

Dynamical systems modeling for epidemic spread and control

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Abstract

Epidemic outbreaks are characterized by complex nonlinear dynamics, with the effects of transmission, recovery, population interactions and interventions. This study introduces a framework for the modelling of epidemics and its assessment of disease control mechanisms based on dynamical systems. The susceptible, exposed, infected and recovered populations are modeled using coupled nonlinear differential equations. A time-varying transmission and recovery rates are indicative of changes in the contact patterns, health care response and control measures. To determine the long-term epidemic behavior, disease-free and endemic equilibrium points are determined. Stability analysis offers insight into about conditions necessary for the elimination of a disease or for its persistence. The framework facilitates assessment of transmission reduction, recovery enhancement, and epidemic stability and offers a flexible mathematical basis to learn about outbreaks and to devise effective public health control measures and policies.

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I. Introduction

Infectious disease outbreaks are still a public health problem due to the complex interactions between populations, environmental conditions, individual response, and intervention strategies that can impact the spread of infectious disease [1, 2]. A useful way of understanding these interactions and predicting the spread of epidemics over time is to use mathematical modelling. Of the methods available, dynamical systems modeling is especially useful in that it captures epidemic dynamics in terms of interconnected states of the population that are modeled with differential equations [3, 4, 5]. Conventional compartmental models distinguish between different categories of individuals in a population, such as susceptible, exposed, infected and recovered. While these models can give useful information, it may be that in more realistic outbreaks with fluctuating epidemic parameters, these will be underrepresented [6, 7]. A variety of contact behaviour, prevention, health services, vaccination, isolation and treatment interventions can have a major effect on epidemic trends. Thus, it is desirable to include non-linear interactions and parameters that vary with time to better represent disease dynamics [8, 9]. In this study, a Susceptible–Exposed–Infected–Recovered model is used to describe and control the spread of epidemics and a dynamical systems framework is developed. The formulation proposed includes time varying transmission and recovery rates and nonlinear differential equations to account for the evolution of the epidemic conditions. The persistence and elimination conditions are examined by studying the disease-free and endemic equilibria [10, 11]. Different population evolution scenarios are studied with the help of stability analysis and numerical simulation for varying parameter and intervention scenarios. The framework is the mathematical foundation of the interpretation of the behavior of epidemics, the assessment of control measures, and the informed decision making for disease management [12].

II. Mathematical Formulation of Epidemic Dynamics

Systematic use of compartmental populations and nonlinear differential equations to mathematically model the epidemic dynamics allows the analysis of disease transmission, progression, recovery, equilibrium behavior and the effects of intervention.

A. Model Assumptions and Parameters

The model is a homogeneous population with compartments representing the different disease states and transmission, incubation, recovery, mortality and intervention rates governing transitions between disease states in a dynamic manner [13], [14].

$$N(t) = S(t) + E(t) + I(t) + R(t) \quad (1)$$

In the equation (1), the total population consists of the susceptible, exposed, infected and recovered people, which provides the fundamental population balance that will be used in the mathematical analysis of the dynamics of an epidemic.

$$\beta_{eff(t)} = \beta^0 [1 - \eta u(t)] \quad (2)$$

The effective contact rate represented by equation (2) reflects the impact of the intervention intensity, and reflects reductions in infectious contacts due to control measures implemented.

$$R^0 = \frac{\beta\sigma}{[(\mu + \sigma)(\mu + \gamma)]} \quad (3)$$

$$R_e(t) = R^0 \left[\frac{S(t)}{N(t)} \right] \quad (4)$$

The basic reproduction number is estimated by (3) which reflects the average number of secondary infections that can be expected to be produced by the initial infected person in an entirely susceptible population in the presence of the epidemic. The remaining susceptible population is included in equation (4), allowing the evaluation of the effectiveness of transmission growth, stability or reduction as time goes on, in a dynamic way.

$$\Lambda(t) = \int_0^t \left[\frac{\beta(\tau)I(\tau)}{N(\tau)} \right] d\tau \quad (5)$$

The first order of the equation (5) sums up the intensity of transmission for the time in question and captures the cumulative exposure of the susceptible population for a period of the considered epidemic.

$$\Psi = \sqrt{\beta^2 + \sigma^2 + \gamma^2 + \mu^2} \quad (6)$$

It is worth noticing that the parameters of transmission, incubation, recovery and mortality are integrated into a single mathematical quantity by the root-based formulation in equation (6), that quantifies the overall strength of the epidemic dynamics.

B. System of Nonlinear Differential Equations

The coupled nonlinear differential equations model changes in the susceptible, exposed, infected and recovered classes in time, along with the interactions between the classes of infected individuals, their progression until they recover, and the transitions among the classes over time.

$$\frac{dS}{dt} = \Lambda - \left[\frac{\beta(t)S(t)I(t)}{N} \right] - \mu S(t) \quad (7)$$

The changes in susceptible individuals are defined by population recruitment, infectious contact and natural mortality in equation (7), showing a progressive loss of susceptible individuals over time due to disease transmission.

$$\frac{dE}{dt} = \left[\frac{\beta(t)S(t)I(t)}{N} \right] - (\sigma + \mu)E(t) \quad (8)$$

The exposed individuals, which are created due to infectious interactions and then moved toward infection, are represented by equation (8) and natural mortality during the incubation period of the disease is expressed as a dynamic process.

$$\frac{dI}{dt} = \sigma E(t) - [\gamma(t) + \mu + \delta]I(t) \quad (9)$$

The change in the number of infected people is shown in (9) as the sum of the number of individuals that become infected and the number of individuals that recover, die naturally, or because of the disease, during the course of the epidemic.

$$\frac{dR}{dt} = \gamma(t)I(t) - \mu R(t) \quad (10)$$

In equation (10) the success of recovery of infected people is represented as growth of the recovered population, and natural mortality is assumed to slowly reduce the number of recovered people in the population.

$$I(t) = I(0) + \int_0^t [\sigma E(\tau) - (\gamma(\tau) + \mu + \delta)I(\tau)] d\tau \quad (11)$$

The differences between newly progressing infections and removals due to recovery and mortality are summed over the observation period considered in equation (11) to express the evolution of the infected population.

$$D(t) = \sqrt{\left[\left(\frac{dS}{dt}\right)^2 + \left(\frac{dE}{dt}\right)^2 + \left(\frac{dI}{dt}\right)^2 + \left(\frac{dR}{dt}\right)^2\right]} \quad (12)$$

The rates of change of all SEIR compartments are combined into a measure of the overall magnitude and intensity of the evolution of the epidemic state in equation (12), which is based on the root.

C. Disease-Free and Endemic Equilibria

Complete elimination of disease is termed disease-free equilibrium while when there is continuous transmission of the disease, it is termed endemic equilibrium. Their occurrence and endurance create conditions for the proper forecasting of the future behavior of epidemics.

Figure 1 shows Disease-Free and Endemic Equilibrium Stability Analysis Framework for Epidemic Dynamics, infected and recovered paths and their stability conditions in the context of epidemic dynamics modelling.

$$E^0 = \left(\frac{A}{\mu}, 0, 0, 0\right) \quad (13)$$

The conditions of the disease-free equilibrium, in which the number of exposed and infected individuals is zero, is represented by the right-hand side of equation (13), corresponding to successful eradication of all epidemic transmission.

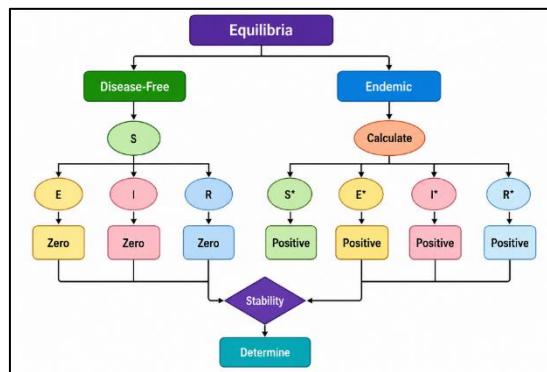


Figure 1

Disease-Free and Endemic Equilibrium Stability Analysis Framework for Epidemic Dynamics

$$S = \frac{N(\sigma + \mu)(\gamma + \mu + \delta)}{(\beta\sigma)} \quad (14)$$

The parameters of transmission, progression, recovery and mortality are used to determine the susceptible population at endemic equilibrium in equation (14) which reflects susceptibility levels when the infection is maintained in the population.

$$E = \left[\frac{(\gamma + \mu + \delta)}{\sigma} \right] I \quad (15)$$

The ratio of exposed population to infected population at equilibrium, as given in equation (15), is linked to the rate of disease progression, recovery and mortality rates when infections are sustained in communities.

$$B^* = \int_0^T [I^*(t) + E^*(t)] dt \quad (16)$$

This is equation (16) used to integrate the exposed and infected populations over the time period of the observation, showing a cumulative disease measure in terms of the epidemic or endemic situation, quantitatively.

$$\lambda = \frac{[-a^1 \pm \sqrt{a^{12} - 4a^0 a^2}]}{(2a^2)} \quad (17)$$

The equilibrium stability equation is used in equation (17) to calculate the characteristic roots; negative real components mean that the behavior is stable, whereas positive real components mean that the behavior is unstable and the epidemics will continue to develop.

III. Proposed Dynamical Systems Modeling Framework

The proposed framework combines compartmental epidemic dynamics, nonlinear mathematical models, time-varying epidemiological parameters, equilibrium analysis, stability analysis, and numerical

simulation to explore the dynamics of diseases and strategies to control them.

A. Susceptible–Exposed–Infected–Recovered Dynamics

The SEIR dynamics illustrate the dynamics of the population going from susceptible to exposed to infected to recovered, which includes incubation effects, transmission interactions, infectious progression, and resolution of the disease over time.

$$\frac{dS}{dt} = \Lambda - \left[\frac{\beta SI}{N} \right] - \mu S \quad (18)$$

Susceptible population variation due to recruitment, infectious transmission and natural mortality is captured in equation (18) as susceptible population decreases with the increase of new exposures from infected individuals.

$$\frac{dE}{dt} = \left[\frac{\beta SI}{N} \right] - (\sigma + \mu)E \quad (19)$$

The changes in exposed individuals in the epidemiological incubation stage that relate to disease transmission and reduction from progression to infection and natural mortality are represented by equation (19).

$$\frac{dI}{dt} = \sigma E - (\gamma + \mu + \delta)I \quad (20)$$

Equation (20) depicts the evolution of infected population as a progression from the exposed population, and the recovery, natural mortality and disease-related mortality all contribute to the decrease in active infection during the epidemic development.

$$R(t) = R(0) + \int_0^t [\gamma I(\tau) - \mu R(\tau)] d\tau \quad (21)$$

The recovered population is recovered cumulatively by integrating up recovered gains and losses of individuals from infection to mortality, as given by equation (21), and is in this sense a temporal accumulation of individuals successfully recovered from infection.

$$C(t) = \int_0^t [E(\tau) + I(\tau)] d\tau \quad (22)$$

Integrating over time exposed and infected populations as in equation (22) yields an overall measure of the burden of the epidemic, which is useful because it accounts for the populations exposed and infected during the course of an epidemic.

$$M(t) = \sqrt{[S^2(t) + E^2(t) + I^2(t) + R^2(t)]} \quad (23)$$

The overall epidemiological system state at time can be measured by an integrated quantitative measure, which is the sum of the susceptible, exposed, infected and recovered population, and is combined by equation (23) by a root based magnitude.

B. Time-Varying Transmission and Recovery Rates

Time varying transmission and recovery rates reflect dynamic changes in the transmission and recovery rates caused by changes in contact patterns, changes in healthcare interventions, changes in the effectiveness of those interventions, changes in behaviour and changes in control measures, and allow realistic modelling of changes over time in epidemic conditions.

$$\beta(t) = \beta^0[1 + \alpha \sin(\omega t)] \quad (24)$$

The periodic fluctuations in the transmission of disease from equation (24) is a good model for the fluctuations that are observed as a result of behavioral, environmental or intervention related factors, and it fits the temporal fluctuations well because it uses baseline transmission, variation amplitude, and frequency as parameters.

$$\gamma(t) = \gamma^0[1 + \rho \cos(\omega t)] \quad (25)$$

The temporal variation of recovery, as in equation (25), represents recovery governed by a baseline rate with periodic modulation, which accounts for fluctuations in treatment effectivity, the healthcare capacity and recovery conditions during epidemics.

$$\bar{\beta}(t) = \left(\frac{1}{t}\right) \int_0^t \beta(\tau) d\tau \quad (26)$$

The average transmission rate (by equation (26)) is a representative measure of overall infectious contact intensity over the course of an epidemic, being an integral of time dependent transmission over the observation period.

$$\Gamma(t) = \int_0^t \gamma(\tau) I(\tau) d\tau \quad (27)$$

$$R_t = \frac{\beta(t)S(t)}{[N(\gamma(t) + \mu + \delta)]} \quad (28)$$

By multiplying recovery rates with infected populations, equation (27) can be used to quantify the number or influence of recovered individuals over time as the epidemic develops, adding up recovery activity over time. Incorporating transmission, susceptibility, recovery and mortality factors in Equation (28) gives an estimate of the reproduction potential of the infection as it changes with time, and shows if the infections are increasing or decreasing as the epidemic evolves.

$$T(t) = \sqrt{\left\{ \left[\frac{\beta(t)}{\beta^0} \right]^2 + \left[\frac{\gamma(t)}{\gamma^0} \right]^2 \right\}} \quad (29)$$

The normalized transmission and recovery rates are combined using the root form of equation (29), which results in an integrated index to assess the overall effects of transmission and recovery on altering epidemic dynamics over time.

IV. Result and Discussion

The variations in susceptible, exposed, infected and recovered populations are shown in the simulation results when the parameters are changed. Minimizing transmission and maximizing recovery helps lower peaks of transmission and helps control epidemics.

Table 1
Epidemic Dynamics Under Different Transmission-Rate Scenarios

Scenario	Transmission Rate β	Recovery Rate γ	Peak Infected (%)	Peak Time (Days)	Final Recovered (%)	Basic Reproduction Number R_0
S1	0.20	0.10	12.4	78	68.5	2.00
S2	0.25	0.10	18.7	65	76.8	2.50
S3	0.30	0.10	25.6	54	83.2	3.00
S4	0.35	0.10	31.8	46	87.6	3.50
S5	0.40	0.10	37.9	39	91.3	4.00

As shown in Table 1, the dynamics of an epidemic are significantly increased if there is a high transmission rate. The peak infection increases from 12.4% to 37.9% as β is raised from 0.20 to 0.40 and peak time decreases from 78 to 39 days. In Figure 2, transmission rates are shown, with the intensity of the epidemics increasing with the scenarios.

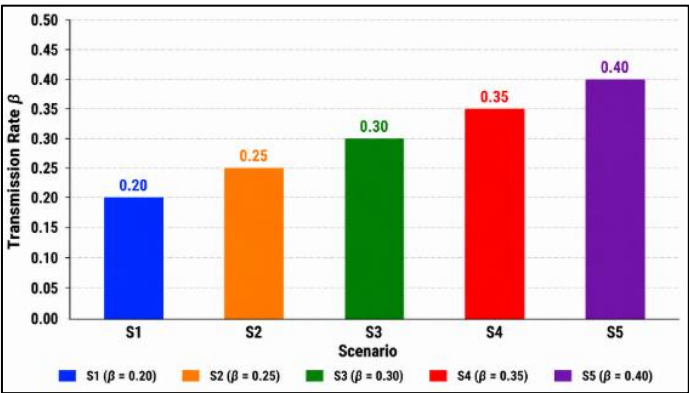


Figure 2
Comparative Analysis of Transmission Rate across Epidemic Transmission Scenarios

The final recovered population is also increased by 68.5%, to 91.3%. In addition, R_0 rises from 2.00 to 4.00, meaning the spread of the disease becomes more and more effective.

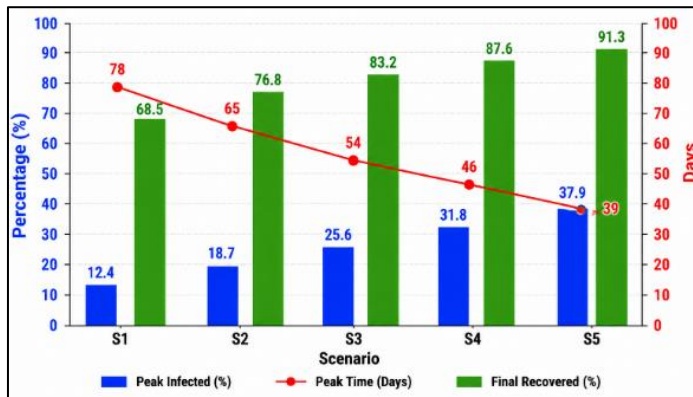


Figure 3

Comparative Analysis of Peak Infection, Peak Time, and Final Recovery Across Epidemic Transmission Scenarios

The infection peaks, timing and recovery are compared across Scenarios in Figure 3. The results support that higher transmission leads to larger peaks of infections, larger overall disease burden and leads to an increased epidemic spread.

V. Conclusion

This study proposed a framework of Dynamical Systems modeling to explain the dynamics of epidemics and test the effectiveness of the disease control measures. The SEIR formulation proposed was based on the interactions of susceptible, exposed, infected, and recovered populations using nonlinear differential equations. By allowing for time varying rates of transmission and recovery, it was possible to accurately describe different epidemic conditions and intervention effects. Equilibria (disease-free and endemic) and stability analysis were used to gain insights into disease elimination and persistence and long-term epidemic behavior. The numerical simulations showed that decreasing transmission rates and increasing recovery rates can significantly decrease the growth of infections and the severity of the epidemic. In general, the developed framework is a flexible mathematical tool to understand the dynamics of an epidemic, compare different scenarios of intervention and aid to the public health planning and management of the disease.

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