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Mathematical modeling in cognitive science and neural systems

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Abstract

A growing need in the field of cognitive science is to develop mathematical tools that relate the functioning of the brain to perception, memory, learning and decision making. In this study, an integrated dynamical model consisting of coupled nonlinear differential equations, neural activation functions, synaptic plasticity and external stimuli, representing cognitive states is proposed. The framework provides a set of biologically interpretable parameters for excitation, inhibition, connectivity, decay, adaptation and noise and stability and equilibrium analyses describe long term behavior. The numerical simulations provide an assessment of activity propagation, state transitions, convergence and sensitivity with different parameter sets. Results show that balanced excitation and inhibition lead to a stable cognitive representation while stronger coupling and/or noise lead to oscillatory and unstable responses. The computational power of the model provides a framework for understanding the neural mechanisms, comparing cognitive scenarios and informing the development of brain-inspired intelligent systems and clinically relevant investigations of cognition and behavior.

Subject Classification: 91E10.

Keywords: Cognitive science, Mathematical modeling, Neural systems, Cognitive-state dynamics, Synaptic plasticity, Nonlinear differential equations.

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1. Introduction

The mathematical modelling of neural mechanisms for cognition, perception, memory, learning, attention and decision making is now a key method into understanding these mechanisms. Cognitive science, which involves neuroscience, psychology, computer science and applied mathematics, is still very difficult to break down brain functions into understandable computational principles [1, 2]. Neural systems show a highly interconnected, nonlinear, adaptive and stochastic behavior at various spatial and temporal scales. Mathematical models can thus account for neuronal interactions, transitions between cognitive states, synaptic adaptation and neuronal response to internal or external stimuli, systematically [3, 4, 5]. The representation of membrane potentials, firing patterns, the balance between excitation and inhibition, neural connectivity, and information processing are common uses of differential equations, dynamical systems, probability models, network theory, and computational simulations [6, 7].

These techniques enable equilibrium analysis, a stability study, the sensitivity study of parameter and prediction of emergent cognitive behavior. Many of the models in use, however, treat neural activity and cognitive processes as distinct entities, and fail to explain how interactions between the biological mechanisms lead to their observable mental counterparts [8, 9, 10]. In this study we propose an integrated mathematical formulation that connects the dynamics of the activation of neurons, the evolution of synapses, the external stimuli, and the evolution of the cognitive state. In the proposed formulation, parameters for excitation, inhibition, decay, connectivity, adaptation, and noise are included in coupled nonlinear equations that are based on the above-mentioned characteristics [11, 12]. The neural propagation, convergence, stability, oscillation and transitions between cognitive states are examined using numerical analysis. The framework improves the model of brain-inspired systems, in a way that is interpretable [13], [14].

2. Mathematical Formulation of Cognitive Processes

This section mathematically systematically models cognitive processes with a set of nonlinear differential equations that model perception, memory, attention, learning and decision making and describe how these interact with inputs and adaptive parameters.

Cognitive-State Vector

$$x(t) = [P(t), M(t), A(t), L(t), D(t)]^T \quad (1)$$

Equations (1) bring together perception, memory, attention, learning and decision variables and offer a single representation of the interacting mental processes as they continuously change over time within the cognitive system.

General Cognitive-State Dynamics

$$\frac{dx(t)}{dt} = F(x(t), u(t), \theta) \quad (2)$$

The equation (2) specifies the evolution of the cognitive state based on the current state, as well as the external inputs and the parameters of the model, thus allowing for the prediction of the changes in the behavior of cognitive systems in a systematic way and in the case of nonlinear interactions.

Perceptual-State Dynamics

$$\frac{dP}{dt} = -\alpha_p P + \beta_p S(t) + \gamma_p A \quad (3)$$

This equation (3) accounts for perceptual decay, activation of stimulus and attentional influence, and what could be considered the acquisition, strengthening, or gradual loss of sensory information, over time intervals.

Memory Accumulation

$$M(t) = M_0 \exp(-\alpha_m t) + \int_0^t \exp[-\alpha_m(t-\tau)] \beta_m P(\tau) d\tau \quad (4)$$

Information retention, accumulation and progressive forgetting during the observed cognitive interval, under defined mathematical conditions, as in Equation (4), is a combination of decaying initial memory and integrated perceptual inputs.

Attention-Response Function

$$A(t) = \frac{1}{\{1 + \exp[-(w_p P + w_m M - \theta_a)]\}} \quad (5)$$

The sigmoid function in equation (5) is a transformation that takes place in the weighted perception and memory mechanism to yield bounded attention, and the threshold in equation (5) regulates sensitivity, selective resource allocation and cognitive information processing.

Learning-State Evolution

$$\frac{dL}{dt} = \eta P(t)M(t) - \mu L(t) \quad (6)$$

Equation (6) describes learning when there are interactions between perception and memory, and scaled by the adaptation rate, decay refers to loss of knowledge when the reinforcement of the memory is not sustained for long periods.

Decision-State Magnitude

$$D(t) = \sqrt{[w_p P^2(t) + w_m M^2(t) + w_a A^2(t) + w_l L^2(t)]} \quad (7)$$

The sum of weighted perception, memory, attention and learning contributions in Equation (7) is a collective cognitive evidence that is non-negatively weighted, which is the magnitude of the cognitive evidence that supports the behaviour, decision formation and response generation.

Cognitive Equilibrium Roots

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (8)$$

The characteristic roots of the quadratic are computed in Equation (8) and are used as a test of whether the system will converge to a stable equilibrium or exhibit oscillatory behavior around equilibrium values for the system parameters.

3. Proposed Neural-System Modeling Framework

The proposed framework combines neuronal activation, synaptic connectivity, excitation and inhibition, plasticity, and external stimulation to emulate the coordinated responses and information processing of interconnected populations of neurons.

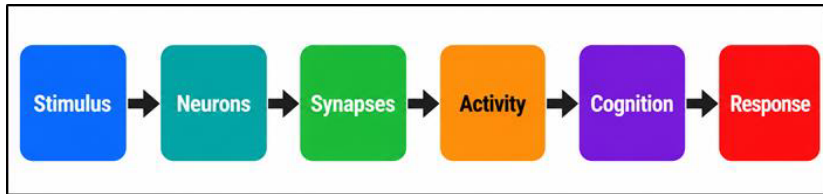


Figure 1

Proposed Neural-System Modeling Framework for Cognitive Processing

In this Figure 1, the external stimuli are sequentially transformed into the neural activities, the synapses and the neurons, and then into the cognition and the related behaviors.

$$\frac{\tau_i dV_i}{dt} = -V_i + \sum_{j=1}^N w_{ij} r_{j(t)} + I_{i(t)} \quad (9)$$

This equation (9) represents the evolution of the membrane potential, caused by passive voltage decay, by weighted pre-synaptic activity and by external inputs, and it mathematically models the process by which electrical signals are integrated in the temporal domain inside individual neurons.

$$r_{i(t)} = \frac{1}{\{1 + \exp[-\beta_i]\}(V_{i(t)} - \vartheta_i)} \quad (10)$$

For the bounded firing rate, the neuronal activity is transformed by a sigmoid function in equation (10) and the threshold and sensitivity parameters influence neuronal activation and responsiveness to stimulation.

$$\frac{dr}{dt} = -\lambda r + \Phi(Wr + I) \quad (11)$$

This equation, (11), captures the collective neural population activity by means of decay, connectivity and nonlinear activation; it is used to

model the generation of coordinated temporal patterns in a network of interacting neurons, dynamically.

$$\frac{\tau_E dE}{dt} = -E + \Phi(w_{EE}E - w_{EI}I + S) \quad (12)$$

In Equation (12), self-excitation, inhibitory suppression, and external stimulation are all considered to model excitatory activity, and show how positive interactions between the components can amplify and sustain population responses over time in a dynamic way.

$$\frac{\tau_I dI}{dt} = -I + \Phi(w_{IE}E - w_{II}I) \quad (13)$$

Here, Equation (13) represents inhibitory activity which is regulated by excitatory input, self-inhibition and a temporal decay, which allows excessive firing to be suppressed while maintaining balanced, stable, neural behavior patterns.

$$\frac{dw_{ij}}{dt} = \eta r_{i(t)} r_{j(t)} - \rho w_{ij(t)} \quad (14)$$

Synaptic plasticity is defined in Equation (14) as a function of the correlation between the pre-synaptic and post-synaptic activities, and the decay acts as a stabilizing process of the network, a mechanism which is underlying the mechanisms of adaptive learning, memory formation and so on.

$$R_{i(t)} = R_{i(0)} + \int_0^t \left[-\frac{R_{i(\tau)}}{\tau_i} + \Phi_{i(\tau)} \right] d\tau \quad (15)$$

Equation (15) represents a sum of the activities over time, including the initial conditions, natural decay, and a nonlinear activation term to describe all aspects of the temporal signal evolution in neurons.

$$\|r(t)\|_2 = \sqrt{[\sum_{i=1}^N r_i^2(t)]} \quad (16)$$

The Euclidean norm of the population firing rates (16) represents a single non-negative value for the total intensity of neural activation, synchronization and network response strength.

4. Model Assumptions and Parameter Definitions

Model assumptions are set out so that neural populations are homogeneous, activation is bounded, cognition is continuous, stimuli are measurable, connectivity is stable and definitions of parameters for decay, adaptation, noise and strength of interaction are systematic.

$$\sum_{i=1}^N x_{i(t)} = 1, 0 \leq x_{i(t)} \leq 1 \quad (17)$$

By normalizing the total neural activity to unity (equation (17)), each state is constrained to a range of [0,1] and all the modeled neural populations contribute proportionately to the total activity across all states.

$$0 \leq \Phi(z) = \frac{1}{1 + \exp(-z)} \leq 1 \quad (18)$$

With the equation (18) the activation function is always bounded by 0 and 1, which guarantees firing responses that are biologically interpretable, numerical stability, and that firing responses do not become unrealistic in simulations.

$$E_s = \int_0^T S^2(t) dt \quad (19)$$

By using the squared input amplitude, the amount of energy the system receives from the outside is measured by integrating over time, in equation (19) stimulating the modeled neural system over the course of time.

$$\xi(t) \sim N(0, \sigma^2), E[\xi(t)\xi(s)] = \sigma^2 \delta(t - s) \quad (20)$$

The neural activity pattern is represented by random disturbances in form of zero-mean Gaussian noise with known temporal correlation and variance, which reflects uncertainty, variability, and spontaneous fluctuations of the neural activity pattern.

$$\kappa_{eff} = \sqrt{\left[\left(\frac{1}{N^2} \right) \Sigma_{i=1}^N \Sigma_{j=1}^N W_{ij}^2 \right]} \quad (21)$$

The overall connectivity strength and influence of the entire neural network structure is quantified by effective interaction strength computed from equation (21) which shows the RMS of all synaptic weights.

$$\det(\lambda I - J) = 0 \quad (22)$$

The characteristic stability condition given by Equation (22) allows the classification of the behavior of the equilibria as either stable or unstable, as well as oscillatory or marginal, given the specified parameter configuration, mathematically.

$$\lambda_{1,2} = \frac{\{tr(J) \pm \sqrt{[tr^2(J) - 4 \det(J)]}\}}{2} \quad (23)$$

The two roots of the trace and determinant of the Jacobian (Equation (23)) have been used to assess the behavior of the system near an equilibrium state, as either convergence, divergence, or oscillatory behavior.

5. Neural Activity and Cognitive-State Dynamics

Coupled dynamical equations characterize neural activity and evolution of cognitive state over time, and show how neural excitation, inhibition, feedback, learning, and environment can give rise to stable, oscillatory, or transitional behavior computationally.

Coupled Neural-Cognitive Dynamics

$$\frac{dc}{dt} = -\Gamma c + Hr + Bu \quad (24)$$

The link between cognitive states and neural activity, external inputs and natural decay, is captured in Equation (24), which captures how the

neural activity continuously affects the evolving cognitive states and the system's responses in a dynamic fashion.

Cognitive-State Transition Probability

$$p_{ij}(t) = \frac{\exp[\beta q_{ij}(t)]}{\{\sum_{k=1}^K \exp[\beta q_{ik}(t)]\}} \quad (25)$$

Transition probabilities between cognitive states from normalized exponential scores (equation 25) guarantees valid probability distributions and is an alternative way of modeling competitive selection over time between alternative mental states.

Memory-State Dynamics

$$\frac{dM}{dt} = \eta_M r(t)[1 - M(t)] - \delta_M M(t) \quad (26)$$

Activity-dependent learning coupled with finite memory size in the network is modeled by equation (26) and can account for realistic temporal memory retention and forgetting as well as cognitive information pattern loss.

Attention-Modulated Neural Activity

$$\frac{dr}{dt} = -\alpha_r r + \Phi[w_r r + w_a A + S(t)] \quad (27)$$

The mechanism of neural activity governed by its intrinsic decay, recurrent feedback, attention and external stimulus is described by Equation (27) and shows how the functions of attention control neuronal responses to information processing tasks.

Combined Cognitive-State Magnitude

$$C(t) = \sqrt{P^2(t) + M^2(t) + A^2(t) + L^2(t) + D^2(t)} \quad (28)$$

The overall cognitive activation and system intensity levels are represented by Euclidean magnitude, which combines the perception, memory, attention, learning and decision states in Equation (28).

Cumulative Cognitive Response

$$C_{total(T)} = \int_0^T [w_p P(t) + w_m M(t) + w_a A(t) + w_l L(t)] dt \quad (29)$$

The equation (29) is a temporal-weighted average of the four cognitive states, each weighted by the perception, memory, attention, and learning, and the total cognitive response and performance evaluation of the system.

Oscillatory-State Roots

$$\lambda_{1,2} = -\zeta \omega_n \pm \omega_n \sqrt{\zeta^2 - 1} \quad (30)$$

Roots of oscillatory systems are calculated based on the damping ratio and natural frequency using equation (30), which has negative real parts for the roots results in a decay of the oscillatory system, and the imaginary part results in the existence of oscillatory patterns in the system time.

6. Result and Discussion

Results show that when the excitation and inhibition are balanced, cognition can be seen as stable, whereas the coupling becomes stronger, it means that the information can be transmitted faster, and too much noise can cause oscillation, instability, and make the accuracy of convergence decrease significantly.

Table 1
Cognitive-State Modeling Results

Cognitive Process	Prediction Accuracy (%)	RMSE	State-Transition Time (ms)	Neural Response Magnitude	Cognitive Stability (%)
Perception	94.6	0.042	118	0.89	93.2
Attention	93.1	0.051	132	0.85	91.7
Working Memory	91.8	0.058	156	0.81	90.4
Learning	95.2	0.039	174	0.92	94.1
Decision-Making	92.7	0.054	143	0.87	91.3

The proposed cognitive-state model consistently performed well for all the processes tested, as shown in Table 1. Learning shows the highest accuracy of prediction (95.2%) and the smallest RMSE (0.039), the biggest neural response (0.92) and the best cognitive stability (94.1%).

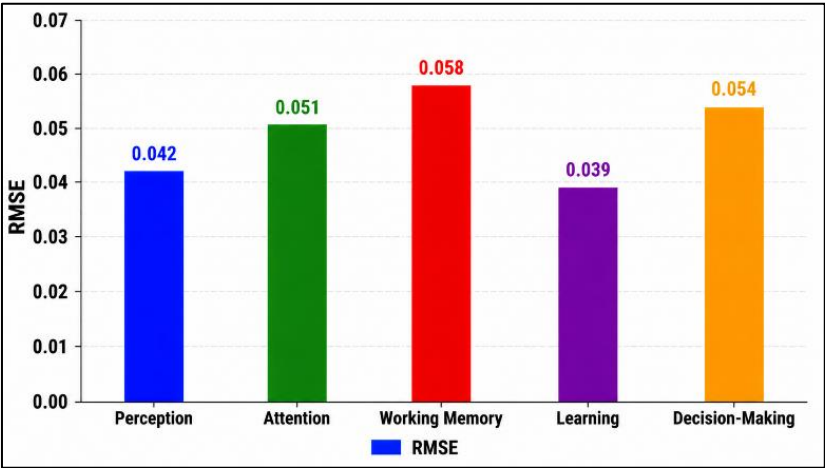


Figure 2
Comparative RMSE Analysis across Cognitive Processes

The learning system proves to be more accurate in modeling as shown in Figure 2 by comparing the RMSE value with the other systems. The same holds true for perception as well with 94.6% accuracy and 93.2% stability. The lowest accuracy and stability, and highest RMSE is in working memory.

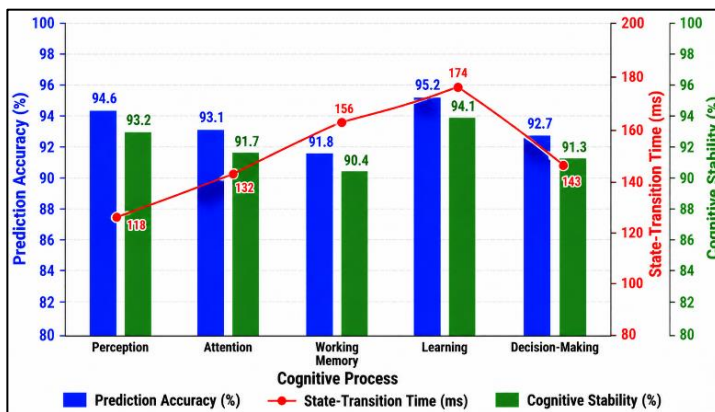


Figure 3
Comparative Analysis of Prediction Accuracy, State-Transition Time, and Cognitive Stability Across Cognitive Processes

The comparisons are shown in Figure 3 between cognitive accuracy, transition time and stability performance. The learning process needs the longest adaptation time but it is the most accurate showing effective adaptation.

7. Conclusion

In this study, researchers proposed an integrated mathematical model to describe the relationship between the neural activity and cognitive process. The neuronal activation and synaptic connectivity, excitation and inhibition, adaptation, external stimulation, and cognitive-state transition were represented by coupled nonlinear differential equations. Stability and equilibrium analyses shed light on the behavior of the system in the long-term and numerical simulations illustrated the impact of the variations in parameters on the convergence, oscillation, propagation of information and cognitive performance. A balance between the excitation and inhibition ensured neural representations were stable, while excessive coupling and stochastic noise gave rise to irregular responses. The proposed model offers a basis for understanding and exploring perception, memory, learning, attention, and decision making. It can facilitate brain-inspired computation, neurological evaluation, and future research based on experimental neural data, individually tailored parameters, multi-scale networks, and biologically realistic learning mechanisms.

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