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A new mathematical model driven by the Theory of Complexity applied to the relation between economic variables, money demand and blockchain crypto-coin application

Gerardo Iovane *
Department of Computer Science
University of Salerno
Fisciano (SA) 84084
Italy

Massimo Ceccobelli †
Department of Business and Management
Libera Università International degli Studi Sociali (L.U.I.S.S.)
Rome
Italy

Abstract

We can say that in the context of Financial Computing, the present article aims to delve into an exploration of the intricate web of relationships among the key economic variables, with a central focus on the concept of money, mapping and connecting different theories through a new definition of velocity of money. In practice, the novelties of the article are based on the following aspects: i) definition of a new concept of velocity of circulation of money based on utility; ii) consequent redefinition of the demand for money; iii) fil rouge found to connect the various theories based on the above: a) quantity theory of money by Friedman, b) Keynesian theory on Money, c) neoclassical theory on consumers choices, d) mean variance theory by Markowitz, Arrow Pratt approximation, e) Purchasing Power Parity, f) prospect theory by Tversky and Kahneman.

The article attempts to identify a common thread among various models related to the concept of money. The concept of money anyway is intended in a broader sense than usual, including the financial instruments that can be used as a financial reserve of value easily convertible in cash. In practice we intend for money financial wealth. When we write in the article the word money we will intend it in this broader sense. The objective is not to create a comprehensive theory of money in the economic and financial fields, but, starting from

* *ORCID-ID:* 0000-0002-3119-4608

* *E-mail:* giovane@unisa.it (Corresponding Author)
† *E-mail:* mceccobelli@luiss.it

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this broader concept of money, to identify possible connections between theories that have so far been treated separately. The new concept of the velocity of money circulation, which depends on the ratio between the utility of purchasable goods and services, and the utility of money, helps in establishing some of these connections.

In the article, when 'goods' are mentioned, it refers to 'goods and services.'

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1. Introduction

Our analysis about Theories on Money starts from the famous Quantity Theory of Money by Milton Friedman [1]. Following this theory changes in the quantity of money in the economy have a proportional impact on the level of prices. This relationship is the basis for the equation of exchange: MV = PQ, where M represents the money supply, V represents the velocity of money, P represents the price level, and Q represents the quantity of goods produced. The Money supply in our concept of Money includes not only money produced by the behavior of the banking sector but also financial instruments produced by the interaction of financial operators in direct markets, It is a theory for the long-run not a detailed analysis of short run. Moreover, demand for money does not depend on interest rates like in the Keynesian theory of money [2]. In fact, The Keynesian Theory for money assesses the utilization of money for three goals: i) transactional use, ii) precautionary (future transactional) use, iii) speculative use. The speculative use depends on interest rates. Use for future transactional (or rather precautionary) purposes and speculative purposes fall within the more general concept of reserve of value. That is, money from a macroeconomic point of view is first used for immediate transactional purposes and then remains in the system as a reserve of value. Of course, also goods can be a reserve of value, but their utility depends mostly on the fact that they can be consumed. The reserve of value is subject to some risks, mainly: a) liquidity risk (risk that the financial instrument does not translate into a means of payment, when necessary, in a short time), b) financial risk (risk of changes in the value of the financial instrument over time). In our understanding, we also consider many assets as a store of value, provided they have low risk liquidity.

Money is recognized as a reserve of value usually when it is backed up by assets (utility of money is also affected by the quality of assets backed up):

In the past century's gold,

More recently assets of banks whose liabilities are money

Then assets of entities producing income (typically companies or the State) whose liabilities are "almost money" held directly by private sector. This is the basis of our concept of money.

Only recently we are assisting to the phenomenon of cryptocurrencies (as in particular bitcoin) not backed up by assets of which we will speak at the end of the article.

The overall wealth of the system can be described as follows:

MV=PQ (Meaning the goods produced Q purchased with transactional money MV at market prices P.

The money M that remains in the economic system as a store of value after transactional uses.

The total wealth W at the economic system level will be:

$$W = PQ + M = MV + M = M(1 + V) \quad (1)$$

The real wealth will be

$$W/P = Q + M/P \quad (2)$$

Essentially, the transactional use and store of value overlaps. The money that party A pays to party B to buy a good is transactional money for party A, but becomes a store of value for party B.

A reduced utility of money as a store of value promotes transactional use.

We will demonstrate that also starting from the Quantity Theory, velocity of money circulation depends by market variables.

We will define in fact an expression for velocity of money circulation depending on utility of goods produced in the economy (Q) and utility of Money in the broader sense. This is obtained through a reinterpretation of the traditional theory of consumers choices that is a fundamental framework in economics that seeks to explain how individuals make rational decisions in allocating their limited resources among different goods [3], [4], [5].

According to this theory, consumers aim to maximize their utility derived from consuming goods, subject to their budget constraints.

We re-interpretate the microeconomic approach in a macroeconomic framework considering choices between goods and money existing in an economy.

This in the context of Cobb Douglas preferences (Cobb-Douglas utility functions).

Cobb-Douglas utility functions are utilized in consumer theory to analyze consumer behavior, as well as in general equilibrium models to examine the allocation of goods and resources in an economy. The dynamics of production is hypothesized following a Cobb Douglas production function with an increasing marginal cost function.

We will then consider exogenous the supply of money as it is derived by banking and market operators' behavior.

Interest will be dedicated to the linkage between microeconomic theory and the quantity theory of money.

We will follow the scenario in which operators with different levels of wealth and preferences purchase baskets of goods with equal percentage distributions. Particularly, it will mathematically elucidate the relationship between individual characteristics (wealth and preferences) production and the formation of equilibrium prices and quantities for goods in the market.

Once having defined the new concept of velocity of money circulation (that in the Cobb-Douglas for good basket (x1) and money (x2) is alpha/beta, we analyze the mean variance theory [6], [7] and in particular the Arrow Pratt approximation [8]. In particular, we will analyze the relationships between interest rates (or better risk premiums), market risk, velocity of money circulation and demand for money. Considering that in the contest of Cobb-Douglas preferences the adversity/propensity to risk is a function of money utility and therefore of money demand. So it is demonstrated that velocity of circulation of money is dependent also by interest rate (or better by risk premiums related to risk levels) and this makes compatible in a certain sense that Friedman' and Keynes' assumptions on money demand as we said before [9].

The last part of the article is finally dedicated to analyze the impact of the above analysis on exchange rate behavior [10], [11], [12], [13] with the help of further theories as the Purchasing Power Parity and the Prospect Theory of Tversky and Kahneman.

The Purchasing Power Parity (PPP) theory compares the relative prices and purchasing power of different currencies in different countries.

We consider PPP in the relative form that considers inflation rates and suggests that changes in exchange rates should reflect the differences in the inflation rates of different countries. If Country A has a higher inflation rate than Country B, the currency of Country A should depreciate to reflect the change in prices.

Prospect Theory, [14], [15], [16], [17], [18], [19], [20], [21] developed by Daniel Kahneman and Amos Tversky in 1979, is a behavioral economics theory that seeks to explain how individuals make decisions under uncertainty challenging the assumption of perfect rationality and incorporating insights from psychology. A specific aspect of Prospect Theory is that it introduces a value function, which describes how individuals subjectively assess gains and losses. The value function is typically concave for losses and convex for gains. This implies that individuals are risk-averse in environment of gains and risk-seeking in an environment of losses.

The above analysis contributes to explain in the last part of the article through a simulation of the behavior of the bitcoin exchange rate vs euro in the last years. Bitcoin is a recent innovation in the context of monetary supply characterized by the fact that it is regulated in the blockchain and therefore not by a central bank but by a decentralized ledger. A characteristic that combines with the fact that the money supply, no longer decided by a central bank, is entrusted to decentralized points that produce money (in an increasingly limited manner) through an algorithm. Even exchanges of money between individuals are not tracked by a banking body but by decentralized points. This mechanism, which escapes the discretion of a central bank and limits the supply of money in the system, is one of the aspects that has determined the enormous growth in value of cryptocurrencies (and for example of bitcoin since its birth). [22] [23] The paper continues with an application to the Bitcoin crypto coin. Finally, the results are generalized by using the MRQF theory and the concept of Financion, considered of quantum of interaction between two financial operators or agents [24].

2. Exchange Equation

Describing the Quantitative Theory of money, we start from the famous equation of exchange according to which (1), where given a basket of goods represented by the global market in a specific economic context we have that:

- P is the price level of goods in the basket in a period
- Q is the quantity purchased in the basket in the same time period
- M is the total amount of money issued in the system
- V is the velocity of money circulation

The monetarists, starting from this equation, obtain that:

If Q is constant (vertical supply curve).

If V is a constant and derives from the operators' habit to use the currency in the time unit.

From this it follows that, if Q and V are substantially constant:

$$\mathrm{P}\bar{Q} = \mathrm{M}\bar{V} \tag{3}$$

There is therefore a close and proportional relationship between the price level P and the quantity of currency issued M. This is a specific case but the dynamics can be multiple.

In particular, it is clear and evident that the assumption of Q as constant can be disproved in economy with unused factors of production if the banks do their credit selection duty correctly. In particular, if for example the banks grant credit (by issuing currencies) to technological operators, their high productivity compared to the market standard can determine a growth of Q with the same production factors which can be greater than the growth of M, thus determining even a decrease in prices despite the growth of the currency (technological deflation).Less obvious is the issue of the velocity of money circulation. A key point of the article is to demonstrate that velocity of money circulation depends on utilities of money and goods.

3. Exchange equation and utilities: Velocity of money circulation

At the microeconomic level, the prices of two goods P (which in our case are basket Q and the currency M) is given by the substitution marginal rate of the first good with the second and therefore by the ratio between the assumed marginal utilities, that are usually decreasing

$$\frac{P_q}{P_m} = \frac{U_m(Q)}{U_m(M)} \tag{4}$$

where:

$U_m(Q)$ is the marginal utility of (Q),

$U_m(M)$. is the marginal utility of (M).

The figure 1 describes marginal (Um) and total (Utot) utility versus quantity of goods.

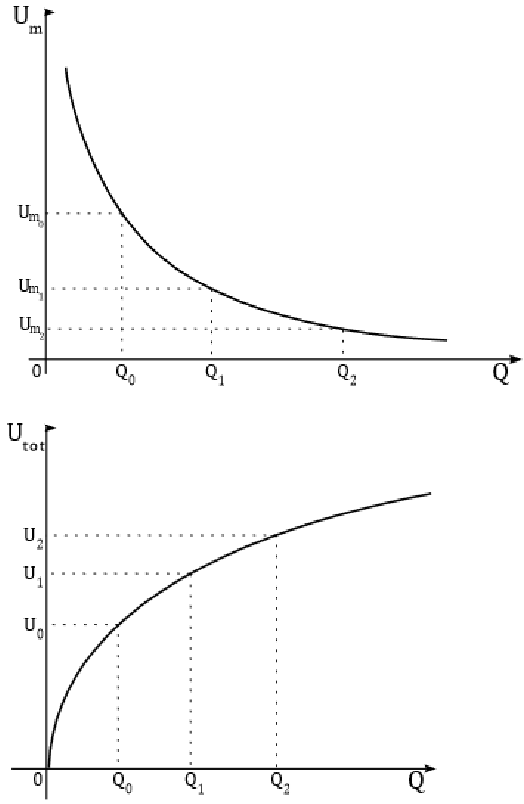

Figure 1
Marginal (Um) and Total (Utot) Utility versus Quantity of Goods.

Let us now analyse the above concepts through the Cobb-Douglas utility function.

If we indicate with

- Q the quantity of goods,
- M_{nom} the nominal quantity of money
- M the real quantity of money (M_{nom}/P_m),
- α a $constant, 0 < \alpha < 1$,
- W the total wealth,
- $U = f(Q, M)$ the utility function,
- $U_{tot}(M)$,the total utility of M (contribution of M to total utility equal to optimal M_{nom} on wealth multiplied for total optimized utility) ,

- $U_{tot}(Q)$,the total utility of Q, (contribution of Q to total utility equal to optimal Q multiplied for P_q on wealth multiplied for total optimized utility)
- P_Q the price level of goods,
- P_m the price of the money (pm=pq)
- $MRS$the marginal rate of substitution,

then,

$$U = Q^{\alpha} M^{(1-\alpha)} \tag{5}$$

$$\frac{\partial\prime\, U}{\partial Q} = \frac{\alpha}{Q} U \tag{6}$$

$$\frac{\partial\prime\, U}{\partial M} = \frac{(1-\alpha)}{M} U \tag{7}$$

$$- MRS \ = P_q/P_m = \frac{\alpha}{Q} \frac{M}{(1-\alpha)} = \frac{U\prime(Q)}{U\prime(M)} \tag{8}$$

$$U_{tot}\ (Q) = (\alpha\ U)$$

$$U_{tot}\ (M) = (1-\alpha)\ U$$

$$\alpha = \frac{P_Q Q}{w}, \text{and } (1-\alpha) = \frac{P_m M}{w}$$

Consequently,

$$\frac{U_{tot}(Q)}{U_{tot}(M)} = \frac{\alpha}{1-\alpha} = \frac{P_Q Q}{M_{mom}},$$

By using (3) we have

$$P\bar{Q} = M_{mom}\ \bar{V}\ \Rightarrow\ \frac{U_{tot}(Q)}{U_{tot}(M)} = \frac{P\bar{Q}}{M_{mom}}\ \Rightarrow\ \bar{V} = \frac{U_{tot}(Q)}{U_{tot}(M)} \tag{9}$$

The above considering Cobb Douglas utility functions. It is interesting to demonstrate that also in the case of quasi linear utility functions the velocity of money circulation is the ratio between total utility of goods and total utility of money.

$$U = \ \alpha \ln(Q) + \ \beta M$$

$$\frac{U^I}{\partial Q} = \ \frac{\alpha}{Q}$$

$$\frac{U^I}{\partial M} \ = \ \beta$$

$$\text{-MRS} = \frac{\alpha}{\beta\ Q} = \frac{P_q}{P_m} = 1$$

$$\text{Q} = \frac{\alpha}{\beta}$$

$$M_{mom} = \mathrm{W} - \frac{\alpha}{\beta}\ P$$

$$\mathrm{V} = \frac{Utot\ Q}{Utot\ M} = \frac{\alpha}{\beta M} \tag{10}$$

$$\mathrm{V}\ M_{mom} = \mathrm{P\ Q}$$

Thus, it proves that, starting from a microeconomic assumption, the velocity of money circulation in the quantity theory of money depends on the ratio between the total utility of Q and the total utility of M.

Of course (1-alpha) that is the percentage of total utility referred to money, depends on the breadth of the monetary aggregate considered. If the aggregate is wider (1-alpha) will be higher and velocity of money circulation lower, and vice versa.

So (1) remain always valid also with a wider concept of money.

Let us now examine these aspects more specifically, with the microeconomic model.

We will now analyze the case of n individuals (with n=3) with Cobb-Douglas utility functions with different parameters α1, α2, α3 > 0 and $\alpha + \beta = 1$ who buy quantities Q1, Q2 and Q3 at a market price P of a basket Q (with Q=Q1+Q2+Q3) in unequal quantities. Let us assume that $\overline{\mathrm{M}}$ the quantity of money supplied in the system is exogenous. Given the initial savings (initial financial wealth or nominal money) of the 3 individuals (M1, M2, M3) it will be M = M1+M2+M3. Let us imagine that the 3 individuals are 2 consumers and a producer/consumer. The producer will initially have in addition to the M3 monetary wealth the goods produced at their market price (p * Q1 + p Q2 + p Q3).

The wealth of 3 individuals can be described as follows:

$$\mathrm{W}_1 = \mathrm{M}_1 \tag{11}$$

$$\mathrm{W}_2 = \mathrm{M}_2 \tag{12}$$

$$\mathrm{W}_3 = \mathrm{M}_3 + \mathrm{p\ Q} = \mathrm{M}_3 + (\mathrm{p\ Q}_1 + \mathrm{p\ Q}_2 + \mathrm{p\ Q}_3) \tag{13}$$

It will be according with Warlasian demand;

$$\alpha_1 \mathrm{W}_1 = \mathrm{p}_1\ \mathrm{Q}_1 = \mathrm{M}_1\ (1 - \alpha_1\ )\ \mathrm{V}_1 = \alpha_1 M_1 \tag{14}$$

$$\alpha_2 \mathrm{W}_2 = \mathrm{p}_2\ \mathrm{Q}_2 = \mathrm{M}_2\ (1 - \alpha_2\ )\ \mathrm{V}_2 = \alpha_2 M_2 \tag{15}$$

$$\alpha_3 \mathrm{W}_3 = \mathrm{p}_3\ \mathrm{Q}_3 = \mathrm{M}_3\ (1 - \alpha_3\ )\ \mathrm{V}_3 + \alpha_1 M_1\, \alpha_3 + \alpha_2 M_2\, \alpha_3 + \ \alpha_3 M_3\, \alpha_3 \tag{16}$$

Then the optimal financial wealth for each individual will be:

$$(1 - \ \alpha_1)\ W_1 = M_{1\ opt} = M_1\ (1 - \ \alpha_1) \tag{17}$$

$$(1 - \ \alpha_2)\ W_2 = M_{2\ opt} = M_2\ (1 - \ \alpha_2) \tag{18}$$

$$(1 - \ \alpha_3)\ W_3 = M_{3\ opt} = M_3\ (1 - \ \alpha_3) + \ \alpha_1 M_1\ (1 - \ \alpha_3)$$

$$+ \alpha_2 M_2(1 - \alpha_3) + \alpha_3 M_3 (1 - \alpha_3) \quad (19)$$

$$Q = Q_1 + Q_2 + Q_3 \quad (20)$$

The system above is in 10 equations (from eq.11 to eq. 20) and 11 unknowns variables.

The unknowns are W1, W2, W3, P, Q, Q1, Q2, Q3, M1opt, M2opt, M3opt the known are $\overline{M}$, M1, M2, M3, α1, α2, α3.

To find an equilibrium point, we use a production function that relates the marginal cost to the quantity produced and equates it with the price of goods.

That function of production is a Cobb Douglas that has the characteristic to have a marginal cost function increasing related to quantity produced if $\alpha + \beta \leq 1$ and $\alpha > 0$ and $\beta > 0$.

The Cobb Douglas production function formula is as follows:

$$Q = A * L\text{^}\alpha\ K\text{^}(1\text{-}\alpha) \quad (21)$$

With Q= production
A = technological coefficient
L = quantity of work
K = quantity of capital
α+β = parameters with the above characteristics.

The marginal cost function is obtained by deriving the formula of total cost (TC = w*L + r*K with w = salaries per unit of work and r = cost of capital) respect to the quantity produced.

The Cobb Douglas marginal cost function will then be:

$$\text{CM} = \frac{w}{\alpha A K^{(1-\alpha)}} * \frac{Q}{A K^{(1-\alpha)}} \text{^} \left(\frac{1}{\alpha} - 1\right) + \frac{r}{(1-\alpha) A L^{\alpha}} * \frac{Q}{A L^{\alpha}} \text{^} \left(\frac{1}{1-\alpha} - 1\right) = P \quad (22)$$

The curve of marginal cost is the 11^{th} equation in the system of 11 unknowns. Tables 1-3 show some useful examples.

Table 1
Initial example

	TOTAL	INDIVIDUAL 1	INDIVIDUAL 2	INDIVIDUAL 3
ALPHA UTILITY FUNCTION		0,60	0,45	0,19
VELOCITY OF CIRCULATION OF MONEY		1,50	0,82	0,23
MONEY	600,0	200,0	210,0	190,0
QUANTITY PRODUCT	77,5	30,0	23,7	23,8
QUANTITY PRODUCT PER PRICE	309,5	120,0	94,5	95,0
OPTIMAL MONEY	600,0	80,0	115,5	404,5
WEALTH	909,5	200,0	210,0	499,5
LEVEL OF PRICES	3,99			
TECNOLOGICAL FACTOR (A)	1,75			
WAGES	2,00			
LABOUR	36,9			
COST OF CAPITAL	1,50			
CAPITAL	50,0			
ALPHA PRODUCTION FUNCTION	0,40			
MARGINAL COST	3,99			

Table 2
Example with reducted velocity circulation of money

	TOTAL	INDIVIDUAL 1	INDIVIDUAL 2	INDIVIDUAL 3
ALPHA UTILITY FUNCTION		0,50	0,40	0,19
VELOCITY OF CIRCULATION OF MONEY		1,00	0,67	0,23
MONEY	600,0	200,0	210,0	190,0
QUANTITY PRODUCT	70,3	26,1	21,9	22,3
QUANTITY PRODUCT PER PRICE	269,5	100,0	84,0	85,5
OPTIMAL MONEY	600,0	100,0	126,0	374,0
WEALTH	869,5	200,0	210,0	459,5
LEVEL OF PRICES	3,84			
TECNOLOGICAL FACTOR (A)	1,75			
WAGES	2,00			
LABOUR	29,0			
COST OF CAPITAL	1,50			
CAPITAL	50,0			
ALPHA PRODUCTION FUNCTION	0,40			
MARGINAL COST	3,84			

Table 3
Example with a reduction of salaries

	TOTAL	INDIVIDUAL 1	INDIVIDUAL 2	INDIVIDUAL 3
ALPHA UTILITY FUNCTION		0,50	0,40	0,19
VELOCITY OF CIRCULATION OF MONEY		1,00	0,67	0,23
MONEY	600,0	200,0	210,0	190,0
QUANTITY PRODUCT	73,3	27,2	22,8	23,3
QUANTITY PRODUCT PER PRICE	269,5	100,0	84,0	85,5
OPTIMAL MONEY	600,0	100,0	126,0	374,0
WEALTH	869,5	200,0	210,0	459,5
LEVEL OF PRICES	3,68			
TECNOLOGICAL FACTOR (A)	1,75			
WAGES	1,80			
LABOUR	32,1			
COST OF CAPITAL	1,50			
CAPITAL	50,0			
ALPHA PRODUCTION FUNCTION	0,40			
MARGINAL COST	3,68			

The grayed out cells represent the varied input data in the example.

In the three examples above the first two individuals are consumers, the third a producer/consumer.

Alpha Utility function is the coefficient from the Cobb Douglas utility function,

$$U = Q^{\alpha}\,(M/P)^{(1-\alpha)}\ (\,see\ eq.5)$$

V = velocity of circulation of money (Alpha/(1-Alpha)),

Money = Initial nominal money,

Mopt = Optimal money,

W = total wealth (including goods at market prices for the individual producer),

Quantity = quantity produced of total and optimal balance for each individual,

Level of price = price level of goods, wages, labor, cost of capital, capital, A and Alpha production cost= parameters of the Cobb Douglas production function,

MARGINAL COST = Marginal cost in equilibrium equal to the price level of goods.

In the first example the intersection between the Warlasian demand curve and the Cobb Douglas supply curve determines an equilibrium level of 77.5 units of quantity produced and sold at the price (equal to the marginal cost of 3.99 per unit of product (see figure 2 – Point A).

In the second example we assume that the preferences of individuals vary with a generalized decrease in the velocity of circulation of money. This determines on the one hand the downward shift of the demand curve which meets the Cobb Douglas supply curve at the point where the new equilibrium price is 3.84 (the decrease in the velocity of circulation of money produces a lower price level), and the new equilibrium quantity is 70.3 (companies at a lower price are willing to produce less, see figure 2 – Point B).

In the third example we assume that the fall in the prices of goods and in any case a specific condition of the labor market can have an effect on the labor market with a significant decrease in wages per work unit (from 2 to 1.8). In this case we will have a downward shift in the supply curve and the new equilibrium point will be at an even lower price level of 3.68 and a quantity produced of 73.3 in partial recovery compared to the previous simulations (see figure 2 – Point C).

In essence, the market finds its own equilibrium which, however, can change in the event of exogenous or endogenous variations in the model parameters.

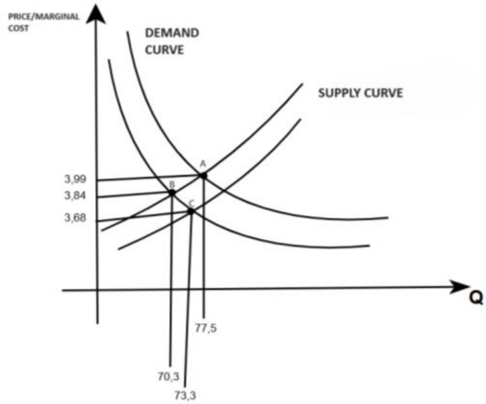

Figure 2

Demand and supplies curves in the three examples (Point A, Point B, Point C).

4. Innovative demand for money

Now it is interesting with the help of mean variance theory (i.e. Markowitz's theory) and in particular according to the Arrow Pratt approximation to analyse the main functions of money related to aversion/propensity to risk, volatility of financial prices and risk premiums (difference between expected interest rates or better financial instruments performances and riskless rates). Furthermore, one can define a utility function $U_{tot}(Q)$ with decreasing marginal utility (as it is usual in the market for goods) and a utility function $U_{tot}(M)$ which can be even a decreasing marginal utility or an increasing marginal utility (typical of individuals that hold speculative currencies like bitcoin for example).

In particular, this derives from the fact that holders of pure speculative currency are not risk averse (as for goods and transactional currencies) but they are risk inclined (as we will see this characteristic has important impacts on the dynamics of the Bitcoin). The success of the Bitcoin also derives from the fact that holders of pure speculative currency are risk inclined (as we have said this characteristic has important impacts on the dynamics of the Bitcoin). In particular, with regard to risk aversion/propensity, we will have from financial theory following Arrow Pratt approximation that:

$$\pi \cong -\frac{1}{2}\sigma^2 \, U''/U' \tag{23}$$

with π corresponding expected risk premium, and $with\, \sigma^2$ corresponding to expected variance of market returns and - U''/U' is the aversion to risk coefficient. In the case of Cobb-Douglas utility function:

$$U = Q^{\alpha}(M/P)^{\beta}$$

$$-\frac{U''}{U'} = (1-\beta)/M \tag{24}$$

$$\pi_{\epsilon} \cong \frac{1}{2}\frac{(1-\beta)}{M}\sigma\%^2\, M^2 = \frac{1}{2}(1-\beta)\,\sigma\%^2 M$$

$$\pi_{\%} \cong \frac{1}{2}(1-\beta)\sigma\%^2 \tag{25}$$

$$2\frac{\pi\%}{\sigma^2} < 1\,, \beta > 0$$

If the utility function is instead a quasi linear -U''/U' would be 0 (indifference to risk). In this case, in fact, money is held as a residual of the independent preference on goods.

Now, starting from the assumptions above, we will try to design an innovative demand for money (always intended in the wider sense of course).

It will be

$$\beta \cong 1 - \frac{2\pi}{\sigma^2}$$

Following Arrow Pratt approximation and the Exchange Equation (3) we can define now a novel demand for money.

$$Y = PQ = MV \;\Rightarrow M_d = \frac{Y}{V} = \frac{Y}{\alpha}\,\beta \cong \frac{Y}{\alpha}\left(1 - \frac{2\pi}{\sigma^2}\right) \tag{26}$$

Demand for money $M_d \cong \frac{PQ}{\alpha}\left(1 - \frac{2\pi}{\sigma^2}\right)$, supply of money $M_s = M_0$

If expected returns on risk increase there are more expectations in the risk reward framework on a financial investment. That can discourage some investor for the higher difficulty to match expectations. So the demand for money decrease.

The Graph of M_e equilibrium in money market will be as follows in Figures 3,4 and 5.

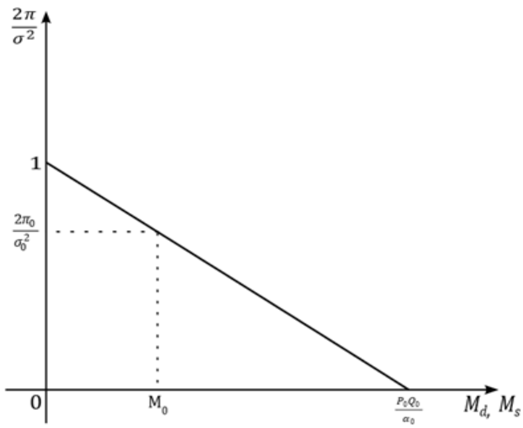

Figure 3
Money market equilibrium

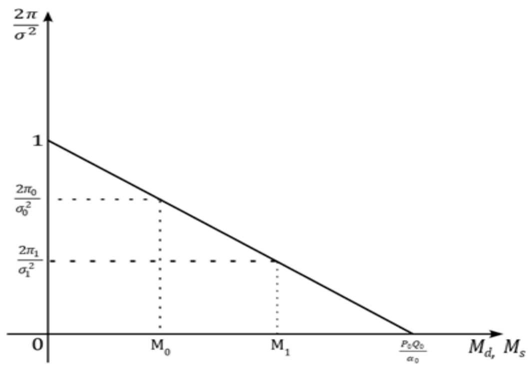

Figure 4
An example of new equilibrium in Money market with change in money offer.

Let us assume that $\frac{2\pi}{\sigma^2}$ decrease to $\frac{2\pi_1}{\sigma_1^2}$ to reach the equilibrium there could be an expansive monetary policy (increase of M_s from M_0 to M_1).

In fact, with lower expected return risk adjusted from financial instruments, demand for money (as reserve for value) increases (compensated in this case by an increase in supply). If for instance $\frac{2\pi}{\sigma^2}$ decreases for an increase in σ^2, monetary authorities can compensate higher market volatility with an injection of money offered.

Another possibility to re-equilibrate the lower $\frac{2\pi}{\sigma^2}$ is the shift (rotation) downward of the money demand due to decreasing Y (P_1 or Q_1) and/or increasing α_1 (utility for goods).

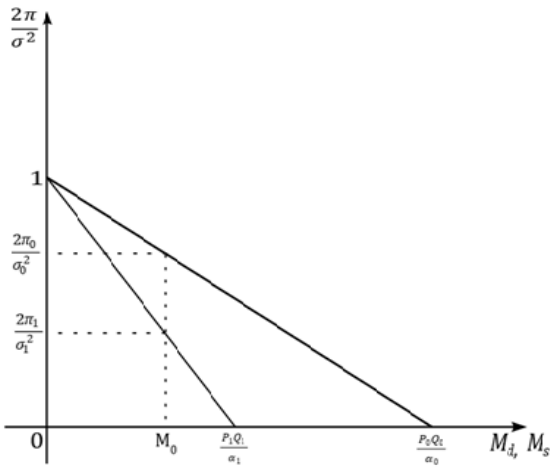

Figure 5
Money market new equilibrium with change in P*Q/alpha

Considering the demand for money above is quite easy to define an innovative LM curve and in general an innovative IS/LM framework. It will be the argument of a possible future article.

5. Prices of goods, Exchange rates and velocity of money circulation

Following Arrow-Pratt's approximation in Cobb-Douglas we have that:

$$\beta \cong 1 - \frac{(2\,\pi)}{\sigma^2}$$

Then we have from Exchange equation:

$$P = \frac{MV}{Q}$$

Now we introduce the Purchasing Power Parity in the relative form:

$$\Delta\%\ \text{Exchange Rate} = \frac{1+\Delta\% P_{Other\ Currency}}{1+\Delta\%\text{P}} - 1 \qquad (27)$$

In a simplifying hypothesis (with no inflation in the other currency) is

$$\Delta\% \text{ Exchange Rate} = \frac{1+\Delta\%P_{Other\ Currency}}{1+\Delta\%\text{P}} - 1 = \frac{P_0}{P1} - 1 \quad (28)$$

We know then that:

$$v = \frac{\alpha}{\beta} \text{ with } \alpha > 0\ e\ \beta > 0$$

Then in the simplified hypothesis of $\Delta\%P_{Other\ Currency} = 0$

$$\frac{P_0}{P_1} = \frac{M_0}{M_1}\frac{Q_1}{Q_0}\frac{v_0}{v_1} \quad (29)$$

$$P_1, M_1, Q_1 and v_1 = values\ at\ time\ 1$$

$$P_0, M_0, Q_0 and v_0 = values\ at\ time\ 0$$

So it will be:

1. If risk-averse individuals with $\beta < 1$ become even more risk-averse, $\beta_1 < \beta_0$ with $\beta_1 = \beta$ at time 1 and $\beta_0 = \beta$ at time 0, then $v_1 > v_0$. Consequently, prices of goods increase, and the currency exchange rate, considering for simplicity the absence of other currencies inflation, depreciates relative to another currency.

   Conversely, if risk-averse individuals with $\beta < 1$ become less risk-averse $\beta_1 > \beta_0$.
2. If risk-seeking individuals with $\beta > 1$ become even more risk-seeking, $\beta_1 > \beta_0$,then $v_1 < v_0$. Consequently, prices of goods decrease, and the currency exchange rate, considering for simplicity the absence of other currencies inflation, appreciates relative to another currency. Conversely, if risk-seeking individuals with $\beta > 1$ become less risk-seeking $\beta_1 < \beta_0$.
3. If risk-averse individuals with $\beta < 1$ become risk-seeking, $\beta_1 > 1$, then $v_1 < v_0$.

   Consequently, prices of goods decrease, and the currency exchange rate, considering for simplicity the absence of other currencies inflation, appreciates relative to another currency. Conversely, if risk-seeking individuals with $\beta > 1$ become risk-averse $\beta_1 < \beta_0$.

In contrast, when individual preferences remain constant, an increase in the quantity of money supplied leads to higher prices and currency depreciation, while an increase in the supply of goods has the opposite effect.

In a specific scenario where consumers exhibit satiety for real goods and a risk-seeking behaviour regarding money ($\alpha < 0\ , 1 - \alpha > 1$), an unconventional case emerges where the velocity of money circulation becomes negative. In this case, an increase in the quantity of money supplied results in lower prices and currency appreciation. This unusual phenomenon occurs due to the unique interplay of preferences.

Beyond this counterintuitive case, in a more typical and realistic situation, it can be concluded that when individuals become more risk-seeking or less risk-averse, the currency appreciates, assuming the inflation rate of the other currency remains unchanged. Conversely, when individuals become more risk-averse or less risk-seeking, the currency depreciates. Additionally, based on the Arrow-Pratt approximation and holding sigma constant, currency appreciation resulting from changes in preferences is associated with a lower expected weighted risk premium, while currency depreciation is associated with a higher expected weighted risk premium. In practice, if changes in preferences lead to currency appreciation, the opposite is true for currency depreciation.

Lastly, in the case of currency appreciation at equilibrium returns and interest rates, one can expect greater volatility accepted by individuals, while the opposite holds true in the case of currency depreciation.

6. An application to crypto coin: the Bitcoin case

Now we analyse the case of Bitcoin behaviour. Let us assume an utility function

$$U = Q^{\alpha}(M/P)^{\beta}$$

with $$0 < \alpha < 1 \text{ e } (\beta) > 1$$

In this case holder's $U(Q)$ is always decreasing marginal utility, but holder's $U(M)$ has increasing marginal utility (people who buy Bitcoin tend to be prone to the speculative currencies risk) $\frac{U''(x)}{U'(x)} > 0$.

Since to Purchasing Power Parity in relative version (PPP) the variation of the exchange rate for two currencies is:

$$\Delta\% \frac{EURO}{BITCOIN} = \frac{1+\Delta\%P_{EURO}}{1+\Delta\%P_{BITCOIN}} - 1 \qquad \text{(see eq. 27)}$$

The great Bitcoin revolution therefore is historically linked to the decrease in Bitcoin prices for goods due to:

1. Variation in expectations of goods purchasable with Bitcoin (Elon Musk and eBay news. The recent surge in Bitcoin was, in part, a result of its official recognition by financial markets, as evidenced by its recent ETF launch.);
2. Variation in individual preferences (Tversky-Kahneman effect of decreasing propensity to risk in gains environment and increasing propensity for risk in a loss environment);
3. Content incremental growth of the money supply (Mining mechanism).

Changes in the perceived possibility to utilise Bitcoin for transactions have occurred, likely due to influential events like announcements from figures like Elon Musk or major platforms such as eBay. These variations in expectations can have significant economic implications for widely acceptability of Bitcoins for purchases, increasing demand for the cryptocurrency, potentially driving up its price. Conversely, the contrary for negative news or doubts about its utility.

Focusing on changes in how people make decisions when faced with financial gains and losses. This concept draws from the work of Tversky and Kahneman-Prospect Theory in behavioural economics. Following this theory when individuals become more risk-averse in situations where they stand to gain, it can lead to more conservative investment choices. Conversely, a heightened willingness to take risks in situations involving losses might impact trading behaviour.

Furthermore, the process of incrementally increasing the money supply, primarily through the mining mechanism commonly associated with cryptocurrencies like Bitcoin has significant implications. The incremental growth of the money supply, controlled by protocols and algorithms, and limited by the increasing energy consumption to produce more money, impact positively the overall stability and value of a currency.

The recent rise in Bitcoin can be attributed in part to its official recognition through inclusion in an ETF. This is perceived in the market as the potential for increased acceptance and as a reduction in velocity of money circulation due to an increase in its utility. So principal movements of Bitcoin exchange rate in the last years can be described by the above theories making adequate assumptions. Table 4 shows a specific simulation on bitcoin.

Table 4
Simulation on bitcoin

Date	N Bitcoin millions	Euro annual inflation	Exchange rate revaluation	Implicit PPP Inflation	Exchange rate vs Euro AVG	Estimated quantity of offered goods	Alfa	Beta	Estimated money circulation velocity	IMPORTANT NEWS
	EMPIRICAL DATA					ASSUMPTIONS EXPERT DRIVEN		SIMULATIONS		
set-20	17,9				9,0	100	0,5	1,00	0,50	
apr-21	18,6	1,2%	429,0%	-80,9%	47,8	300	0,7	2,53	0,28	E. MUSK
lug-21	18,7	1,9%	-39,6%	68,6%	28,9	150	0,5	2,16	0,23	E. MUSK
nov-21	18,8	3,6%	83,7%	-43,6%	53,1	200	0,5	2,89	0,17	RUMORS EBAY
gen-22	18,9	4,7%	-31,7%	53,2%	36,3	200	0,5	1,89	0,26	
giu-22	19	7,8%	-36,5%	69,8%	23,0	100	0,5	2,24	0,22	
giu-23	19,4	6,0%	10,7%	-4,2%	25,5	100	0,5	2,39	0,21	
apr-24	19,7	2,6%	143,0%	-57,8%	62,0	300	0,7	2,68	0,26	ETF

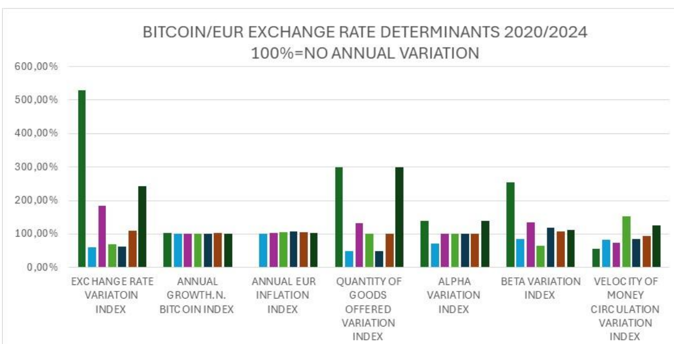

Figure 6
Behaviour in the time series of bitcoin exchange rate determinants

In the simulation above we try to explain ex post the dynamics of the BITCOIN/EUR exchange rate in recent years.

To do this we use what was stated previously in the article.

We remember that:

$$\Delta\% \frac{EURO}{BITCOIN} = \frac{1+\Delta\% P_{EURO}}{1+\Delta\% P_{BITCOIN}} - 1 = (1 + \Delta\% P_{EURO}) * \frac{P0}{P1} - 1$$

And the price of goods in Bitcoin, although unobservable, can be derived from the following formula.

$$\frac{P0}{P1} = \frac{M0}{M1}\frac{Q1}{Q0}\frac{V0}{V1}$$

$$\Delta\% P_{EURO},\ \Delta\% \frac{EURO}{BITCOIN}, M0\ ed\ M1 \text{ are empirical variables}$$

For Q0 and Q1 we made assumptions expert driven linked to the above discussed news (Elon Musk, Ebay, ETF),

As a dependent variable we obtained the velocity of circulation of Bitcoin from the historical series starting from an assumed value for 2020 of 0.5. As expected, we obtain that v falls when the prices of goods fall and therefore the exchange rate revalues. Vice versa in the opposite case.

Furthermore, since in the Cobb Douglas v=α/β with α and β unobservable, we tried to hypothesize α increasing as Q increases and vice versa, with a hypothesized value for 2020 of 0.5 thus obtaining the values of β which we remember is a factor, among other things, of risk aversion or propensity. We have thus found that in periods of rally the exchange rate β tends to be as high as can be expected and vice versa. Furthermore, after periods of strong rally β tends to decrease and vice versa as Prospect Theory suggests.

In conclusion we can see from figure 6 that:

- money and euro inflation are quite stable in the period (annual index very near to 100%),
- news on potential utilization of bitcoin to purchase goods have a strong impact on exchange rate behavior (very high correlation),
- alpha is quite stable,
- beta and velocity of money circulation follow the behavior described above.

7. Energetic model and time dependance

Following the MRQF theory (i.e. Multiscale Relative Quantum Finance theory) by Gerardo Iovane et al [22], markets can be described thanks to energetic models. In fact we can consider the total energy E as conservative law thanks to each the money demand\offer is full connected to the product quantities and disponibilities. With this vision if we call T the kinetic energy corresponding to the money quantity which can be transited, that is T=$\frac{MV^2}{2}$, and if we call the potential the quantity U=$\frac{PQ\frac{U_{tot(Q)}}{U_{tot(M)}}}{2}$, we find the traditional conservation law E=T+U . For instance, we can interpretate part of the algorithms of this article in that sense. We have that:

$$Energy = \frac{MV^2}{2} = \frac{PQ\frac{U_{tot}(Q)}{U_{tot}(M)}}{2} = \frac{PQ\frac{\alpha}{\beta}}{2} = \frac{PQ\alpha}{2\left(1-\frac{2\pi}{\sigma^2}\right)} \tag{30}$$

Than, the total energy is the total wealth of the system until the system can be considered isolated, that is until a new contribution of money or products is conferred to the total system. In other words, in a fixed instant for a financial operator we could find all the energy in the kinetic form that is money to be invested or on the other hand all the energy in potential form, that is assets represented by bought products. In general, a financial operator will have a mix between the two quantities.

So, we will have that energy will be greater if the following increase:

- Income (P or Q),
- Utility of goods (α),
- Expected risk premium (π),
- Velocity of money circulation (v).

Energy, on the other hand, will decrease if the variance (σ^2) increases, that is, the entropy of the system.

Let us consider the following relations, where as usual the subscript after the comma means partial derivative with respect to the variable following the comma.

$$U_T = Q^{\alpha} M^{1-\alpha} \tag{31}$$

$$U_{T,Q} = \alpha Q^{\alpha-1} M^{1-\alpha} = \frac{\alpha}{Q} U_T \tag{32}$$

$$U_{T,M} = \frac{(1-\alpha)Q^{\alpha}M^{1-\alpha}}{M} = \frac{(1-\alpha)U_T}{M} \tag{33}$$

where M = M_{mom} /P. From (32)

$$QU_{T,Q} = \alpha U_T$$

From (33)

$$MU_{T,M} = (1-\alpha)U_T$$

Dividing the (33) with the (32) we have:

$$\frac{MU_{T,M}}{QU_{T,Q}} = \frac{(1-\alpha)}{\alpha}$$

So

$$\alpha MU_{T,M} - (1-\alpha)QU_{T,Q} = 0 \tag{34}$$

Generally, for $U_T = U_T(Q, M)$

$$\gamma_1 MU_{T,M} + \gamma_2 QU_{T,Q} = S(M, Q, t) \tag{35}$$

where $\gamma_1, \gamma_2 \in \mathbb{R}$ and the source $S = S(M, Q, t)$. The Cobb-Douglas formula (31) is a special case of (35) that we have when $\gamma_1 = \alpha, \gamma_2 = -(1-\alpha) \wedge S = 0$ that is in the absence of a source.

General case with explicit tangential temporal dependence of U_T of oscillating sinusoidal type.

$$U_T = U_T(t, Q, M) \tag{36}$$

$$U_T = Q^\alpha M^{1-\alpha} \sin(\omega t) \tag{37}$$

$$U_{T,t} = Q^\alpha M^{1-\alpha}\ \omega\ cos(\omega t) \tag{38}$$

$$U_{T,t,t} = -Q^\alpha M^{1-\alpha} \omega^2 \sin(\omega t) \tag{39}$$

Thus, in time the harmonic equation holds:

$$U_{T,t,t} + \omega^2 U_T = 0 \tag{40}$$

$$U_{T,Q} = \frac{\alpha}{Q} U_T \tag{41}$$

$$U_{T,M} = \frac{(1-\alpha)U_T}{M} \tag{42}$$

So, the $U_{T,t,t}$ becomes

$$U_{T,t,t} = Q^\alpha M^{1-\alpha}\ \omega\ cos(\omega t) \frac{\sin(\omega t)}{\sin(\omega t)} = \omega \cot(\omega t) U_T$$

Then

$$U_T = \frac{1}{\omega}\ tg(\omega t) U_{T,t} \tag{43}$$

Replacing the (43) in the (41) and (42) we have:

$$QU_{T,Q} = \alpha U_T = \frac{\alpha}{\omega}\ tg(\omega t) U_{T,t} \tag{44}$$

$$MU_{T,M} = \frac{1-\alpha}{\omega}\ tg(\omega t) U_{T,t} \tag{45}$$

By adding the (44) and (45) we have:

$$QU_{T,Q} + MU_{T,M} = \frac{1}{\omega}\ tg(\omega t) U_{T,t} \tag{46}$$

General case with implicit time dependence:

$$U_T = Q^\alpha(t) M^{1-\alpha}(t)$$

$$U_{T,t} = \alpha \frac{Q^\alpha}{Q}(t) Q_t M^{1-\alpha}(t) + (1-\alpha) Q^\alpha \frac{M^{1-\alpha}}{M} M_t =$$

$$= \frac{\alpha}{Q} Q_t U_T + \frac{1-\alpha}{M} M_t U_T$$

$$U_{T,t} = \left(\frac{\alpha}{Q} Q_t + \frac{1-\alpha}{M} M_t\right) U_T \tag{47}$$

General case with linear time dependence:

$$U_T = Q^\alpha M^{1-\alpha} \omega t$$

$$U_{T,t} = \omega Q^\alpha M^{1-\alpha} = \frac{\omega}{t} U_T \tag{48}$$

and

$$U_{T,Q} = \frac{\alpha}{Q} Q^\alpha M^{1-\alpha} \omega t = \frac{\alpha}{Q} U_T \tag{49}$$

and

$$U_{T,M} = \frac{1-\alpha}{M} U_T \tag{50}$$

Obtaining U_T from the (48), we have U_T in function of $U_{T,t}$:

$$U_T = \frac{1}{\omega}\, t\, U_{T,t}$$

Then from the (46), (49) and (50) we have:

$$QU_{T,Q} + MU_{T,M} = \frac{t}{\omega} U_{T,t} \tag{51}$$

8. Conclusions

Finally, we can say that considering the concept of money in a broader sense like a transactional instrument but also a reserve of value easily convertible in cash, we can find a linkage between macro and microeconomic models. In particular, in the Cobb Douglas utility function framework, we have the parameter β is at the same time a function of utility of the money, velocity of money circulation, adversity to risk and demand for money and it can have an impact on exchange rate dynamics.

The final part of the article is dedicated to a simulation that making reasonable assumptions for data not officially available, try to explain ex post the dynamic of the bitcoin in the last years. Finally the model is included in a Relativistic Quantum energetic framework and generalized for time dependence.

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Disclaimer/Publisher's Note:

Supplement

I. *The Quantity Theory of Money*

The Quantity Theory of Money is an economic model that explains the relationship among the following variables:

1. **Quantity of Money**: The total quantity of money (M) in circulation in an economy.
2. **Velocity of Circulation**: The velocity of circulation of money (V).
3. **Real Output**: The level of goods in the economy (Q).
4. **Prices**: The level of the prices of goods.

The relationship among these variables is explained in the quantity equation: M * V = P * Q.

The above relationship implies that:

- Increase in the quantity of money (M) determines a proportional increase in the price level (P), assuming Q and V remain constant.
- Variation in the quantity of money can modify real output in the short erm, but in the long term it will modify only prices.

II. *Keynesian theory of money demand (transactional, precautionary, and speculative use)*

Keynes illustrated the three categories of money usage by people: transactional, precautionary, and speculative.

1. **Transactional:** money as a medium of exchange to facilitate daily transactions.
2. **Precautionary Use:** money as an emergency reserve to pay unexpected expenses or financial uncertainty.
3. **Speculative Use:** money held in order to obtain a financial return from changes in its value.

III. *Classical theory of consumer choice*

The classical theory of consumer choice is based on the idea that consumers try to maximize their utility when making purchasing decisions. Purchasing decisions are based on individual preferences, available budget, and the prices of goods.

Preferences: The utility function represents the consumer's preferences in purchasing available goods. It is expressed as U = f(x, y), where x and y are the goods considered.

Available Budget: Consumers must make purchasing decisions considering budget constraints. The amount of goods that can be purchased must not exceed the available income. The budget constraint is expressed as $pX + qY \leq M$, where p and q are the relative prices of goods X and Y, and M represents the consumer's available income (or wealth).

Equilibrium of consumers: the equilibrium occurs when consumers maximize their utility given the budget constraints. It is obtained when the indifference curve is tangent to the budget constraint line.

IV. *Cobb-Douglas utility function*

The Cobb-Douglas utility function for two variables is given by:

$$\mathrm{U}(\mathrm{x1};\mathrm{x2}) = \mathrm{x1}^{\alpha}\,\mathrm{x2}^{(1-\alpha)}$$

Given a Cobb-Douglas utility function, the corresponding Walrasian demand function, which represents the level of consumption of good (es. X1) for each given combination of prices and wealth *(w)* that maximizes the utility function under the budget constraint, is given by:

$$\mathrm{Q} = \frac{\alpha * \mathrm{W}}{p}$$

The Cobb-Douglas utility function has diminished Marginal Utility. It means that the marginal utility of each good is positive but decreases as its consumption increases.

V. *The Cobb-Douglas production function*

The Cobb-Douglas production function represents the production of goods as

$$\mathrm{Q} = \mathrm{A} * \mathrm{L}^{\mathrm{a}} * \mathrm{K}^{\mathrm{b}}, where$$

- Q = production

- A = technological factor
- L = labor
- K = capital
- a,b = parameters

With a cost function:

$$C = w * L + c * K$$

A key property of the Cobb-Douglas production function is diminishing marginal productivity: increasing one factor of production while keeping the others constant leads to an increase in total output, but at a decreasing rate if a + b = 1, a > 0, and b > 0. This implies an increasing marginal cost.

VI. *Mean-Variance Theory and Arrow-Pratt Approximation*

- The Mean-Variance theory, developed by Harry Markowitz, is a method that can be used in order to select optimal financial portfolios.
- The model assumes that investors are rational and risk averse.
- Then they make investment decisions looking for the optimal combination of the mean (expected return) and variance (risk) of asset returns with the objective is to obtain a portfolio that minimizes risk (variance) for a given level of expected return, or that maximizes expected return for a given level of risk.
- The result is the efficient frontier, which represents the optimal portfolios in terms of mean and variance.

Arrow-Pratt approximation:

- The Arrow-Pratt approximation is a method for measuring an individual's risk aversion. Anyway, it can be used also in order to measure risk propension for individual inclined to risk.
- The absolute and relative risk aversion can be expressed respectively as:
  - Absolute risk aversion: $-\frac{U''(x)}{U'(x)}$
  - Relative risk aversion: $-\frac{x\,U''(x)}{U'(x)}$
- This is a quantification of an individual's attitude towards risk.

VII. *The Purchasing Power Parity*

The Purchasing Power Parity (PPP) theory try to express the exchange rate between two currencies based on different purchasing power in the two countries where the currencies are utilized.

Here are the main points of the PPP theory:

Forms of PPP:

- **Absolute PPP**: The exchange rate should be equal to the ratio of the general price levels of the two countries.
- **Relative PPP**: The exchange rate should vary in proportion to the differences in inflation rates between the two countries.

The PPP theory is based on the international arbitrage of goods. The limitations to the theory are given by various elements such as transportation costs, trade barriers, and market segmentation. In any case, despite its limitations, the PPP theory remains a useful method in order to analyze and compare purchasing power and price levels between different countries.

VIII. *The theory of prospect (Tversky & Kahneman)*

The Prospect Theory by Tversky and Kahneman is an economic theory that describes the way people make decisions involving risk. It suggests that individuals evaluate trading opportunities to their current situation, and that they have a tendency to be risk-averse when it comes to gains and risk-seeking when it comes to losses. This theory has had a significant impact on the field of economics and psychology.

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