

Mathematical modeling of decision-making processes in cognitive science

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Abstract

Decision-making is a basic cognitive function that includes alternative assessment, the incorporation of information and choosing an appropriate action when uncertainty exists. In this study, the mathematical modeling framework of differential equations and computational methods are used to analyze the decision making process in cognitive science. The proposed model accounts for interactions between cognitive parameters, such as stimulus strength, evidence accumulation, evidence threshold, memory influence and uncertainty about the decision. Temporal dynamics of cognitive states are mathematically described and behavioural reaction is predicted. Sensitivity analysis is used to assess the impact of the important parameters on the decision making process, and numerical techniques are used to give approximate solutions for the complicated dynamics of the model. A computational algorithm is designed to predict systematically decisions made under different cognitive conditions. The results of the simulations show the impact of the parameter variations on decisions accuracy, response behaviour and convergence behaviour. The framework offers a quantitative method of understanding cognitive mechanisms, and can be used to facilitate computational investigations of human decision behaviour under various experimental conditions.

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I. Introduction

The process of making decisions is a basic cognitive process by which an individual considers the available information, makes a comparison of alternatives and chooses a set of actions based on internal preferences and environment [1, 2]. It has a crucial role in cognitive science for human behaviors are continually affected by perception, memory, attention, uncertainty, learning, and past experience [3]. Systematic approaches that can capture the temporal dynamics and nonlinear interactions between the cognitive variables are needed to understand these interacting mechanisms. Mathematical modeling is a useful tool to quantitatively study decision making processes [4, 5, 6]. The learning and decision-making processes of accumulating evidence and forming a choice can be captured by differential equations, dynamical systems, probability models and computational methods [7, 8]. Important parameters such as the intensity of the stimulus, the rate of evidence accumulation, the decision criterion, memory effect, cognitive bias, and uncertainty can be represented using such models [9, 10]. They also allow investigation of the impact of changes in these parameters on response time, decision accuracy and behavioral outcomes. Mathematical models in cognitive science can be used to translate theoretical statement and assumptions into measurable and testable relationships [11, 12]. These situations are quite complex where analytical solutions are hard to come by, and numerical simulations can be used to investigate the situation further. Sensitivity analysis is useful to find out the parameters of cognition that have the most significant effect on anticipated decisions.

II. Mathematical Formulation of Decision-Making Processes

The formulation of decision making is achieved by differential equations that capture the evidence accumulation, cognitive states, memory influence, uncertainty and thresholds for responses, which are used to quantitatively describe the changing behavioral choices over time [13, 14].

Cognitive Evidence Accumulation

$$\frac{dE(t)}{dt} = \alpha S(t) - \beta E(t) + \sigma \xi(t) \quad (1)$$

The cognitive evidence accumulation by cognitive evidence model (represented in equation (1) below) accounts for stimulus strength, evidence decay and cognitive noise to model the development of information in the formation of a decision.

Cognitive State Dynamics

$$\frac{dC(t)}{dt} = \gamma E(t) - \delta C(t) + \eta M(t) \quad (2)$$

Equation (2) models the dynamics of cognitive state, depending on the evidence accumulated, memory influence and natural cognitive decay.

Memory Evolution

$$\frac{dM(t)}{dt} = \lambda C(t) - \mu M(t) \quad (3)$$

Equations (3) shows the evolution of memory, in which the cognitive activity plays a role in determining the formation of memory and the previously stored information decays over time.

$$\frac{dD(t)}{dt} = w^1 E(t) + w^2 C(t) + w^3 M(t) - \theta \quad (4)$$

$$P(t) = \frac{1}{[1 + \exp(-kD(t))]} \quad (5)$$

The equation (4) represents the dynamics of the decision variables, based on the idea of weighted evidence, cognitive state and memory contributions with respect to a pre-defined decision threshold. The decision variable is transformed to a probability using a logistic function in equation (5) which allows probabilistic interpretations of different alternatives.

Uncertainty Dynamics

$$\frac{dU(t)}{dt} = \rho[1 - P(t)] - \omega U(t) \quad (6)$$

The dynamics of uncertainty described in equation (6) show that uncertainty grows if the decision maker has low confidence in the decision and diminishes by any means of cognitive resolution.

Response-Time Model

$$T_r = T^0 + \frac{\theta}{[|E(t)| + \varepsilon]} \quad (7)$$

Finally, Equation (7) is dependent on the strength of the evidence accumulated and the decision threshold, with the implication that the more evidence that is accumulated the faster the response, and the lower the decision threshold the more evidence that is required to reach a decision.

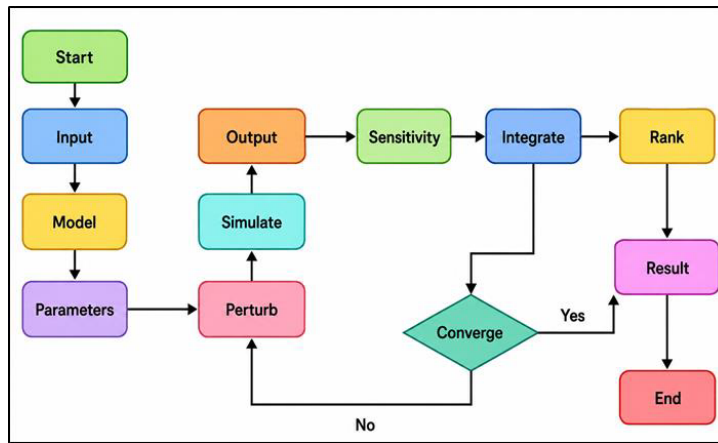
III. Model Analysis and Solution Methodology

The formulated cognitive model is then analyzed mathematically and computationally, to assess the stability, dynamic behavior, the effects of the model parameters and numerical solutions under different decision conditions.

A. Sensitivity Analysis of Cognitive Parameters

Sensitivity analysis examines the changes in cognitive parameters to see how they affect decision outcomes and results in identification of dominant parameters that influence response accuracy, convergence, uncertainty and behavioral dynamics.

The process of systematic sensitivity analysis in assessing the cognitive parameters, measuring the variations, ranking the influences and accurately determining the impacts on the cognitive process of decision making is illustrated in the Figure 1.

**Figure 1**

**Sensitivity Analysis Framework for Cognitive Parameters in
Decision-Making Processes**

Normalized Parameter Sensitivity

$$S_i = \left(\frac{\partial D}{\partial p_i} \right) \left(\frac{p_i}{D} \right) \quad (8)$$

Equation (8) shows the normalized parameter sensitivity as a fraction of the relative change in the decision response to a proportional change in one of the individual cognitive parameters. Normalization allows to compare parameters which have different numerical scales.

Finite-Difference Sensitivity

$$S_{i(t)} \approx \frac{[D(t, p_i + \Delta p_i) - D(t, p_i - \Delta p_i)]}{(2\Delta p_i)} \quad (9)$$

In Equation (9) the finite difference approach is used to estimate the temporal sensitivity by perturbing the parameter upwards and downwards from a baseline value.

Integrated Sensitivity Measure

$$I_i = \int_0^T \left| \frac{\partial D(t)}{\partial p_i} \right| dt \quad (10)$$

This brings in Equation (10), which has an integrated sensitivity measure which integrates the influence of the parameter throughout the time period observed.

Mean Temporal Sensitivity

$$\bar{S}_i = \left(\frac{1}{T} \right) \int_0^T S_{i(t)} dt \quad (11)$$

To obtain the mean temporal sensitivity, the time dependent sensitivity is averaged over time in equation (11), which gives an overall representation of the importance of the parameters in cognitive processing.

Sensitivity of Decision Probability

$$S_{p,i} = \frac{\partial P}{\partial p_i} = kP(t)[1 - P(t)] \left(\frac{\partial D}{\partial p_i} \right) \quad (12)$$

The sensitivity of decision probability is evaluated by equation (12) and link the change of the parameters to the change of probabilistic decision outcomes by using the logistic decision function.

Global Cognitive Sensitivity Index

$$G_i = \frac{\left[\int^0 \{D(t, p_i + \Delta p_i) - D(t, p_i)\}^2 dt \right]}{\left[\int^0 D^2(t, p_i) dt \right]} \quad (13)$$

Finally, Equation (13) defines the global cognitive sensitivity index by comparing the differences in decisions with baselines and the integrated deviations in the decisions. Together, these formulations pinpoint key cognitive factors, measure local and global effects on parameters, and enable systematic cognitive interpretation of the robustness of the decision model across different scenarios of behavior and across different cognitive conditions.

B. Numerical Solution of the Mathematical Model

Numerical methods provide an approximation of the model solutions over a finite number of time steps, allowing for the evaluation of the complex cognitive dynamics, time scales of decision making, convergence under various circumstances and responses.

Euler Approximation of Evidence

$$E_{n+1} = E_n + h[\alpha S_n - \beta E_n + \sigma \xi_n] \quad (14)$$

The accumulated evidence at the discrete time points is updated in accordance with equation (14) using the stimulus intensity, evidence decay and cognitive noise and step size.

Euler Approximation of Cognitive State

$$C_{n+1} = C_n + h[\gamma E_n - \delta C_n + \eta M_n] \quad (15)$$

The same applies for equation (15) which also represents an approximation for the cognitive state, involving evidence, memory influence and cognitive decay.

Runge-Kutta First Stage

$$K_1 = f(t_n, X_n) \quad (16)$$

The first Runge-Kutta stage is provided in equation (16) and it is used to determine the first step in the trajectory of the cognitive state for each numerical step.

Fourth-Order Runge-Kutta Solution

$$X_{n+1} = X_n + \left(\frac{h}{6} \right) (K_1 + 2K_2 + 2K_3 + K_4) \quad (17)$$

The fourth order Runge-Kutta method is used to construct the equation (17) which involves four intermediate slope estimates; this helps

to ensure that the numerical accuracy and stability are improved, as compared with the Euler's approach.

$$X(t) = X(0) + \int^0 f(\tau, X(\tau), p) d\tau \quad (18)$$

$$X_{n+1} = X_n + \left(\frac{h}{2}\right) [f(t_n, X_n) + f(t_{n+1}, X_{n+1})] \quad (19)$$

The dynamics of cognition are represented in (18) in integral form, that is, as the current level of cognition is the sum of the initial state of cognition and the totality of changes over time. Approximation of this accumulation is done using trapezoidal rule in (19) in which the derivative values are taken at consecutive times.

Numerical Approximation Error

$$e_n = |X(t_n) - X_n| \quad (20)$$

Lastly, Equation (20) is a numerical approximation error and it measures the absolute difference between the exact states and computed states. These equations provide a stable simulation, convergence checking, error assessment and trustworthy numerical interpretation of complex decision making processes under different cognitive situations.

C. Computational Algorithm for Decision Prediction

The computational algorithm works in an iterative manner, updating the decision states, checking the threshold conditions, predicting future behavioral choices based on defined decision rules by a systematic integration of the numerical solutions and the decision rule.

Weighted Cognitive Input

$$Z_n = w_{1E_n} + w_{2C_n} + w_{3M_n} - w_{4U_n} \quad (21)$$

To calculate the weighted cognitive input, accumulated evidence, cognitive state, memory influence and uncertainty will be added to each other based on the weighting coefficients of each factor in Equation (21).

Recursive Decision-State Update

$$D_{n+1} = D_n + h(Z_n - \theta) \quad (22)$$

The decision state is updated recursively at each time step as shown in Equation (22) and according to the cognitive input and the pre-defined decision threshold. This formulation allows the continuous tracking of the evolution of decisions while thinking.

Accumulated Decision Evidence

$$A(t) = \int^0 [w_{1E(\tau)} + w_{2C(\tau)} + w_{3M(\tau)} - w_{4U(\tau)}] d\tau \quad (23)$$

The integral in equation (23) is accumulated decision evidence which takes into account the cumulative effect of the cognitive variables over the course of the decision interval.

$$P_n = \frac{1}{[1 + \exp(-kD_n)]} \quad (24)$$

$$C_{D(t)} = \left(\frac{1}{t}\right) \int_0^t P(\tau) d\tau \quad (25)$$

The updated decision state is converted to a probabilistic estimate by applying the logistic transformation to it in equation (24), to represent the probability of selecting a certain decision. The overall confidence is set as a stable measure of confidence and is determined by equation (25), which is obtained by averaging the values of the probability over time.

Prediction Error Function

$$E = \left(\frac{1}{N}\right) \sum_{n=1}^N (Y_n - \hat{Y}_n)^2 \quad (26)$$

Lastly, in Equation (26), prediction error is simply the average squared difference between the actual and predicted decisions. These equations can be used in a systematic way for computational frameworks of cognitive integration, temporal updating, confidence estimation and quantitative evaluation of decision-prediction performance.

IV. Result and Discussion

The simulation results show relations between the cognitive parameters and the decisions predicted, highlighting variations in the way responses behave, the parameters' sensitivity and stability, and convergence to the proposed mathematical framework.

Table 1
Sensitivity Analysis of Cognitive Parameters on Decision Performance

Cognitive Parameter	-20% Decision Accuracy (%)	Baseline Accuracy (%)	+20% Decision Accuracy (%)	Sensitivity Index
Decision Threshold	85.31	89.72	92.46	0.199
Memory Influence	86.42	89.72	91.83	0.151
Stimulus Strength	83.76	89.72	93.54	0.273
Cognitive Bias	88.14	89.72	87.36	0.109
Uncertainty Level	92.18	89.72	84.63	0.211

The sensitivity of the decision performance to variations of major cognitive parameters is shown in Table 1. The highest sensitivity was for stimulus strength with an index of 0.273 that suggests a considerable influence on decision accuracy. The change of the cognitive parameters is compared with the accuracy of the decision making in Figure 2.

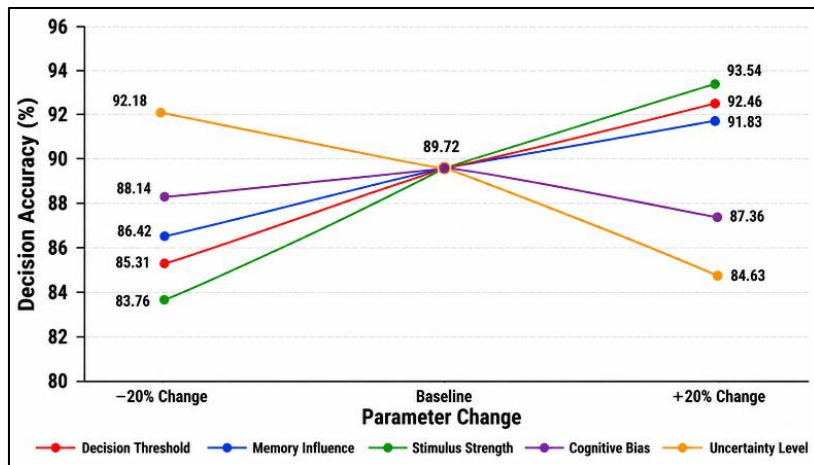


Figure 2

Sensitivity Analysis of Cognitive Parameters on Decision-Making Accuracy

The sensitivity index for decision threshold is 0.199 and for memory influence, the changes in performance are moderate. Cognitive bias has the lowest value for the sensitivity index (0.109) and the higher the number, the less accurate. Overall, the results show that the strength of the stimulus and the decision threshold are the most important parameters to control the model prediction performance.

V. Conclusion

This study introduced a mathematical model of cognitive science decision-making process, which combined the cognitive parameter, differential equation, numerical method and prediction technology of the computing method. It modeled dynamic interaction between evidence accumulation, thresholds, memory influence, stimulus strength and decision uncertainty. The sensitivity analysis was used to identify influential parameters and how changing these parameters can change decision paths and behaviour. A combination of numerical solution methods allowed effective evaluation of cognitive dynamics that analytical solutions might be hard to obtain. In addition, the computational algorithm offered a systematic process of changing cognitive states and making predictions for decisions as conditions changed. The results indicate that mathematical modelling can be a source of useful quantitative information of complex cognitive mechanisms and can help to interpret the decision behaviour. Experimental cognitive information, adaptive parameters, stochastic factors and machine learning methods can be added to the proposed framework in future studies to improve the accuracy of prediction and the applicability of the proposed framework.

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