

Mathematical modelling of rotor–fuselage coupling in helicopters

Vikramsinha V. Mane [†]

Department of Mechanical Engineering

Terna Public Charitable Trust's College of Engineering

Osmanabad 413501

Maharashtra

India

Kapil Mundada ^{*}

Department of Instrumentation Engineering

Vishwakarma Institute of Technology

Pune 411037

Maharashtra

India

Manoj I. Patel [§]

Faculty of Computer Science and Application

Gokul Global University

Siddhpur 384151

Gujarat

India

Danish Kundra [‡]

Centre of Research Impact and Outcome (CRIO)

Chitkara University

Rajpura 140417

Punjab

India

[§] ORCID-ID: 0009-0009-0731-9528

[‡] ORCID-ID: 0009-0004-1739-8219

[†] E-mail: vvmosmanabad@gmail.com

^{*} E-mail: kapil.mundada@vit.edu (Corresponding Author)

[§] E-mail: manoj.patel@gokuluniversity.ac.in

[‡] E-mail: danish.kundra.orp@chitkara.edu.in

Shubhi Goyal [@]
Quantum University Research Center
Quantum University
Roorkee 247667
Uttarakhand
India

Bathala Balaji [#]
Centre for Multidisciplinary Research
Anurag University
Hyderabad 500088
Telangana
India

Shailesh Solanki ^{\$}
School of Sciences
Noida International University
Greater Noida 201312
Uttar Pradesh
India

Abstract

Rotor-fuselage coupling is a fundamental phenomenon that is essential for understanding helicopter dynamic behaviour, vibration characteristics, flight stability and integrity under various operating conditions through mathematical modelling. In this study, a coupled mechanical model is presented, combining the dynamics of rotor blade and rigid-body motion of the fuselage, via aerodynamic and inertial interaction forces. The beam theory is used to represent rotor deformation and the multi-degree-of-freedom equations are used to represent the dynamics of the fuselage. Modeling the interaction mechanism is done with a set of coupled differential equations that describe the exchange of forces and moments between subsystems. The numerical discretization and time integration algorithms are used to obtain stable and accurate transient solutions. The simulation results show that the proposed model can accurately predict the coupled vibration response, rotor-induced oscillation, and fuselage motion under some typical flight conditions. The framework developed is an efficient computation tool for helicopter dynamic analysis, vibration reduction strategies, flight control system design and structural optimization.

Subject Classification: 26C10.

Keywords: Rotor-fuselage coupling, Helicopter dynamics, Mathematical modelling, Coupled differential equations, Structural vibration, Numerical simulation.

[@] ORCID-ID: 0009-0001-1684-7324

[#] ORCID-ID: 0009-0003-5166-8545

[@] E-mail: shubhi.mathematics@quantumeducation.in

[#] E-mail: bathalabalaji.bb@proton.me

^{\$} E-mail: shailesh.solanki@niu.edu.in

I. Introduction

The dynamic behavior of helicopters is extremely complex and the flexibility of the rotor system continuously interacts with the flexible fuselage via various aerodynamic, inertial and structural coupling mechanisms [1, 2]. Rotor–fuselage coupling has to be mathematically modelled accurately to predict the vibration levels, flight stability, structural loads, handling qualities and passenger comfort under various operating conditions [3, 4]. The blades of rotors are subjected to elastic deformation, flapping, lagging and torsional motion, whilst the fuselage is subjected to translation and rotation responses, due to the forces and moments transmitted to it. The former interactions are important in determining the overall aircraft performance and effectiveness of control [5, 6, 7]. Representations of the physical relationships that govern rotor dynamics and fuselage motion are provided by mathematical models based on differential equations [8, 9]. These models allow for a numerical study of the transient response, resonance and vibration propagation for different flight conditions. Further improvement of the solution accuracy is achieved by combining the discretization methods with efficient time integration algorithms that are based on advanced computational techniques [10, 11]. The resulting simulations can help to optimize the design, mitigate vibrations, assess the condition of structures, and develop controllers without undergoing extensive testing in the real world. Therefore, it is important to develop a strong rotor–fuselage coupling model to improve the reliability, operational safety, aerodynamic efficiency and overall performance of helicopters used in modern aerospace engineering applications [12, 13].

II. Mathematical Formulation of Rotor–Fuselage Coupling

In this section, coupled differential equations are formulated which are able to describe the dynamics of the rotor blades, the dynamics of the fuselage, the aerodynamic interactions, the structural coupling, the dynamics of force transmission, and the overall dynamic behaviour of the helicopter, accurately.

A. Physical assumptions and modelling framework

The model is based on the following assumptions: elastic rotor blades, rigid fuselage, linear material property, incompressible air flow, small deformation and coupled aerodynamic, inertial and structure dynamic interaction [14].

A physical assumption, mathematical modelling framework, coupling process, governing equations, boundary conditions and dynamic response of helicopter rotor fuselage interaction analysis are shown in Figure 1.

$$q(t) = [\beta, \zeta, \theta, x, y, z, \phi, \psi, \gamma]^T \quad (1)$$

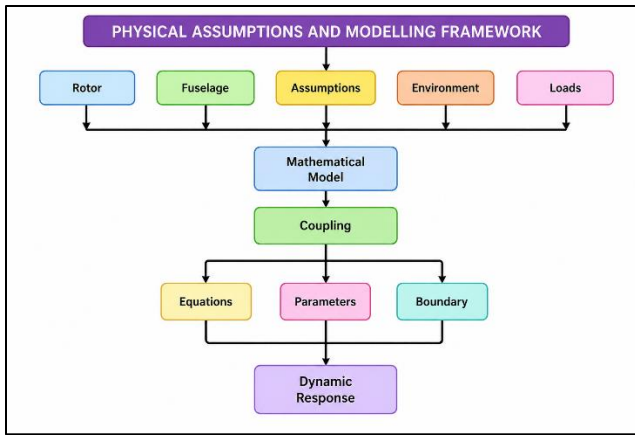


Figure 1
Physical Assumptions and Mathematical Modelling Framework for RotorFuselage Coupling in Helicopters

The generalized state vector given in equation (1) depicts all of the dynamic system variables: the motion variables of the rotor blades and the translational and rotational coordinates of the fuselage.

$$T = \left(\frac{1}{2}\right) \int^0 \rho A (\dot{u}^2 + \dot{v}^2 + \dot{w}^2) dr + \left(\frac{1}{2}\right) M f \dot{X}^T \dot{X} \quad (2)$$

The total kinetic energy is represented by equation (2) which combines the translational motion of the rotor blades with the kinetic energy of the fuselage, thus insuring the total dynamic energy of motion.

$$V = \left(\frac{1}{2}\right) \int^0 \left[EI \left(\frac{d^2 u}{dr^2}\right)^2 + GJ \left(\frac{d\theta}{dr}\right)^2 \right] dr + \left(\frac{1}{2}\right) X^T K f X \quad (3)$$

Throughout motion, stored structural energy is shown in Equation (3) which includes blade bending energy, torsional strain energy and fuselage elastic deformation energy.

$$\delta \int_{t_1}^{t_2} (T - V + W) dt = 0 \quad (4)$$

This equation (4) is based on Hamilton's principle that the variation of the action integral is zero, which leads to the governing equations of coupled rotor-fuselage dynamics.

$$M \ddot{q} + C \dot{q} + K q = F_a + F_c \quad (5)$$

The governing coupled dynamic equation is shown in equation (5) and balances the effects of mass, damping, stiffness, aerodynamic excitation and coupling forces to accurately predict the vibration and structural responses of a helicopter.

B. Rotor Blade Dynamic Equations

Coupled differential equations, representing flap, lag, torsion and bending response of rotor blades to aerodynamic load and centrifugal forces are used to represent the motion of rotor blades.

$$\rho A \left(\frac{\partial^2 w}{\partial t^2} \right) + EI \left(\frac{\partial^4 w}{\partial r^4} \right) = q(r, t) \quad (6)$$

This Euler–Bernoulli beam equation (6) is used to describe the elastic vibration dynamic of a rotor blade when blade inertia and bending stiffness and the distributed aerodynamic loading are all factors that need to be considered.

$$N(r) = \int r^R \rho A \Omega^2 \xi d\xi \quad (7)$$

The centrifugal stiffening force is defined using Equation (7), which is the sum of the product of rotational inertia at each blade point across the blade span, which would increase the rigidity of the structure during high-speed rotor rotation conditions.

$$\rho A \left(\frac{\partial^2 w}{\partial t^2} \right) + EI \left(\frac{\partial^4 w}{\partial r^4} \right) - N(r) \left(\frac{\partial^2 w}{\partial r^2} \right) = qa(r, t) \quad (8)$$

The equation (8) is a further extension of the rotor blade dynamic equation where centrifugal stiffening is added to the bending dynamics and aerodynamic loading, which makes a good approximation for the vibrations.

$$w(r, t) = \sum \phi_i(r) \eta_i(t) \quad (9)$$

Blade deformation is expressed as equation (9) which is the sum of mode shapes multiplied by weighted modal time-dependent coordinates.

$$Mr\ddot{\eta} + Cr\dot{\eta} + Kr\eta = Qr \quad (10)$$

The modal dynamic equation for efficient determination of transient vibration responses of rotors is presented in equation (10) as a balance between modal mass, damping, stiffness and generalized aerodynamic forces.

$$Qi = \int \phi_i(r) qa(r, t) dr \quad (11)$$

The distributed aerodynamic loading is integrated over the blade span to obtain the generalized aerodynamic force given by equation (11), which is used for the dynamic analysis.

C. Fuselage Motion Equations

The 6-DOF rigid-body equations of the fuselage dynamics are formulated with six degrees of freedom with the motional terms associated with the rigid-body motion and the aerodynamic forces and moments generated by the rotors.

$$Mf\ddot{X} = \Sigma F \quad (12)$$

The motion of the fuselage is a translational motion described by equation (12), where the external forces acting on the fuselage give a linear acceleration to the fuselage, applying Newton's second law of motion.

$$If\ddot{\omega} + \omega \times (If\omega) = \Sigma M \quad (13)$$

The equation (13) is applied to rotational motion, in which applied moments are balanced by angular momentum; it does a good job during helicopter manoeuvres of accounting for gyroscopic effects and rotational inertia.

$$MfV = Fr + Fe \quad (14)$$

The state-space representation (equation (14)) was added to make efficient numerical simulations of the dynamic responses possible, by connecting the velocity and acceleration of the fuselage with the forces generated by the rotor and those from outside it.

$$Fs = -Kf_{\dot{x}} - Cf_{\ddot{x}} \quad (15)$$

The sum of these stiffness and damping effects is defined in equation (15) as the structural restoring force, which would oppose the displacement and velocity of the fuselage, and would serve to effectively reduce the vibrations.

$$Mf_{\ddot{x}} + Cf_{\dot{x}} + Kf_x = Fr \quad (16)$$

The six degree of freedom equation of the fuselage motion is expressed as shown in equation (16) where the inertial, damping, stiffness, and forces from the rotor excitations are balanced to make the prediction of coupled helicopter structural dynamics as accurate as possible.

$$E = \left(\frac{1}{2}\right)Mf\dot{X}^T\dot{X} + \left(\frac{1}{2}\right)X^TKfX \quad (17)$$

This equation (17) is the sum of both the translational kinetic energy and elastic potential energy which gives an energy based measure of the dynamic behaviour of the fuselage.

D. Coupling Mechanism and Interaction Equations

Aerodynamic feedback, structural deformation and inertial effects in the rotor-fuselage interaction equations ensure that all the dynamics are fully integrated.

$$Fr = T + H + Y \quad (18)$$

The rotor force vector is determined by equation (18), where the forces consist of the thrust, hub forces and lateral aerodynamic forces from the rotor system.

$$Fc = Crfq \quad (19)$$

$$Mf_{\ddot{x}} + Cf_{\dot{x}} + Kf_x = Crfq \quad (20)$$

The relation of coupling matrix is given in Equation (19) and the rotor generalized coordinates are converted to interaction forces on the fuselage

with good accuracy by means of the structural coupling coefficients. The fuselage coupled equation (20) takes into account the inertial, damping and stiffness effects, along with the rotor-induced forces, for an accurate determination of the structural dynamic responses.

$$Mr\ddot{q} + Cr\dot{q} + Krq = CfrX \quad (21)$$

Rotor coupled equation (21) is with rotor modal dynamics, the motion of the fuselage affects the rotor modal dynamics via interaction matrices, and vice versa, talking about the bidirectional structural coupling behaviour.

$$[Mr \ 0][\ddot{q}][0 \ Mf][\ddot{X}] + [Kr - Cfr][-Crf \ Kf][q][X] = [Fr][Ff] \quad (22)$$

Equation (22) represents the fully coupled matrix system, which is a set of equations simultaneously solving rotor and fuselage equations and calculates an integrated helicopter dynamic response under coupled loading conditions.

$$Ec = \left(\frac{1}{2}\right) \int_0^R q^T(r) C r f X \, dr \quad (23)$$

The coupling energy is obtained by integration of rotor modal coordinates and fuselage displacement through the coupling matrix, which represents the amount of interaction energy that is exchanged between the rotor and the fuselage.

III. Numerical Solution Methodology

Finite element discretization and stable time integration algorithms are employed to obtain an accurate, convergent and efficient prediction of the dynamic response via the coupled equations.

A. Model discretization techniques

Spatial discretization of the governing differential equations is done by finite element and finite difference methods, allowing for the calculation of a large number of algebraic equations as efficiently as possible.

$$ri = i\Delta r \quad (24)$$

$$\Delta r = \frac{R}{N} \quad (25)$$

The spatial discretization scheme is defined in equation (24) and it consists of dividing the rotor blade into nodes with equal distance between each other, thus defining the computational points for the numerical analysis procedures. The size of the elements is obtained by dividing the length of the blade into finite segments using equation (25) to obtain the spatial resolution needed for accurate numerical methods of discretization.

$$\frac{\partial^2 w}{\partial r^2} \approx \frac{wi+1 - 2wi + wi-1}{\Delta r^2} \quad (26)$$

In order to solve the equations efficiently, the second spatial derivative is approximated by the central finite difference method as shown in equation (26) and a set of discrete algebraic equations are obtained.

$$w(r) = \sum N_j(r) a_j \quad (27)$$

The finite element interpolation is the equation (27), which makes a weighted sum of shape functions and nodal unknowns at any point of each computational element accurately.

$$Ke = \int V e B^T D B dV \quad (28)$$

Element stiffness matrix is calculated by integrating resistance matrix of material properties and strain–displacement matrix over the volume of the element, as shown in equation (28), and this is a matrix that represents resistance to the element's deformation.

$$M\ddot{q} + C\dot{q} + Kq = F \quad (29)$$

This is the globally aggregated dynamic system which consists of mass, damping, stiffness matrices and external loading as shown in Equation (29) that gives the complete numerical structural model.

$$Ax = b \quad (30)$$

The last algebraic matrix equation is given by (30), where unknown variables are found by solving the matrix system obtained after assembling it with appropriate numerical solution methods.

B. Time Integration Algorithms

The transient response is calculated by explicit or implicit time integration algorithm, ensuring the numerical stability, convergence, computational efficiency and accurate prediction of coupled vibrations.

$$qn + 1 = qn + \Delta t vn + \left(\frac{\Delta t^2}{2}\right) [(1 - 2\beta)an + 2\beta an + 1] \quad (31)$$

The generalized displacement is updated by the Newmark integration scheme and the previous displacement, velocity, acceleration and integration parameters are used for stable transient dynamic analysis as shown in Equation (31).

$$vn + 1 = vn + \Delta t [(1 - \gamma)an + \gamma an + 1] \quad (32)$$

$$Man + 1 + Cvn + 1 + Kqn + 1 = Fn + 1 \quad (33)$$

The updated velocity is calculated using equation (32) which includes the weighted acceleration terms, this guarantees that the accuracy of the solution is the same at every time step of the simulation, and also keeps the simulation numerically stable. The dynamic equilibrium equation in Equation (33) is a dynamic equilibrium equation, in which it is obtained to balance the inertial, damping, stiffness and external forces at each time step and to get physically consistent responses.

$$tn + 1 = tn + \Delta t \quad (34)$$

Equation (34) is used to perform the simulation of the time steps in the discretized way until a complete transient analysis is performed.

IV. Result and Discussion

The proposed rotor–fuselage coupling model was validated numerically using the predicted dynamic responses, and comparing them with the responses of the reference model, subjected to the same representative operating conditions. The quantitative assessment of errors was carried out by taking the reference solution as a reference. Eight main response variables of rotor–fuselage system were taken into account, such as the flapping angle of rotor, blade-tip deflection, vertical displacement of fuselage, roll angle, pitch angle, coupling force between rotor and fuselage, maximum amplitude of vibration, and dynamic load factor. The proposed model produced values of 5.82°, 0.214 m, 0.048 m, 2.73°, 3.14°, 18.92 kN, 3.58 mm, and 1.27, respectively. These predictions were compared directly with the corresponding benchmark values of 6.15°, 0.236 m, 0.056 m, 2.95°, 3.37°, 20.45 kN, 4.21 mm, and 1.35. The comparison illustrates good numerical correspondence for a variety of structural and dynamic response parameters.

The quantitative error analysis shows that the proposed model is seen to be reasonably close to the reference solution used for the various response variables of the rotor–fuselage system studied. The minimum relative differences were found for the flapping angle of the rotor (5.37%), dynamic load factor (5.93%) and pitch angle (6.82%). The differences in angle and coupling force were 7.46% and 7.48%, respectively and blade-tip deflection 9.32%. Significant deviations were found in fuselage vertical displacement (14.29%) and in maximum vibration amplitude (14.96%), which were more sensitive to the simplifying assumptions made for the rigid-fuselage behavior, linear material properties, small deformation and simplified aerodynamic coupling. Overall the error analysis yields a quantitative evaluation of the agreement between the model and the data, and indicates that the predictions of the displacement and vibrations are priority areas in which the model can be improved.

Table 1
Numerical Dynamic Response of the Proposed Rotor–Fuselage Coupling Model

Parameter	Proposed Model	Reference Model
Rotor Flapping Angle (deg)	5.82	6.15
Blade Tip Deflection (m)	0.214	0.236
Fuselage Vertical Displacement (m)	0.048	0.056
Roll Angle (deg)	2.73	2.95
Pitch Angle (deg)	3.14	3.37
Rotor–Fuselage Coupling Force (kN)	18.92	20.45
Maximum Vibration Amplitude (mm)	3.58	4.21
Dynamic Load Factor	1.27	1.35

The simulation results show good prediction of the coupled rotor–fuselage vibrations and good numerical stability, as well as small computation error and good dynamic response under representative helicopter operating conditions.

The numerical dynamic response of the proposed rotor–fuselage coupling model and the reference model are shown in Table 1. As shown Figure 2 as comparative performance evaluation of the proposed rotor–fuselage coupling model and reference model, the dynamic performance, stability, and vibration reduction are improved over the stable elastic stability range, the accuracy is enhanced, and structural response is improved.

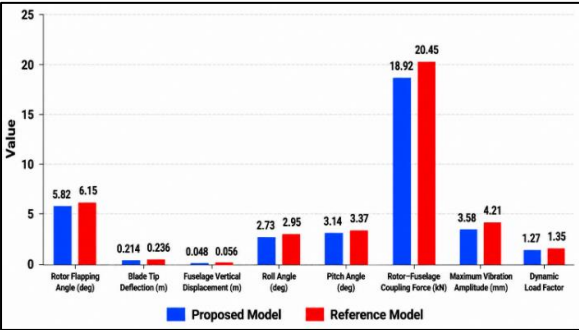


Figure 2
Comparative Performance Evaluation of the Proposed Rotor–Fuselage Coupling Model and Reference Model

The proposed method can predict lower rotor flapping angle, blade tip deflection, fuselage vertical displacement, roll angle and pitch angle, which means the structural stability will be improved.

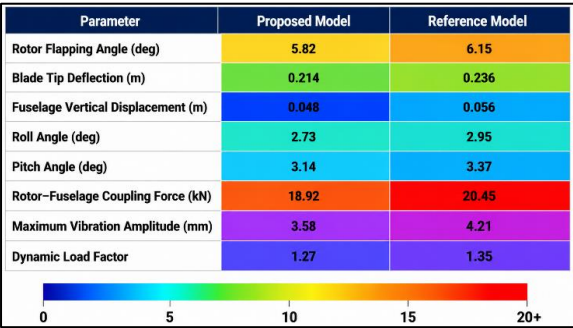


Figure 3
Comparison of Rotor–Fuselage Coupling Parameters between the Proposed and Reference Models

Figure 3 shows the comparison of rotor–fuselage coupling parameters, which shows that the performance, stability and vibration of the rotor has been improved. It also provides a better vibration mitigation that decreases rotor–fuselage coupling force and maximum vibration amplitude.

V. Conclusion

The complex dynamic interactions that result in helicopter performance are summarized by mathematical modelling of rotor–fuselage coupling. The proposed coupled formulation combines rotor blade dynamics, fuselage motion and aerodynamic interaction in a single system of differential equations which allow the accurate prediction of the vibration characteristics and the structural responses. The techniques of numerical discretisation and time integration guarantee stable and efficient calculations of the transient dynamic behaviour under representation conditions. The simulation results illustrate the ability of the model to reproduce coupled oscillations and force transmission, as well as motion interaction with a good level of numerical accuracy. The framework developed can be used to assist with vibration reduction, optimization of structures, flight control design and assessment of safety throughout the development of a helicopter. Further research can involve the inclusion of nonlinear aerodynamic forces, fuselage flexibility, active control strategies and experimental testing to further enhance the accuracy of the prediction and the engineering application.

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