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On new type of Δ-open sets

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Abstract

We initiated a generalization of Δ-open sets, namely pre Δ-open set. Indeed, we studied the following notions such as pre Δ-interior, and pre Δ-closure and discussed their properties. Furthermore, we utilize this concept to provide certain types of functions which known as faintly pre Δ continuous, several characterizations about this function are obtained and some relations with other known functions are discussed.

Subject Classification: *Primary 93A30, Secondary 49K15.*

Keywords: *Pre Δ-open set, Pre Δ-interior, Pre Δ-closure, Faintly pre Δ continuous.*

1. Introduction

Since Mashhour, El-Monsef and El-Deeb, [1] proposed the conception of pre-open, it has been widely researched in number of publications see [2], [3], [4]. In 2020, Ganesan [5] explored a new type of Δ- open namely $\alpha\Delta$-open. Also, he created new type of generalized closed known as generalized Δ- closed yields to remarkable results about this concept. Nour and Jaber [6] examined a

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semi Δ- open as a generalization of Δ- open sets and studied the properties of semi-Δ interior and semi-Δ closure. Recently, Marabeh [7] gave analogue to Δ -exterior, Δ-derived, Δ-boundary and Δ-dense besides to Δ-interior and Δ-closure. In this article, we initiate a novel type of Δ- open namely pre Δ- open such as a numerous characterizations of this set are emphasized. Also, several characterizations about Δ- open and Δ-closed product topology and subspace topology are provided. As further works, we apply this concept to different spaces see [10-13].

2. Preliminaries

Definition 2.1: If we have the space(X,τ) . then set K in X named

1) Semi-open [8], [10] in case that $K \subseteq c\text{ѣ}\ \text{ᶖʃ}t\ (K)$.
2) Pre-open [1], in case that $K \subseteq \text{ᶖʃ}t\ c\text{ѣ}\ (K)$.

Definition 2.2: [9] A contained set K for the space (X,τ) is θ-open, where for all $\text{ѣ} \in K$, $\exists$ open set H s.t $\text{ѣ} \in B \subseteq c\text{ѣ}(B) \subseteq K$.

Definition 2.3: [5] A contained set K of a space (X,τ) called Δ- open where $\exists$open L_1 & L_2 s.t $K = L_1 \Delta\, L_2$ & the assortment for each Δ- open subsets for X is signified as $\Delta O(X)$.

Definition 2.4: [6] the contained set K of the space (X,τ) called semi Δ- open set where $\exists$ semi-open sets H_1 & H_2 s.t $K = (H_1 - H_2) \cup (H_2 - H_1)$ and the assortment for each semi Δ- open subset for X have been signified by $S\Delta O(X)$.

Definition 2.5: [10] if we have a space (X,τ) & $H \subseteq X$ is known Pg-closed if $Pc\text{ѣ}(H) \subseteq W$, where $H \subseteq W$ & $W \in PO(X)$.

Proposition 2.6: [10] the space (X,τ) called pre $T_{\frac{1}{2}}$- space iff each singleton either be pre-open or be pre-closed.

Definition 2.7: [11] the space (X,τ) is asserted to be θ-connected where two nonempty disjoint θ-open sets cannot be united to form X.

3. Pre Δ $-$open set

This section displayed the novel concept known as Pre Δ-open set and its relations with other well-known sets had been discussed. In addition, properties of $p\Delta$-closure and $p\Delta$-interior had been obtained.

Definition 3.1: the subsets K in the space (M,τ) named pre Δ-open set where $\exists$two pre-open sets L_1 & L_2 s.t $K = (L_1 - L_2) \cup (L_2 - L_1)$. Also, the assortment for each pre Δ- open subset in X be signified by $P\Delta O(M)$.

Example 3.2: Assume that $M = \{\varkappa, \text{ʃ}, \text{ᶖ}, \text{ɋ}\}$ equipped with a topology $\tau = \{\phi, M, \{\varkappa, \text{ʃ}, \text{ᶖ}\}, \{\text{ʃ}, \text{ᶖ}, \text{ɋ}\}, \{\text{ɋ}\}, \{\text{ʃ}, \text{ᶖ}\}\}$, then $O(M) = \{\phi,\ M, \{\text{ʃ}\}, \{\text{ᶖ}\}, \{\text{ɋ}\}, \{\varkappa, \text{ʃ}\}, \{\varkappa, \text{ᶖ}\},$

$\{ſ, ʝ\}, \{ſ, q\}, \{c, q\}, \{\varkappa, ſ, ʝ\}, \{\varkappa, ſ, q\}, \{\varkappa, ʝ, q\}, \{ſ, ʝ, q\}\}$. And $P\Delta O(M) = \{\phi, M, \{\varkappa\},$ $\{ſ\}, \{ʝ\}, \{q\}, \{\varkappa, ſ\}, \{\varkappa, ʝ\}, \{\varkappa, q\}, \{ſ, ʝ\}, \{ſ, q\}, \{ʝ, q\}, \{\varkappa, ſ, ʝ\}, \{\varkappa, ſ, q\}, \{\varkappa, ʝ, q\}, \{ſ, ʝ, q\}\}$.

Proposition 3.3: All pre-open is pre Δ- open set.

Proof: if P pre-open subsets in (X, τ). Since P & ϕ are pre-open sets and since $P\Delta\phi = (P - \phi)\cup(\phi - P) = P$, then P is pre Δ- open set.

However, the next example points out the opposite of previous proposition might not be true.

Example 3.4: Let $M = \{\varkappa, ђ, ѣ\}$ equipped with a topology $\xi = \{\phi, X, \{\varkappa\}\}$, then $PO(X) = \{\phi, X, \{\varkappa\}, \{\varkappa, ђ\}, \{\varkappa, ѣ\}\}$ and $P\Delta O(X) = \{\phi, X, \{\varkappa\}, \{ђ\}, \{ѣ\}, \{\varkappa, ђ\},$ $\{ђ, ѣ\}, \{\varkappa, ѣ\}\}$. Clearly, $\{ƃ\} \in P\Delta O(M)$ but $\{ђ\} \notin PO(X)$.

Proposition 3.5: Every pre-closed is pre Δ- open set.

Proof: assume that P is pre-closed in $\mathcal{X}$, so $\mathcal{X} - P$ be pre-open. Since $(\mathcal{X} - P)\Delta\mathcal{X} = [(\mathcal{X} - P)\cup\mathcal{X}] - [(\mathcal{X} - P)\cap\mathcal{X}] = \mathcal{X} - (\mathcal{X} - P) = P$. Hence, $P \in P\Delta O(\mathcal{X})$.

Proposition 3.6: Each Δ-open be pre Δ-open.

Proof: This comes from knowing that every open is pre-open.

A next example points out the opposite of previous proposition might not be true.

Example 3.7: Let $M = \{\varkappa, ђ, ѣ\}$ endowed with topology $\xi = \{\phi, M, \{\varkappa, ђ\}\}$, then $PO(M) = \{\phi, M, \{\varkappa\}, \{ђ\}, \{\varkappa, ђ\}, \{ђ, ѣ\}, \{\varkappa, ѣ\}\}$. Consequently, $P\Delta O(M) = \{\phi, X,$ $\{\varkappa\}, \{ђ\}, \{ѣ\}, \{\varkappa, ђ\}, \{ђ, ѣ\}, \{\varkappa, ѣ\}\}$ and $\Delta O(M) = \{\phi, X, \{ѣ\}, \{\varkappa, ђ\}\}$. Hence $\{\varkappa\} \in P\Delta O(M)$ but $\{\varkappa\} \notin \Delta O(M)$.

Remark 3.8: the intersection of pre Δ- open possibly won't be pre Δ- open set.

Example 3.9: Let $M = \{\varkappa, ƃ, ѣ, y\}$ equipped with a topology $\tau = \{\phi, M, \{\varkappa\}, \{ƃ\}, \{\varkappa, ƃ\}\}$, then the following families are found $PO(X) = \{\phi, X, \{\varkappa\}, \{ƃ\}, \{\varkappa, ƃ\}, \{\varkappa, ƃ, ѣ\}, \{\varkappa, ƃ, y\}\}$ and $P\Delta O(M) = \{\phi, X, \{ƃ\}, \{ѣ\}, \{y\},$ $\{ƃ, ѣ\}, \{ƃ, y\}, \{\varkappa, ѣ\}, \{\varkappa, y\}, \{ѣ, y\}, \{ƃ, ѣ, y\}, \{\varkappa, ѣ, y\}\}$. Clearly, $\{\varkappa, ѣ\}, \{\varkappa, y\} \in P\Delta O(M)$ while $\{\varkappa, ѣ\}\cap\{\varkappa, y\} = \{\varkappa\} \notin P\Delta O(M)$.

Proposition 3.10: Let Ҥ and Ҟ are subsets in X. If Ҥ be open set and Ҟ is pre Δ-open set, then Ҟ∩Ҥ is pre Δ- open set.

Proof: since Ҟ is pre Δ- open, so ∃pre open $\mathcal{N}$ & Ɱ consequently $K = (\mathcal{N} - Ɱ)\cup(Ɱ - \mathcal{N})$. Consequently, $Ҟ\cap Ҥ = [(\mathcal{N} - Ɱ)\cup(Ɱ - \mathcal{N})]\cap Ҥ = [(\mathcal{N} - Ɱ)\cap Ҥ]\cup[(Ɱ - \mathcal{N})\cap Ҥ] = (\mathcal{N}\cap Ҥ - Ɱ\cap Ҥ)\cup(Ɱ\cap Ҥ -$

$\mathcal{N}\cap$Ң). Accordingly, $\mathcal{N}\cap$Ң & Ɱ$\cap$Ң pre-open set for X. Hence, Ꝁ$\cap$Ң is pre Δ-open.

Proposition 3.11: If K be Δ-open & L be pre-closed, then $K-L$ is pre Δ-open set.

Proof: Since K be Δ-open set, so there are two open sets ϖ and Θ so that $K=(\Theta-\varpi)\cup(\varpi-\Theta)$. It follows $K-L=[(\varpi-\Theta)\cup(\Theta-\varpi)]\cap L^c=[(\varpi\cap L^c)\cup(\Theta\cap L^c)]-[(\varpi\cap L^c)\cap(\Theta\cap L^c)]$. so, $\varpi\cap L^c$ and $\Theta\cap L^c$ are pre-open sets in X. Hence, $K-L$ is a pre Δ- open set.

Proposition 3.12: Every singleton set in pre $T_{\frac{1}{2}}$-space is pre Δ- open set.

Proof: Assume (W,ς) is a pre $T_{\frac{1}{2}}$-space, consequently $\{s\}$ be either preopen or be preclosed for each $s\in W$. If $\{x\}$ is pre-open, then, $\{s\}\in P\Delta O(W)$. Otherwise, $\{s\}$ is pre-closed. so, $\{s\}\in P\Delta O(W)$.

Proposition 3.13: if (ρ,ς_ρ) is open subspace for a space (W,ς). If K is pre Δ-open in Y, then K is pre Δ- open in X.

Proof: Since $V\in P\Delta O(\rho)$, then $V=H\cap\rho$ for some $H\in P\Delta O(W)$. Since ρ be open , so we have $V\in P\Delta O(W)$.

Proposition 3.14: Assume that (ρ,ς_ρ) is open sub space for topological space (W,ς) & $\mathrm{T}\subseteq\rho$. If $\mathrm{T}\in P\Delta O(W)$, then $\mathrm{T}\in P\Delta O(\rho)$.

Proof: Since $\mathrm{T}\in P\Delta O(W)$, so $\exists$ two pre-open set H & L in W s.t $\mathrm{T}=(S-\mathcal{L})\cup(\mathcal{L}-S)$. It follows $\mathrm{T}=\mathrm{T}\cap\rho=[(S-\mathcal{L})\cup(\mathcal{L}-S)]\cap\rho=[(S-\mathcal{L})\cap\rho]\cup[(\mathcal{L}-S)\cap\rho]=[S\cap\rho-\mathcal{L}\cap\rho]\cup[\mathcal{L}\cap\rho-S\cap\rho]$ where $S\cap Y$ and $\mathcal{L}\cap\rho$ are pre-open set in ρ.

Definition 3.15: the subset K in (W,ς) named pre Δ-closed where $X-K$ be pre Δ-open.

Definition 3.16: Assume that (W,ς) be a topological space & $K\subseteq M$, so union for each pre Δ- open in X embedded in K is named as pre Δ-interior for K which be denoted as $p\Delta$ịʄ$t(K)$.

Definition 3.17: if we have the space (W,ς) & $K\subseteq X$, a intersected for every pre Δ-open in X. A superset K is named as pre Δ-closure of K which is denoted by $p\Delta c$ƀ(K).

Proposition 3.18: In pre $T_{\frac{1}{2}}$-topological space, (W,ς),$T=p\Delta$ịʄ$t(T)$

Proof: Assume that $T \subseteq W$, then $p\Delta\mathfrak{i}\mathfrak{f}t(T) \subseteq T$. Let $w \in T$. Since (W, ς) is pre $T_{\frac{1}{2}}$-space, then $\{w\}$ is pre Δ-open. hence $w \in \{w\} \subseteq T$. Consequently, $x \in p\Delta\mathfrak{i}\mathfrak{f}t(T)$. Hence, $T = p\Delta\mathfrak{i}\mathfrak{f}t(T)$.

Definition 3.19: if (W, ς) is space & $T \subseteq W$, then T named pre Δ-dense where $p\Delta c\text{ъ}(T) = W$.

Proposition 3.20: Every proper subset of pre $T_{\frac{1}{2}}$-space is not pre Δ-dense set.

Proof: Given T a proper subset of pre $T_{\frac{1}{2}}$-space$W$, then by $p\Delta c\text{ъ}(T) = T \subset W$ which not the pre Δ-dense.

Definition 3.21: if we have the space(W, ς), then $T \subseteq W$, is pre Δ no where dense set where $p\Delta\mathfrak{i}\mathfrak{f}t\left(p\Delta c\text{ъ}(T)\right) = \phi$.

Proposition 3.22: Every proper subset of pre $T_{\frac{1}{2}}$-space is not pre Δ nowhere-dense set.

Proof: if O is the set in pre $T_{\frac{1}{2}}$-topological space, W, so $p\Delta\mathfrak{i}\mathfrak{f}t\ (p\Delta c\text{ъ}(O)) = p\Delta\mathfrak{i}\mathfrak{f}t\ (O) = O \neq \phi$. Hence, K is not pre Δ nowhere-dense set.

Proposition 3.23: Given O be pre Δ- open subset of space (W, ς) and let $T \subseteq O$. If T is pre Δ nowhere-dense in O, then T is pre Δ nowhere-dense in W.

Proof: Assume that T is pre Δ nowhere-dense in O, and suppose that T is not pre Δ nowhere-dense in W, then $x \in p\Delta\mathfrak{i}\mathfrak{f}t\left(p\Delta c\text{ъ}(T)\right)$. Accordingly, $P \in P\Delta O(W)$ with $x \in P \subseteq p\Delta c\text{ъ}(T)$. It follows $P \cap O \subseteq p\Delta c\text{ъ}(T) \cap O$. Since T is pre Δ nowhere-dense in O, then preΔ-interior of $p\Delta c\text{ъ}(T) \cap O$ is empty set. consequently, $P \cap O = \phi$. This implies $x \notin p\Delta c\text{ъ}(O)$ and so $x \notin p\Delta c\text{ъ}(T)$.

Proposition 3.24: Let (W, ς) be a topological space and $O \subseteq W$, then O is pre Δ nowhere-dense set in W iff $p\Delta c\text{ъ}\left(p\Delta\mathfrak{i}\mathfrak{f}t(W - O)\right) = W$.

Proof: Given O is pre Δ nowhere-dense iff $p\Delta\mathfrak{i}\mathfrak{f}t\left(p\Delta c\text{ъ}(O)\right) = \phi \Leftrightarrow W - p\Delta\mathfrak{i}\mathfrak{f}t\left(p\Delta c\text{ъ}(O)\right) = X \Leftrightarrow p\Delta c\text{ъ}\left(W - p\Delta c\text{ъ}(O)\right) = W \Leftrightarrow p\Delta c\text{ъ}\big(p\Delta\mathfrak{i}\mathfrak{f}t(W - O)\big) = W$.

4. Faintly pre Δ Continuous

This section emphasizes on new type of mapping by using pre Δ-open namely faintly pre Δ continuous such as several characterizations had been investigated.

Definition 4.1: A mapping $\zeta : (W, \varsigma) \longrightarrow (Y, \rho)$ is named to be faintly pre Δ continuous if $\zeta^{-1}(T) \in p\Delta O(W, \varsigma)$ for every $T \in \theta O(Y, \rho)$.

Example 4.2: Let $W = \{\varkappa, \mathfrak{b}, ƀ\}$ with $\varsigma = \{\phi, X, \{\varkappa, \mathfrak{b}\}\}$ and $Y = \{q, v, c\}$ equipped with $\rho = \{\phi, Y, \{q\}, \{v, c\}\}$. Construct a function ζ from (X, τ) into (Y, ρ) by $\zeta(\varkappa) = q$, $\xi(\mathfrak{b}) = v$ and $\xi(ƀ) = c$, then ζ is slightly pre Δ continuous.

Definition 4.3: A function $\zeta : (W, \varsigma) \longrightarrow (Y, \rho)$ is said to be faintly Δ continuous iff $\zeta^{-1}(T) \in p\Delta O(W, \varsigma)$ for every $T \in CLO(Y, \rho)$.

Proposition 4.4: Every faintly Δ continuous is faintly pre Δ continuous.

But the opposite direction of Proposition 4.4 could also not valid as showing below:

Example 4.5 Assume $W = \{\varkappa, \mathfrak{b}, ƀ\}$ with a topology $\tau = \{\phi, X, \{\varkappa, \mathfrak{b}\}\}$ and $Y = \{a, b, c\}$ equipped with $\rho = \{\phi, Y, \{a\}, \{b, c\}\}$. Construct a function ζ from (X, τ) into (Y, ρ) by $\zeta(\varkappa) = a$, $\xi(\mathfrak{b}) = b$ and $\xi(ƀ) = c$. Clearly, ζ is slightly pre Δ continuous, while $\{a\}$ is clopen set in Y and $\zeta^{-1}(\{a\}) = \{\varkappa\} \notin \Delta O(X)$. Hence, ζ is not slightly Δ continuous.

Proposition 4.6: A function $\zeta : (W, \varsigma) \longrightarrow (Y, \rho)$ is slightly pre Δ continuous iff $\zeta^{-1}(O) \in p\Delta C(W, \varsigma)$ for every $\theta C \in (Y, \rho)$.

Proof: if O is clopen set in Y, so $Y - O$ is also clopen in Y. By definition 4.1, $\zeta^{-1}(Y - O)$ is pre-Δ open. But $\zeta^{-1}(Y - O) = W - \zeta^{-1}(O)$. Consequently, $\vartheta^{-1}(O)$ is pre-Δ-closed in W.

Proposition 4.7: Every function ζ from pre $T_{\frac{1}{2}}$-topological space, (X, τ), into any topological space, (Y, ρ), is faintly pre Δ continuous function.

Proof: Assume that O be θ-open subset of space (Y, ρ), then $\zeta^{-1}(O)$ either is preopen or be preclosed . Consequently, $\zeta^{-1}(O)$ is pre-Δ open set.

Proposition 4.8: For a function $\zeta : (W, \varsigma) \rightarrow (Q, \rho)$, the below equivalents:

1) ζ be faintly pre-Δ continuous.
2) $p\Delta cƀ\left(\zeta^{-1}(B)\right) \subseteq \zeta^{-1}\left(cƀ_{\theta}(B)\right)$ for each subsets B in Q.
3) $\zeta^{-1}\left(\text{ịʃ}t_{\theta}(O)\right) \subseteq p\Delta\text{ịʃ}t\left(\zeta^{-1}(O)\right)$ for each subsets O of Q.

Proof: 1⇒2 assume that we have $B \subseteq Q$ is subsets for Y, so $cƀ_{\theta}(B)$ is θ-closed in Q. Because ζ be faintly pre-Δ continuous, so $\zeta^{-1}\left(cƀ_{\theta}(B)\right)$ be pre -Δ-closed set in W. Accordingly, $\zeta^{-1}\left(cƀ_{\theta}(B)\right) = P\Delta cƀ\left(\zeta^{-1}\left(cƀ_{\theta}(B)\right)\right)$. Since $B \subseteq cƀ_{\theta}(B)$, then $\zeta^{-1}(B) \subseteq \zeta^{-1}\left(cƀ_{\theta}(B)\right)$. That is $P\Delta cƀ\left(\zeta^{-1}(B)\right) \subseteq P\Delta cƀ \left(\zeta^{-1}\left(cƀ_{\theta}(B)\right)\right) = \zeta^{-1}\left(cƀ_{\theta}(B)\right)$.

2⇒3 Assume that O be a subset of Q, thenQ$/O$ is also subset of Y. By applying (2), $\Delta c\text{ƀ}\big(\zeta^{-1}(Q-O)\big) \subseteq \zeta^{-1}\big(c\text{ƀ}_{\theta}(Q-O)\big) \Leftrightarrow W - P\Delta\text{ỉ}\text{ft}\big(\zeta^{-1}(Q-O)\big) \subseteq \zeta^{-1}\big(Y-\text{ỉ}\text{ft}_{\theta}(O)\big) \Leftrightarrow W - P\Delta\text{ỉ}\text{ft}\big(\zeta^{-1}(Q-O)\big) \subseteq W - \zeta^{-1}\big(\text{ỉ}\text{ft}_{\theta}(Q)\big)$. That is $\zeta^{-1}\big(\text{ỉ}\text{ft}_{\theta}(O)\big) \subseteq p\Delta\text{ỉ}\text{ft}\big(\zeta^{-1}(O)\big)$.

3⇒1 Assume that D be θ-open set in Q, then by applying (3), $\zeta^{-1}(D) = \zeta^{-1}\big(\text{ỉ}\text{ft}_{\theta}(D)\big) \subseteq p\Delta\text{ỉ}\text{ft}\big(\zeta^{-1}(D)\big)$ and since $p\Delta\text{ỉ}\text{ft}\big(\zeta^{-1}(D)\big) \subseteq \zeta^{-1}(D)$, then $\zeta^{-1}(D) \in P\Delta O(W,\varsigma)$. Hence, ζ is faintly pre-Δ continuous.

Definition 4.9: The space (W,ς) is said to be $P\Delta$-space, where any $P\Delta$-open in W be open set.

Proposition 4.10: If $\varphi:(X,\tau) \to (Y,\rho)$ is faintly pre-Δ cont. & K is open in X, then $\varphi_K:(K,\tau_K) \longrightarrow (Y,\rho)$ is faintly pre-Δ continuous.

Proof: Give D be a θ-open. As φ be faintly pre-Δ continuous, $\varphi^{-1}(D)$ be $P\Delta$-open. Because K be open , so we have $\varphi^{-1}(D)\cap K \in P\Delta O(X,\tau)$ implies, ${\varphi_K}^{-1}(D) = \varphi^{-1}(D)\cap K \in P\Delta O(K,\tau_K)$. Hence, φ_K is faintly pre-Δ continuous.

Proposition 4.11: Let $\zeta:(W,\varsigma) \to (Y,\rho)$ and $\psi:(Y,\rho) \longrightarrow (Z,\sigma)$ be two mappings. Then we have:

1) If ζ is faintly pre-Δ continuous and ψ is θ-irresolute, then $\psi\circ\zeta:(W,\varsigma) \to (Z,\sigma)$ is faintly pre-Δ continuous.
2) If $\zeta$is pre-Δ irresolute and ψ is faintly pre-Δ continuous, then $\psi\circ\zeta:(W,\varsigma) \to (Z,\sigma)$ is faintly pre-Δ continuous.

Proof:

(1) Assume O is θ-open . Because ψ be θ-irresolute function, so $\psi^{-1}(O)$ be θ-open . But φ is faintly pre-Δ continuous, therefore $\zeta^{-1}\big(\psi^{-1}(O)\big) = (\psi\circ\zeta)^{-1}(O)$ is pre-Δ-open.

(2) Suppose H is θ-open. Since ψ be faintly pre-Δ continuous, so $\psi^{-1}(H)$ be pre-Δ-open n (Y,ρ). becaus φ be preΔ irresolute, so $\zeta^{-1}\big(\psi^{-1}(O)\big) = (\psi\circ\zeta)^{-1}(O)$ is pre-Δ-open set in W.

Definition 4.12: topological space, (W,ς), is named pre-Δ-connected if W can not be as the union for two nonempty disjoint pre-Δ open set.

Proposition 4.13: If $\zeta:(W,\varsigma) \to (Y,\rho)$ is faintly pre-Δ continuous and (W,ς) is a pre-Δ-connected, then (Y,ρ) is θ-connected.

Proof: Let (Y,ρ) not θ-connected, so ∃ two nonempty θ -open H_1 & H_2 s.t $H_1\cap H_2 = \phi$ & $H_1\cup H_2 = Y$. It follows that $\zeta^{-1}(H_1)\cap\zeta^{-1}(H_2) = \phi$ & $\zeta^{-1}(H_1)\cup\zeta^{-1}(H_2) = X$. Since H_1 and H_2 are θ -open sets and since ζ is faintly pre-Δ continuous, so $\zeta^{-1}(H_1)$ and $\zeta^{-1}(H_2)$ are pre-Δ open sets. Consequently, (W,ς) not pre-Δ-connected which contradiction. Hence, (Y,ρ) is θ-connected.

5. Conclusions

This work initiates a totally novel for Δ-sets namely pre Δ-open set and give several assessments about this and study many features such as preΔ-interior, preΔ-closure, preΔ-exterior and give several about these concepts. Finally, we apply a concept for preΔ-open to establish a new kind for functions, named faintly pre Δ-continuous mapping and by applying some new notions, several characterizations are obtained.

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