots$ Hence, $\sum_{n=1}^{\infty}E_\delta$ $(K_n$, ß)=$E_\delta\,(K_1,ß)+E_\delta\,(K_2,ß)+\ldots+\infty+\ldots$, that is $\sum_{n=1}^{\infty}E_\delta$ $(K_n$, ß)=∞. Then E_δ ($\bigcup_{n=1}^{\infty}K_n$, ß)=$\sum_{n=1}^{\infty}E_\delta$ $(K_n$, ß). Therefore, $(\aleph,F,E_\delta)$ AMS.

Theorem 3.7: *Assume that* $(\aleph,F_1,E_{s1})$, $(\aleph,F_2,E_{s2})$ *be two semi approach measure spaces. If* $F_1\cup F_2$ *is σ-filed then* $(\aleph,F_1\cup F_2,E_\delta)$ *is AMS. where* $E_\delta(K\cup B, C)=E_{s1}(K,C)+E_{s2}(ß,C)$ *for all* $K\in F_1$, $ß\in F_2$, $C\in 2^X$.

Proof: We must proof E_δ is App-measure:

1) Assume that $C\in 2^{\aleph}$, E_δ (∅,C)= E_{s1} (∅,C) + E_{s2} (∅,C)=0+0=0. 1
2) Assume that $K\in F_1$, $ß\in F_2$, $C\in 2^X$ such that $K\cup B=C^c$.
   We must proof E_δ (K∪ B, C)=∞, Since $K\subseteq K\cup ß$ then $K\subseteq C^c$ and $ß\subseteq K\cup ß$ then $ß\subseteq C^c$.Therefor E_{s1} (K,C)=∞and E_{s2} (ß, C)=∞ therefore E_δ (K∪B,C)=$\infty+\infty=\infty$
3) Assume that $\{K_n\cup ß_n\}_{n=1}^{\infty}$ be sequence of disjoint sets in $F_1\cup F_2$where $A_n\in F_1$, $ß_n\in F_2$, $C\in 2^X$ n=1,2,…
   that is E_δ ($\bigcup_{n=1}^{\infty}(K_n\cup ß_n)$, C)=$E_\delta$ ($\bigcup_{n=1}^{\infty}K_n\cup\bigcup_{n=1}^{\infty}ß_n$, C)
   and so E_δ ($\bigcup_{n=1}^{\infty}(K_n\cup ß_n)$, C)=$E_{s1}$ ($\bigcup_{n=1}^{\infty}K_n$,C) + E_{s2} ($\bigcup_{n=1}^{\infty}ß_n$, C)
   E_δ ($\bigcup_{n=1}^{\infty}(K_n\cup ß_n)$,C)=$\sum_{n=1}^{\infty}E_{s1}$ (K_n, C) + $\sum_{n=1}^{\infty}E_{s2}$ $(ß_n, C)$.
   E_δ ($\bigcup_{n=1}^{\infty}(K_n\cup ß_n)$, C)=$\sum_{n=1}^{\infty}(E_{s1}(K_n, C)+E_{s2}(ß_n, C)$.

Then, E_δ ($\bigcup_{n=1}^{\infty}(K_n \cup ß_n)$, C)=$\sum_{n=1}^{\infty}$ E_δ ($K_n \cup ß_n$, C)

Hence, E_δ is app.measure. ($\aleph$, $F_1 \cup F_2, E_\delta$) is AMS.

4. Restriction of semi approach measure

Theorem 4.1: *Assume that $(\aleph, F, E_\delta)$ be semi approach measure space and $Y \in F$, if we define $E_\delta \downharpoonleft_Y : F \times 2^{\aleph} \to [0,\infty]$ by $E_\delta \downharpoonleft_Y(A, ß)$=$E_\delta$ $(K \cap Y, ß)$, $A \in F$, $ß \in 2^{\aleph}$, Then $E_\delta \downharpoonleft_Y$ is approach measure on $\aleph$ known the Y-restriction of E_δ.*

Proof: We must proof $\mu_\delta \downharpoonleft_Y$ is App-measure:

1) $E_\delta \downharpoonleft_Y(\emptyset\ ß)$=$E_\delta$ $(\emptyset \cap Y, ß) = E_\delta(\emptyset, ß)$ that is $E_\delta \downharpoonleft_Y(\emptyset, ß)$=0.
2) Assume that $K \in F$, $ß \in 2^X$ such that $K = ß^c$, $K \neq \emptyset$. Since $E_\delta \downharpoonleft_Y(K, ß)$=$E_\delta$ $(K \cap Y, ß)$ and $K \cap Y \subseteq K$ then $K \cap Y \subseteq ß^c$ Then E_δ $(K \cap Y, ß)$=∞ (since E_δ is a semi approach measure)$E_\delta \downharpoonleft_Y$ (K, ß)=∞
3) If $\{K_n\}_{n=1}^{\infty}$ a sequence of disjoint sets in $F, E_\delta \downharpoonleft_Y$ $(\bigcup_{n=1}^{\infty} K_n, ß)$=$E_\delta$ $(\bigcup_{n=1}^{\infty} K_n \cap Y, ß)$.That is $E_\delta \downharpoonleft_Y(\bigcup_{n=1}^{\infty} K_n, ß)$=$E_\delta$ $(\bigcup_{n=1}^{\infty}(K_n \cap Y), ß)$.Then, $E_\delta \downharpoonleft_Y$ $(\bigcup_{n=1}^{\infty} K_n, ß)$=$\sum_{n=1}^{\infty} E_\delta$ $(K_n \cap Y, ß)$, and $E_\delta \downharpoonleft_Y$ $(\bigcup_{n=1}^{\infty} K_n, ß)$= $\sum_{n=1}^{\infty} E_\delta \downharpoonleft_Y$ $(K_n, ß)$ Then $E_\delta \downharpoonleft_Y$ is approach measure on $\aleph$.

Theorem 4.2: (Approach measure in terms of restriction) *Assume that $(\aleph, F_1, E_{s1})$,$(\aleph, F_2, E_{s2})$ be two semi approach measure spaces,if $F_1 \cup F_2$ σ -filed and $Y \in F_1 \cap F_2$ then $(\aleph, F_1 \cup F_2, M)$ is approach measure space,where E_δ $(K \cup ß, C)$= $E_{s1} \downharpoonleft_Y (K, C)$ + $E_{s2} \downharpoonleft_Y (ß, C)$ $K \in F_1, ß \in F_2, C \in 2^{\aleph}$.*

Proof: We must proof E_δ is Appr-measure:

We can proof the first and third conditions of appr-measure by the same way of the proof in theorem (3.7)

3) Assume that $K \in F_1, ß \in F_2, C \in 2^X$ such that $A \cup ß = C^c$ We must proof E_δ (K∪ß, C)=∞ Since E_δ (K∪ß, C)= $E_{s1} \downharpoonleft_Y$ (K, C) + $E_{s2} \downharpoonleft_Y$ (ß, C) then E_δ (K∪ß, C)= μ_{s1} $(K \cap Y, C)$ + E_{s2} $(ß \cap Y, C)$, Since $K \subseteq K \cup ß$ and $ß \subseteq K \cup B$ then $K \subseteq C^c$ and $ß \subseteq C^c$. Since $K \cap Y \subseteq K$ and $ß \cap Y \subseteq ß$ then $K \cap Y \subseteq C^c$ and $ß \cap Y \subseteq C^c$. Therefore μ_{s1} $(A \cap Y, C) = \infty$ and μ_{s2} $(ß \cap Y, C)) = \infty$. Hence E_δ (K∪ß,C)=∞+∞ $= \infty$. Then E_δ is approach measure, therefore $(\aleph, F_1 \cup F_2, E_\delta)$ AMS.

5. Application of Approach measure on Dynkin class

Proposition 5.1: Assume that $(\aleph, F, E_\delta)$ be AMS, if $Ɓ \subseteq K \in F$ then E_δ $(K - Ɓ$, C)=E_δ (K, C)-E_δ (Ɓ, C) where $K, Ɓ \in F, C \in 2^{\aleph}$.

Proof: It is clear that.

Theorem 5.2: *Assume that $(\aleph, F, E_{\delta_1})$, $(\aleph, F, E_{\delta_2})$ be two AMS where $E_{\delta_1}(\aleph, ß)$=$E_{\delta_2}(\aleph, ß)$, If F is σ- field generated by (Đ) where Đ is $\pi - system$*

of sub sets of ℵ *and* $E_{\delta_1}(K,ß)$ *=*$E_{\delta_2}(K,ß)$ *for each* $K \in Đ$, $ß \in 2^{ℵ}$ *then* $E_{\delta_1}(K,ß)$ *=*$E_{\delta_2}(K,ß)$ *for each* $K \in F$, $ß \in 2^{ℵ}$

Proof: Define Ţ$=\{K \in F: E_{\delta_1}(K,ß) =, \text{if } = E_{\delta_2}(K,ß)\}$

We must proof Ţ is d-system

1) Since $\in F$, $E_{\delta_1}(\emptyset,ß) = E_{\delta_2}(\emptyset,ß)=0$. Then $\emptyset \in$ Ţ.

2) Assume that $K \in$ Ţ, we must proof $K^C \in$ Ţ, $E_{\delta_1}(K^C,ß) = E_{\delta_1}(ℵ - K,ß)$

By Proposition (5.1) we get:

$E_{\delta_1}(K^C,ß) = E_{\delta_1}(ℵ - K,ß) = E_{\delta_1}(ℵ,ß)-E_{\delta_1}(K, ß)$since $K \subseteq ℵ$, $E_{\delta_1}(ℵ,ß)=E_{\delta_2}(ℵ, ß)$ and $K \in$ Ţ.

Then $E_{\delta_1}(K^C,Ɓ) = E_{\delta_2}(ℵ,ß)-E_{\delta_2}(K, ß)$ Therefor $E_{\delta_1}(K^C,ß) = E_{\delta_2}(K^C,Ɓ)$. Hence $K^C \in$ Ţ.

3) Assume that $\{K_n\}_{n=1}^{\infty}$ be a family of disjoint sets in Ţ $E_{\delta_1}(\bigcup_{n=1}^{\infty} K_n,ß) = \sum_{n=1}^{\infty} E_{\delta_1}(K_n,ß)$, Since $K_n \in$ Ţ for n=1,2,..Then $\sum_{n=1}^{\infty} E_{\delta_1}(K_n,ß) = \sum_{n=1}^{\infty} E_{\delta_2}(K_n,ß)$ Hence $E_{\delta_1}(\bigcup_{n=1}^{\infty} K_n,ß) = E_{\delta_2}(\bigcup_{n=1}^{\infty} K_n,ß)$. Therefor $\bigcup_{n=1}^{\infty} K_n \in$ Ţ, Ţ is d-system Since $E_{\delta_1}(K,ß)=E_{\delta_2}(K,ß)$ for each $K \in Đ$, $ß \in 2^{ℵ}$. Then Đ $\subseteq$ Ţ Since F is σ - field generated by (Đ), by theorem (2.6) we get F $\subseteq$ Ţ.

Hence $E_{\delta_1}(K,ß)=E_{\delta_2}(K,ß)$ for each $K \in F$, $ß \in 2^{ℵ}$.

Theorem 5.3: *Assume that* $(ℵ,F,E_\delta)$ *be AMS and* $K \in F$ *then the set*

Ɏ$=\{ß \in F: E_\delta(K \cap ß,C) = E_\delta(K,C)E_\delta(ß,C)\} \cup \{ß^C\}$ is d-system, $C \in 2^{ℵ}$

Proof:

1) Since $\emptyset \in F$, $E_\delta(K \cap \emptyset,C) = E_\delta(\emptyset,C)=0=E_\delta(K,C)E_\delta(\emptyset,C)$ Then $\emptyset \in$ Ɏ.

2) Assume that $ß \in$ Ɏ then $ß^C \in$ Ɏ by definition of the set Ɏ

3) Assume that $\{ß_n\}_{n=1}^{\infty}$ be a family of disjoint sets in Ɏ.

$E_\delta(K \cap \bigcup_{n=1}^{\infty} ß_n,C) = E_\delta(\bigcup_{n=1}^{\infty} K \cap ß_n,C)$ $E_\delta(K \cap \bigcup_{n=1}^{\infty} ß_n,C) = \sum_{n=1}^{\infty} E_\delta(K \cap ß_n,C)$ Since $ß_n \in$ Ɏ then $E_\delta(K \cap \bigcup_{n=1}^{\infty} ß_n,C) = \sum_{n=1}^{\infty} E_\delta(K,C)\, E_\delta(ß_n,C)$ $E_\delta(K \cap \bigcup_{n=1}^{\infty} ß_n,C) = E_\delta(K,C) \sum_{n=1}^{\infty} E_\delta(ß_n,C)$ $E_\delta(K \cap \bigcup_{n=1}^{\infty} ß_n,C) = E_\delta(K,C)E_\delta(\bigcup_{n=1}^{\infty} ß_n,C)$ Therefor $\bigcup_{n=1}^{\infty} K_n \in$ Ɏ Hence Ɏ is d-system

6. Conclusion

This research is to create a new structure for the concept of measurement that differs from classical measurement and find relationship between them, semi approach measure space and construct the restriction of semi approach measure approach measure is Dynkin Class under conditions and construct some results in this manner.

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