

Mathematical modeling of epidemic spread using nonlinear dynamical systems

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Abstract

The spread of an epidemic is a complex nonlinear process that is affected by the process of transmission, the process of recovery, the interaction of the population and the temporal variation in the epidemiological parameters. This study aims to develop mathematical framework for the modelling of the dynamics of epidemics with the help of non-linear dynamics. The population is broken up into epidemiological compartments and the dynamics of the disease spread and recovery are represented by a set of nonlinear differential equations. The framework proposed analyzes the behavior of infection growth, the peak of an epidemic, the recovery pattern, the equilibrium behavior and the stability of the system for varying parameters. The simulation results show that the nonlinear modelling is a powerful tool to capture epidemic dynamics and gain quantitative understanding of transmission dynamics, assessing control strategies, and making epidemic management decisions.

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I. Introduction

Diseases that are infectious are always a challenge to public health systems, as they depend on the interaction of susceptible, infected and recovered populations. These interactions are crucial to predict disease progress, estimate the peak of the epidemic and implement adequate intervention [1, 2]. Mathematical modelling can be used to give a systematic representation of the mechanisms of transmission and give a quantitative assessment of the behaviour of an epidemic under different epidemiological circumstances. Although traditional compartmental models have made a significant contribution to the analysis of epidemics, the actual context of disease transmission shows that there are several factors that are not captured by these models: population mixing, behavioral responses, environmental effects, immunity and time-dependent intervention measures, which all introduce certain nonlinearities into the transmission process [3, 4, 5]. Nonlinear dynamical systems work well as a mathematical model of such complex epidemic dynamics. They are defined by a set of interconnected population compartments using a set of nonlinear differential equations that describe temporal changes in the process of infection, transmission and recovery [6, 7]. The behaviors of the system and the equilibrium conditions are significantly affected by the epidemiological parameters like transmission rate, recovery rate, contact frequency and initial population distribution. To account for evolving disease and control conditions, parameter values may also be made time-varying, which further improves the model's representation of changing disease and control conditions [8, 9]. This research builds a mathematical dynamic modelling framework based on a nonlinear dynamical system to model the spread of an epidemic. The proposed approach involves the compartmental representation, the nonlinear differential equations, time varying transmission and recovery rate, and the numerical simulation procedures [10, 11, 12].

II. Mathematical Formulation of Epidemic Dynamics

The mathematical formulation is based on the interconnection of population states, epidemiological parameters, and governing equations which represent the behavior of an epidemic, which can then be investigated systematically with respect to disease transmission, recovery, progression, and patterns of stability.

A. Model Assumptions and Epidemiological Parameters

The model has the following assumptions: homogeneous population mixing, measurable epidemiological parameters such as transmission, recovery, contact and progression rate parameters that describe the nonlinear dynamics of epidemics over time [13, 14] and defined initial conditions.

$$N(t) = S(t) + E(t) + I(t) + R(t) \quad (1)$$

The sum of the susceptible, exposed, infected and recovered people in the population is represented by equation (1) and is the basic population balance needed to investigate the transition between epidemiological compartments in the course of the disease.

$$\beta_{eff} = \beta \left[\frac{S(t)}{N(t)} \right] \quad (2)$$

Actual infection potential is measured by equation (2) which reflects the altered conditions of exposure by combining the baseline transmission with the proportion of susceptible population at various stages of epidemic development.

$$R^0 = \frac{\beta}{\gamma} \quad (3)$$

$$R_{e(t)} = R^0 \left[\frac{S(t)}{N(t)} \right] \quad (4)$$

The potential transmission of the current epidemic is calculated in Equation (3, 4) which modifies the basic reproduction number by accounting for susceptible availability, such that whether the prevalence of infection will rise or fall over time is determined.

$$\lambda = \sqrt{\beta^2 + \gamma^2 + \sigma^2} \quad (5)$$

In equation (5) a combination of transmission, recovery and progression rates is being put together in one bundle which is providing a quantitative measure to characterize the overall intensity as well as the dynamic behavior of the development of an epidemic.

$$\Phi = \sqrt{\left[\frac{\beta\sigma}{(\gamma + \mu)} \right]} \quad (6)$$

Equation (6) combines the transmission, disease progression, recovery and removal effects into a single equation that allows a consolidated mathematical means of comparing the behavior of the epidemic in various epidemiological or intervention scenarios.

B. Compartmental Representation of Epidemic Spread

The population is segmented into epidemiological compartments and can be systematically followed to see the movement of people both between susceptible and exposed compartments and among infected and recovered compartments during outbreaks [15].

The Compartmental Framework for Epidemic Spread and Population State Transitions shown in Figure 1 as the progression of an epidemic in susceptible, exposed, infected and recovered compartments, showing the dynamic nature of transmission, progression to infection, recovery, mortality and immunity lost.

$$S(t) = N - E(t) - I(t) - R(t) \quad (7)$$

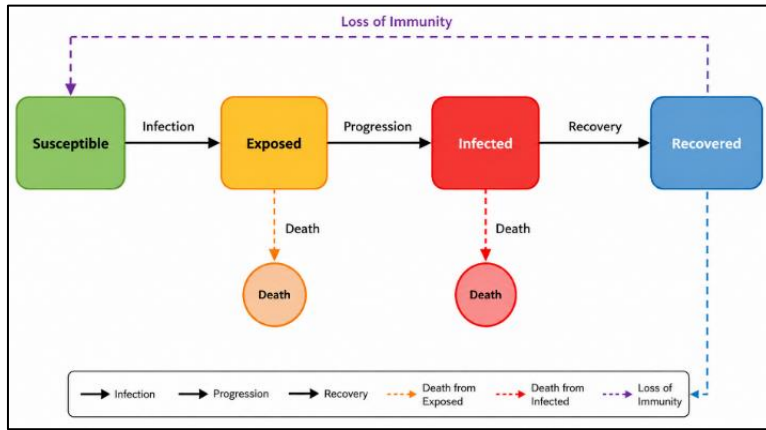


Figure 1
Compartmental Framework for Epidemic Spread and Population State Transitions

The susceptibility of a population is calculated by subtracting out the exposed, infected, and recovered individuals from the total population, as shown in the equation below, and is kept constant throughout the modeled epidemic throughout time.

$$E(t) = E(0) + \int_0^t \left[\frac{\beta S(\tau) I(\tau)}{N} - \sigma E(\tau) \right] d\tau \quad (8)$$

In equation (8) the exposure from the susceptible to the infectious stage within outbreaks is captured through the cumulative changes in exposed individuals by integrating new infections and disease progression.

$$I(t) = I(0) + \int_0^t [\sigma E(\tau) - (\gamma + \mu) I(\tau)] d\tau \quad (9)$$

This is for evolutions of infected population, derived from equation (9), which assumes evolutions of infected populations as a result of exposed population progression, reduction because of recovery and removal, respectively, as infectious prevalence changes throughout the epidemics development periods.

$$R(t) = R(0) + \int_0^t \gamma I(\tau) d\tau \quad (10)$$

$$X(t) = \sqrt{[S^2(t) + E^2(t) + I^2(t) + R^2(t)]} \quad (11)$$

Equation (10) is the equation used to calculate accumulated recovered individuals, which is the integral of the recovery contribution from infected populations, a progressive removal from the infectious states and an increasing immunity during the whole epidemic period dynamically. All the populations of the compartments are combined through a root-based magnitude in equation (11), giving an integrated quantitative measure of

the state of the overall epidemiological system and its temporal evolution during outbreaks.

$$B(t) = \sqrt{[E^2(t) + I^2(t)]} \quad (12)$$

The exposed and infected populations are combined in equation (12), which is a root function that measures the overall level of active disease burden and can be used to compare the severity of epidemics in various epidemic situations.

C. Formulation of Nonlinear Differential Equations

To mathematically model changes in the epidemiology of an compartment over time, nonlinear differential equations include the interactions of transmission, recovery and population transitions.

$$\frac{dS}{dt} = -\frac{\beta SI}{N} \quad (13)$$

The instantaneous incidence of infection depends on transmission rate and population size, and is described by Equation (13) which models the susceptible population decreasing due to the non-linear interactions with infected population.

$$\frac{dE}{dt} = \frac{\beta SI}{N} - \sigma E \quad (14)$$

Changes in exposed individuals in equation (14) are accounted for by new infections and to infectiousness via a latent stage of infection that governs the dynamics of subsequent infectious transmission in populations.

$$\frac{dI}{dt} = \sigma E - (\gamma + \mu)I \quad (15)$$

The Equation (15) represents the evolution of infected population in the course of an epidemic because of progression and due to reduction caused by recovery and removal, respectively.

$$\Psi(t) = \sqrt{\left[\left(\frac{dE}{dt}\right)^2 + \left(\frac{dI}{dt}\right)^2\right]} \quad (16)$$

Rates of change of the exposed and infected populations are combined in equation (16) via a root based measure, thereby giving an integrated measure of the nonlinear infection activity in the epidemic system.

$$\lambda = \frac{-(\sigma + \gamma + \mu) \pm \sqrt{\{(\sigma - \gamma - \mu)^2 + \frac{4\sigma\beta S}{N}\}}}{2} \quad (17)$$

The characteristic roots around an epidemic equilibrium are given by (17) which can be used to assess the stability of the equilibrium by the behaviour of the eigenvalues with negative real parts usually represent stability.

III. Proposed Nonlinear Dynamical Systems Modeling Framework

The proposed framework will be based on the integration of the nonlinear epidemic equations, time-varying epidemiological parameters, stability principles, and computational simulations, which will be used to study the dynamics evolution of the epidemic and to analyze the responses to the epidemic under different scenarios.

A. Time-Varying Transmission and Recovery Dynamics

The time-varying transmission and recovery rates capture dynamic changes in the behaviour of the population, interventions, immunity, and the effectiveness of treatments throughout the course of an outbreak, allowing realistic characterization of the dynamics of an epidemic.

$$\beta(t) = \beta^0 e^{-k\beta t} \quad (18)$$

The variation in transmission rate with time is expressed as exponential decay in Equation (18) which reflects a reduction in disease transmission due to interventions, behavioural changes, alterations in distance and prevention control.

$$\gamma(t) = \gamma^0 + \gamma^1 [1 - e^{-k\gamma t}] \quad (19)$$

The progressively increasing recovery rate over time shown in equation (19) reflects increased effectiveness of treatment, increased responses to healthcare, improved clinical management, and increased capacity to recover during the epidemic period.

$$B(t) = \int_0^t \beta(\tau) I(\tau) d\tau \quad (20)$$

In this case, equation (20) includes the temporal distribution of transmission and infected population sizes, giving an estimate of how much infection pressure has been accumulated and thus an estimate of the total transmission influence over the period of the epidemic observed.

$$G(t) = \int_0^t \gamma(\tau) I(\tau) d\tau \quad (21)$$

$$S(t) = S(0) - \int_0^t \left[\frac{\beta(\tau) S(\tau) I(\tau)}{N} \right] d\tau \quad (22)$$

Equation (21) combines the time varying recovery rate and infected population data, capturing the total amount of recovery activity and showing the recovery contribution throughout an extended period of the evolving epidemic in a dynamic way. Using Equation (22) allows for the development of the susceptible population to be modeled as a summation of transmission losses, reflecting nonlinear transmission interactions over time and determining progressive loss of susceptible individuals as epidemics progress.

$$I(t) = I(0) + \int_0^t [\sigma E(\tau) - \gamma(\tau) I(\tau)] d\tau \quad (23)$$

These rates of change in the infected population are captured in Equation (23) which accounts for the progression of infected population

from the exposed population, and the losses from infected population due to time-dependent recovery, making it possible to assess the status of the infected population over time during an epidemic.

$$R(t) = R(0) + \int_0^t \gamma(\tau)I(\tau)d\tau \quad (24)$$

The recovered population growth equation (24) is the sum of time-dependent recovery contributions due to recovered individuals which describe the cumulative recovery and progressive movement of infected individuals towards the recovered epidemiological compartment.

B. Numerical Solution and Simulation Procedure

Numerical methods are used to approximate solutions of nonlinear epidemic equations with discrete time intervals, allowing the simulation of the susceptible, infected and recovered populations in the computer to be done accurately for a variety of scenarios of epidemiological parameters.

$$S_n^{+1} = S_n - h \left[\beta \frac{(t_n)S_n I_n}{N} \right] \quad (25)$$

Newly transmitted infections are removed from susceptible population at the next time step as given by equation (25), allowing to simulate depletion of susceptible population and keep track of it discretely in numbers during the epidemic.

$$E_n^{+1} = E_n + h \left[\beta \frac{(t_n)S_n I_n}{N} - \sigma E_n \right] \quad (26)$$

A discrete, numerical estimate of the dynamics of latent disease is derived from equation (26) using newly generated exposures and progression losses in each simulation time step.

$$I_n^{+1} = I_n + h[\sigma E_n - \gamma(t_n)I_n] \quad (27)$$

By adding progression from exposed population and losses to the recovered population, Equation (27) establishes a set of numerical equations that allow tracking of the dynamics of active infection during outbreaks.

$$R_n^{+1} = R_n + h\gamma(t_n)I_n \quad (28)$$

During each simulation interval, equation (28) recovers all the individuals by summing their recovery contributions, and the individuals move from infectious to recovered epidemiological states in discrete steps, numerically.

$$X(t) = X(0) + \int_0^t F(\tau, X(\tau))d\tau \quad (29)$$

According to (29), the evolution of the system is represented as an integral of the epidemic function that governs the system, which gives a general numerical base to the accurate numerical approximation of the population states, given the initial conditions.

$$\int_{t_n}^{t_{n+1}} F(t, X)dt \approx \left(\frac{h}{2}\right) [F(t_n, X_n) + F(t_{n+1}, X_{n+1})] \quad (30)$$

The endpoint function values are used on each time interval for the integral, which is an approximation that ends up giving a good estimate of the integral over the numerical ranges during the simulation process, by averaging the function values on the consecutive times.

$$E_{RMSE} = \sqrt{\left[\left(\frac{1}{m}\right) \sum_{i=1}^m (X_i - \hat{X}_i)^2\right]} \quad (31)$$

The root mean square error (RMSE) between computed and reference epidemics is given by equation (31), and is used as a quantitative measure of numerical accuracy and reliability of computed model solutions.

IV. Result and Discussion

The simulated infection peaks, recovery curves, and time to the end of an epidemic vary depending on the parameters, showing that this model captures the essential features of the nonlinear transmission dynamics and interventions in an epidemic.

Table 1
Epidemic Dynamics Under Different Transmission Scenarios

Scenario	Transmission Rate (β)	Recovery Rate (γ)	R_0	Peak Infected (%)	Peak Time (Days)	Final Recovered (%)
S1	0.18	0.12	1.50	10.8	82	61.4
S2	0.22	0.12	1.83	15.6	71	69.7
S3	0.26	0.12	2.17	21.9	59	77.8
S4	0.30	0.12	2.50	28.7	48	84.5
S5	0.34	0.12	2.83	35.4	39	89.2

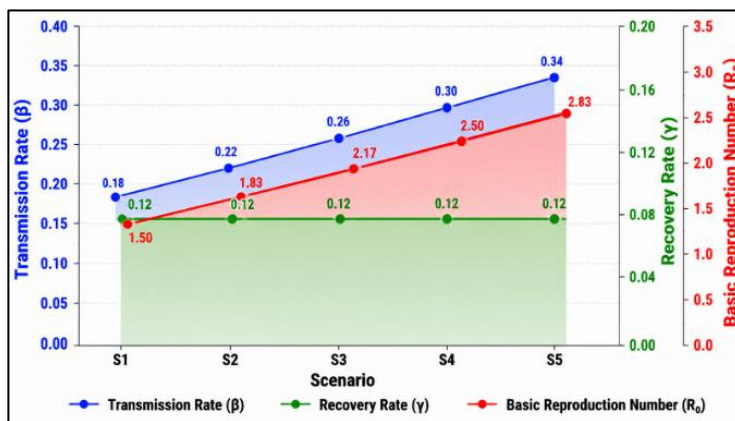


Figure 2
Comparative Analysis of Transmission Rate, Recovery Rate, and Basic Reproduction Number Across Epidemic Scenarios

As seen in Table 1 the Dynamics Under Different Transmission Scenarios, an increase in the transmission rate markedly affects the severity of the epidemic. As β rises from 0.18 to 0.34, R_0 increases from 1.50 to 2.83, while peak infections increase from 10.8% to 35.4%. Figure 2 shows the variations in transmission, recovery and reproduction rates between scenarios.

At the same time peak time reduces from 82 to 39 days, which means the progress of the epidemic is increasing. The number of fish recovered by the end is raised from 61.4% to 89.2%.

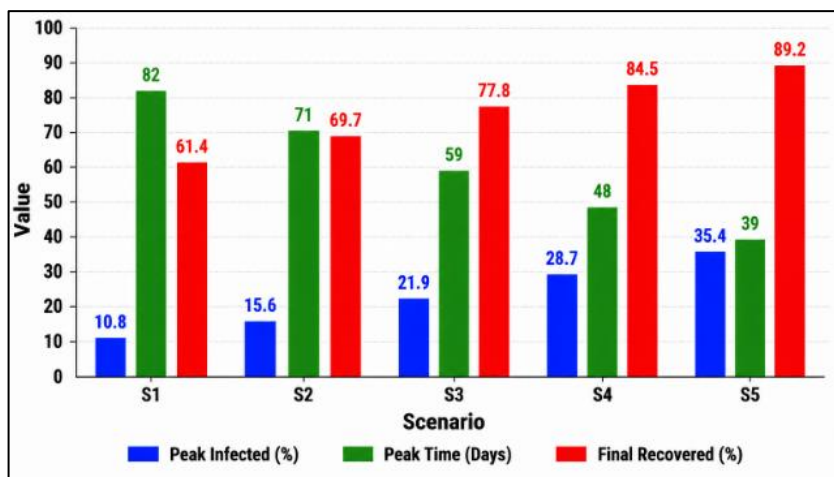


Figure 3

Comparative Analysis of Peak Infection, Peak Time, and Final Recovery Across Epidemic Transmission Scenarios

Figure 3 shows Comparative Analysis of Peak Infection, Peak Time, and Final Recovery Across Epidemic Transmission Scenarios, the difference between the peak infection, timing, and recovery in scenarios. These results indicate that increased transmission rate leads to more exposure of the population during outbreaks, higher epidemic peaks, and more spread of the infection, hence confirming that there is a link between increased transmission rate and increased exposure of the population during an outbreak.

V. Conclusion

In this study, a mathematical framework was offered for analysing the spread of epidemics based on nonlinear dynamical systems. The suggested method combined the population representation of the compartments, the use of nonlinear differential equations, changing rates of transmission, recovery, and numerical simulation procedures as a function of time. The

model was able to describe the temporal evolution of susceptible, exposed, infected and recovered populations in various epidemiological scenarios. Using a simulation analysis the authors show that variations in the parameters of transmission and recovery have important implications for the peak of the infection, the length of the epidemic, the shape of the recovery curve, and the general course of the epidemic. Nonlinear dynamics gave insight on complex interactions of the behaviour of epidemics and the stability of systems. The framework created can be utilized for epidemic forecasting, evaluation of interventions, planning of healthcare resources and informed decision making. Vaccination, demographic heterogeneity, mobility patterns, stochastic effects and real-time epidemiological data can be considered for further research.

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