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On fuzzy K-skew (μ – ϗ) quasi binormal operator

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Abstract

The purpose of that study is to provide many ideas related to the fuzzy k-skew (μ–ϗ) quasi binormal operator that is defined on fuzzy Hilbert space. Additionally, several significant properties of this operator are discussed, and certain theorems of operations concerning this concept have been provided.

Subject Classification: *Primary 47S40, Secondary 03E72.*

Keywords: *Fuzzy set, Fuzzy k-skew binormal operator, Fuzzy k-skew (μ–ϗ) quasi binormal operator.*

1. Introduction

The functional analysis, algebra, topology, geometry, and statistics are fundamental concepts in pure mathematics. Mathematicians have studied these concept for hundreds of years, and they are a powerful tool for finding solutions to a mathematical problems that is abstract and ambiguous [1-4].

On the other hand, Zadeh [5] introduced the concept of fuzzy sets in mathematics and proposed important operations on these sets. The concept of fuzzy sets has been extended to include diverse fields, such as [6,7] by many researchers over the years. Fuzzy Normed spaces have also been studied by many mathematicians, each from their own perspectives, such as Kastsars [6], who introduced the definiton of a fuzzy normed space. Note that the concept of aFuzzy Inner Product Space (FIP-Space) is nothing but an extension of an inner product space. Note that Biswas [7], and Kohel and Kumar [8], also provided several defintions of fuzzy normed spaces. Goudarzi and Vaezpour [9]initially developed the notion of a fuzzy Hilbert space,. Other mathematicians, such as [3, 10, 11], provided other interpretations of fuzzy Hilbert space. The idea of an afuzzy operator was, Adopted by a number of mathematicians, including[8, 10]

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adopted the concept of a fuzzy operator. Additionally, self – adjoint fuzzy operator$\mathfrak{S}$ which introduced by Radharamani [12] if$\mathfrak{S}^* = \mathfrak{S}$. Additionally, it commutes with its adjoint fuzzy operator. where $\mathfrak{S}^*\mathfrak{S} = \mathfrak{S}\mathfrak{S}^*$,with this $\mathfrak{S}$ is referred to as a fuzzy normal operator.

This work introduces a new family of bounded. operators on fuzzy Hilbert space, FHS ($\mathcal{H}$). such as the Fuzzy k-skew ($\mu - \mathfrak{J}$) quasi binormal operator. we study this operator and given several important characteristics of this feature.

2. Fuzzy k-skew ($\mu - \mathfrak{J}$) quasi binormal operator

Definition 2.1: Consider a fuzzy bounded operator $\mathfrak{S}$ on a fuzzy Hilbert space FHS ($\mathcal{H}$), $\mathfrak{S}$ is defined as amapping from $\mathcal{H}$ to $\mathcal{H}$, Therefore, $\mathfrak{S}$ is claimed to be:

1. Fuzzy k-skew binormal operator, if and only if $(\mathfrak{S}^*\mathfrak{S}\mathfrak{S}\mathfrak{S}^*)\mathfrak{S}^k = \mathfrak{S}^k(\mathfrak{S}\mathfrak{S}^*\mathfrak{S}^*\mathfrak{S})$.where k is nonnegative integers.
2. Fuzzy ($\mu - \mathfrak{J}$) quasi binormal operator, if and only if $(\mathfrak{S}^*)^\mu(\mathfrak{S}^*\mathfrak{S}\mathfrak{S}\mathfrak{S}^*) = \mathfrak{J}(\mathfrak{S}\mathfrak{S}^*\mathfrak{S}^*\mathfrak{S})(\mathfrak{S}^*)^\mu$, where $\mathfrak{J}$ signifies an operator with fuzzy bounds, μ is nonnegative integers.

Definition 2.2: Suppose that $\mathcal{H}$ represent afuzzy Hilbert space (FHS($\mathcal{H}$)), and $\mathfrak{S}$ denote a fuzzy bounded operator mapping from $\mathcal{H}$ to $\mathcal{H}$ then $\mathfrak{S}$ namely fuzzy k-skew($\mu - \mathfrak{J}$) quasi binormal operator, if and only if $(\mathfrak{S}^*)^\mu(\mathfrak{S}^*\mathfrak{S}\mathfrak{S}\mathfrak{S}^*)\mathfrak{S}^k = \mathfrak{J}\mathfrak{S}^\kappa(\mathfrak{S}\mathfrak{S}^*\mathfrak{S}^*\mathfrak{S})(\mathfrak{S}^*)^\mu$, where $\mathfrak{J}$.a fuzzy bounded operator, μ, k are nonnegative integers.

Remarks 2.3

1. The fuzzy k-skew ($\mu - \mathfrak{J}$) quasi binormal operator is fuzzy ($\mu - \mathfrak{J}$) quasi binormal operator if k =0.
2. The fuzzy k-skew binormal operator is fuzzy ($\mu - \mathfrak{J}$) quasi binormal operator if $\mathfrak{J}$ is unitary operator and $\mu = 0$

Theorem 2.4: *Let $\mathfrak{S}$ be a fuzzy k-skew ($\mu - \mathfrak{J}$) quasi binormal operator on FHS ($\mathcal{H}$), then $\mathfrak{S}^n$ is a fuzzy k-skew ($\mu - \mathfrak{J}$) quasi binormal operator.*

Proof: We prove by applying mathematical induction. Suppose that $\mathfrak{S}$ be a fuzzy k-skew ($\mu - \mathfrak{J}$) quasi binormal operator, then. When n=1 the result is true, $(\mathfrak{S}^*)^\mu(\mathfrak{S}^*\mathfrak{S}\mathfrak{S}\mathfrak{S}^*)\mathfrak{S}^k = \mathfrak{J}\mathfrak{S}^k(\mathfrak{S}\mathfrak{S}^*\mathfrak{S}^*\mathfrak{S})(\mathfrak{S}^*)^\mu$, Suppose the result is true when n=m. $[(\mathfrak{S}^*)^\mu(\mathfrak{S}^*\mathfrak{S}\mathfrak{S}\mathfrak{S}^*)\mathfrak{S}^k]^m = [\mathfrak{J}\mathfrak{S}^k(\mathfrak{S}\mathfrak{S}^*\mathfrak{S}^*\mathfrak{S})(\mathfrak{S}^*)^\mu]^m$,

Then we prove the result for n=m+1.

$$\left[(\mathfrak{S}^*)^\mu(\mathfrak{S}^*\mathfrak{S}\mathfrak{S}\mathfrak{S}^*)\mathfrak{S}^k\right]^{.m+1}$$

is equel= $[(\mathcal{S}^*)^\mu(\mathcal{S}^*\mathcal{S}\mathcal{S}\mathcal{S}^*)\mathcal{S}^k]^m[(\mathcal{S}^*)^\mu(\mathcal{S}^*\mathcal{S}\mathcal{S}\mathcal{S}^*)\mathcal{S}^k]$, thus we obtain $= [\mathfrak{F}\,\mathcal{S}^\kappa\cdot(\mathcal{S}\mathcal{S}^*\mathcal{S}^*\mathcal{S})(\mathcal{S}^*)^\mu]^{m+1}$. Therefore, the results correct for n=m+1. Thus, $\mathcal{S}^n$ is a fuzzy k-skew ($\mu-\mathfrak{F}$) quasi binormal operator.

Theorem 2.5: *If $\mathcal{S}$ be a fuzzy k-skew ($\mu-\mathfrak{F}$) quasi binormal operator on FHS($\mathcal{H}$), then $\mathcal{S}^*$ is fuzzy k-skew ($\mu-\mathfrak{F}$) quasi binormal operator if $(\mathcal{S}^*\mathcal{S}\mathcal{S}\mathcal{S}^*)$ commute with $(\mathcal{S}^*)^\mu$ and $\mathcal{S}^k$, also($\mathcal{S}\mathcal{S}^*\mathcal{S}^*\mathcal{S}$) commute with $(\mathcal{S}^*)^\mu$ and $\mathcal{S}^k$.*

Proof: Suppose that $\mathcal{S}$ is a fuzzy k-skew ($\mu-\mathfrak{F}$) quasi binormal operator, than we have $((\mathcal{S}^*)^*)^\mu((\mathcal{S}^*)^*\mathcal{S}^*\mathcal{S}^*(\mathcal{S}^*)^*)(\mathcal{S}^*)^k$,This indicates this. $= [(\mathcal{S}^*)^\mu(\mathcal{S}^*\mathcal{S}\mathcal{S}\mathcal{S}^*)(\mathcal{S}^\kappa)]^*$ which impleis $= \left[\mathfrak{F}\,\mathcal{S}^\kappa(\mathcal{S}\mathcal{S}^*\mathcal{S}^*\mathcal{S})(\mathcal{S}^*)^\mu\right]^*$ this equl to $=\left[(\mathcal{S}^*)^\mu(\mathcal{S}\mathcal{S}^*\mathcal{S}^*\mathcal{S})\mathfrak{F}\mathcal{S}^k\right]^*$,therefore, we obtain $=(\mathfrak{F}\mathcal{S}^\kappa)^*(\mathcal{S}\mathcal{S}^*\mathcal{S}^*\mathcal{S})^*$ $(\mathcal{S}^*)^\mu)^*$; So, we get the following result $\mathfrak{F}(\mathcal{S}^*)^\kappa[\cdot\left(\mathcal{S}^*(\mathcal{S}^*)^*(\mathcal{S}^*)^*\mathcal{S}^*\right)(\mathcal{S}^*)^*)^\mu]$, Hence. $\mathcal{S}^*$ is a fuzzy k-skew ($\mu-\mathfrak{F}$) quasi binormal operator.

Proposition 2.6: Let $\mathcal{S}$ be a fuzzy k-skew ($\mu-\mathfrak{F}$) quasi binormal operator, and $\check{\rho}$ be a real scalar. then $\check{\rho}\mathcal{S}$ is a fuzzy k-skew ($\mu-\mathfrak{F}$) quasi binormal operator.

Proof: $(\check{\rho}^{*^\mu}\mathcal{S}^{*^\mu})(\check{\rho}^*\mathcal{S}^*)\left(\check{\rho}\mathcal{S}\right)\left(\check{\rho}\mathcal{S}\right)\left(\check{\rho}^*\mathcal{S}^*\right)\left(\check{\rho}^k\mathcal{S}^k\right)$,since $\mathcal{S}$ is a fuzzy k-skew ($\mu-\mathfrak{F}$) quasi binormal operator, then we have,$\mathfrak{F}\left(\check{\rho}^k\mathcal{S}^k\right)[\acute{\rho}\mathcal{S}\acute{\rho}^*\acute{\mathcal{S}}^*\acute{p}^*\mathcal{S}^*\check{\rho}\mathcal{S}]$ $\left(\check{p}^{*^\mu}\mathcal{S}^{*^\mu}\right)$. So, we get the following result= $\mathfrak{F}\,(\check{\rho}\mathcal{S})^k(\dot{\check{\rho}}\mathcal{S})^*\dagger\mathcal{S}\check{\rho}\mathcal{S}(\check{p}\mathcal{S})^*(\check{\rho}\mathcal{S})^*)^\mu$, Hence, $\check{\rho}\mathcal{S}$ is a fuzzy k-skew ($\mu-\mathfrak{F}$) quasi binormal operator.

Theorem 2.7: *If $\mathcal{S}^{-1}$ exist and $\mathcal{S}$ be a two fuzzy k-skew ($\mu-\mathfrak{F}$) quasi binormal operators on FHS($\mathcal{H}$), such that $(\mathcal{S}^*\mathcal{S}\mathcal{S}\mathcal{S}^*)$ commute with $(\mathcal{S}^*)^\mu$ and $\mathcal{S}^k$, also($\mathcal{S}\mathcal{S}^*\mathcal{S}^*\mathcal{S}$) commute with $(\mathcal{S}^*)^\mu$ and $\mathcal{S}^k$, then $\mathcal{S}^{-1}$ is a fuzzy k-skew ($\mu-\mathfrak{F}$) quasi binormal operator.*

Proof: Suppose that $\mathcal{S}$ is a fuzzy k-skew ($\mu-\mathfrak{F}$) quasi binormal operator, thus$(\mathcal{S}^{-1})^*)^\mu\left((\mathcal{S}^{-1})^*\mathcal{S}^{\cdot-1}\mathcal{S}^{-1}(\mathcal{S}^{-1})^*\right)(\mathcal{S}^{-1})^k$, is equel = $(\mathcal{S}^*)^\mu)^{-1}((\mathcal{S}\mathcal{S}^*)^{-1}$ $(\mathcal{S}^*\mathcal{S})^{-1})(\mathcal{S}^k)^{-1}$*is equel to* $= (\mathcal{S}^*)^\mu)^{-1}\left((\mathcal{S}^*\mathcal{S}\mathcal{S}\mathcal{S}^*)^{-1}\right)(\mathcal{S}^k)^{-1}$, which h imples that = $\left((\mathcal{S}^*\mathcal{S}\mathcal{S}\mathcal{S}^*)(\mathcal{S}^*)^\mu)^{-1}\right)(\mathcal{S}^k)^{-1}$ after which have$[(\mathcal{S}^k)(\mathcal{S}^*\mathcal{S}\mathcal{S}\mathcal{S}^*)$ $(\mathcal{S}^*)^\mu]^{-1}$, this implies = $[(\mathcal{S}^*)^\mu(\mathcal{S}^*\mathcal{S}\mathcal{S}\mathcal{S}^*)(\mathcal{S}^\kappa)]^{-1}$ thus we obtain =$\mathfrak{F}(\mathcal{S}^{-1})^\kappa$ $\left[\left(\mathcal{S}^{-1}(\mathcal{S}^{-1})^*(\mathcal{S}^{-1})^*\mathcal{S}^{-1}\right)(\mathcal{S}^{-1})^*)^\mu\right]$ This means, $\mathcal{S}^{-1}$ is afuzzy k-skew ($\mu-\mathfrak{F}$) quasi binormal operator.

Theorem 2.8: *Let $\mathcal{S}$ and $\dot{\nu}$ be a two fuzzy k-skew ($\mu-\mathfrak{F}$) quasi binormal operators on a FHS($\mathcal{H}$), such that, $\mathcal{S}^*\dot{\nu}=\mathcal{S}\dot{\nu}^*, \mathcal{S}\dot{\nu}=\dot{\nu}\mathcal{S}, \dot{\nu}^*\mathcal{S}^{*^\mu}=\mathcal{S}^{*^\mu}\dot{\nu}^*$. Then $\mathcal{S}\dot{\nu}$ is fuzzy k-skew ($\mu-\mathfrak{F}$) quasi binormal operator.*

Proof: Since $\acute{\mathcal{S}}$, and $\grave{\nu}$ are two fuzzy k-skew ($\mu - \mathfrak{F}$) quasi binormal operator, then $((\acute{\mathcal{S}}\grave{\nu})^*)^{\mu}[((\acute{\mathcal{S}}\grave{\nu})^*(\acute{\mathcal{S}}\grave{\nu})(\acute{\mathcal{S}}\grave{\nu})(\acute{\mathcal{S}}\grave{\nu})^*](\acute{\mathcal{S}}\grave{\nu})^k$

$= \grave{\nu}^{*^{\mu}}\acute{\mathcal{S}}^{*^{\mu}}\left[\grave{\nu}^*\acute{\mathcal{S}}^*\acute{\mathcal{S}}\grave{\nu}\acute{\mathcal{S}}\grave{\nu}\grave{\nu}^*\acute{\mathcal{S}}^*\right]\acute{\mathcal{S}}^k\grave{\nu}^k$

$= (\grave{\nu}^{*^{\mu}}\grave{\nu}^*\acute{\mathcal{S}}^{*^{\mu}}\acute{\mathcal{S}}^*\acute{\mathcal{S}}\grave{\nu}\acute{\mathcal{S}}\grave{\nu}^*\grave{\nu}\acute{\mathcal{S}}^*)\acute{\mathcal{S}}^k\grave{\nu}^k = (\grave{\nu}^{*^{\mu}}\grave{\nu}^*\grave{\nu}\ \acute{\mathcal{S}}^{*^{\mu}}\acute{\mathcal{S}}^*\acute{\mathcal{S}}\acute{\mathcal{S}}\acute{\mathcal{S}}^*\grave{\nu}\grave{\nu}^*)\acute{\mathcal{S}}^{\kappa}\grave{\nu}^{\kappa}$

this implise

$= (\grave{\nu}^{*^{\mu}}(\grave{\nu}^*\grave{\nu}\grave{\nu}\grave{\nu}^*)\grave{\nu}^{\kappa})(\acute{\mathcal{S}}^{*^{\mu}}\acute{\mathcal{S}}^*\acute{\mathcal{S}}\acute{\mathcal{S}}\acute{\mathcal{S}}^*)\acute{\mathcal{S}}^{\kappa}$

$= \mathfrak{F}\left(\grave{\nu}^k(\grave{\nu}\grave{\upsilon}^*\grave{\nu}^*\grave{\nu})\grave{\nu}^{*^{\mu}}\right)\left(\acute{\mathcal{S}}^{\kappa}\left(\acute{\mathcal{S}}\acute{\mathcal{S}}^*\acute{\mathcal{S}}^*\acute{\mathcal{S}}\right)\acute{\mathcal{S}}^{*^{\mu}}\right),$

Therefore we obtain $= \mathfrak{F}\acute{\mathcal{S}}^{\kappa}\grave{\nu}^{\kappa}\left(\grave{\nu}\acute{\mathcal{S}}\ \grave{\upsilon}^*\acute{\mathcal{S}}^*\grave{\nu}^*\acute{\mathcal{S}}^*\grave{\nu}\acute{\mathcal{S}}\right)\grave{\nu}^{*^{\mu}}\acute{\mathcal{S}}^{*^{\mu}} = \mathfrak{F}(\acute{\mathcal{S}}\grave{\nu})^k\ (\acute{\mathcal{S}}\grave{\nu}\ (\acute{\mathcal{S}}\grave{\nu})^*(\acute{\mathcal{S}}\grave{\nu})^*\acute{\mathcal{S}}\grave{\nu})(\acute{\mathcal{S}}\grave{\nu})^*)^{\mu}$

Hence, $\acute{\mathcal{S}}\grave{\nu}$ is fuzzy k-skew ($\mu - \mathfrak{F}$) quasi binormal operator.

Theorem 2.9: *Let* $\acute{\mathcal{S}}$ *and* $\acute{\nu}$ *be two fuzzy k-skew* ($\mu - \mathfrak{F}$) *quasi binormal operator, such that,* $\acute{\mathcal{S}}^*\grave{\nu} = \acute{\nu}\acute{\mathcal{S}}^* = \acute{\nu}^*\acute{\mathcal{S}} = \acute{\mathcal{S}}\ \acute{\nu}^{\cdot *} = \acute{\mathcal{S}}\acute{\nu} = \acute{\nu}\acute{\mathcal{S}} = \acute{\mathcal{S}}^{\cdot *^{\mu}}\acute{\nu}^{\cdot *} = \acute{\nu}^{\cdot *^{\mu}}\acute{\mathcal{S}}^*\ = \acute{\nu}^{\ *^{\mu}}\acute{\nu}^* = 0$. *Then* $\acute{\mathcal{S}} + \acute{\nu}$ *is fuzzy k-skew* ($\mu - \mathfrak{F}$) *quasi binormal operator.*

Proof: Since $\mathcal{S}'$, and $\acute{\nu}$ are two fuzzy k-skew ($\mu - \mathfrak{F}$) quasi binormal operator, then $((\acute{\mathcal{S}} + \acute{\nu})^*)^{\mu}[(\acute{\mathcal{S}} + \acute{\nu})^*(\acute{\mathcal{S}} + \acute{\nu})(\acute{\mathcal{S}} + \acute{\nu})(\acute{\mathcal{S}} + \acute{\nu})^*](\acute{\mathcal{S}} + \acute{\nu})^k$ .This implis that $= (\acute{\mathcal{S}}^{*^{\mu}} + \mu(\acute{\mathcal{S}}^*)^{\mu-1}\acute{\nu}^* + \cdots + \acute{\nu}^{\cdot *^{\mu}})\left[(\acute{\mathcal{S}} + \acute{\nu})^*(\acute{\mathcal{S}} + \acute{\nu})(\acute{\mathcal{S}} + \acute{\nu})(\acute{\mathcal{S}} + \acute{\upsilon})^*\right]$ $(\acute{\mathcal{S}}^k + k(\acute{\mathcal{S}}^{k-1}\acute{\nu}\ \acute{\nu} + \cdots + \ \acute{\nu}^k)$ is equel.

$= (\acute{\mathcal{S}}^{*^{\mu}} + \acute{\nu}^{\cdot *^{\mu}})\left[(\acute{\mathcal{S}}^* + \acute{\upsilon}^*)(\acute{\mathcal{S}} + \acute{\nu}\ )(\acute{\mathcal{S}} + \acute{\nu}\ )(\acute{\mathcal{S}}^* + \acute{\nu}^*)\right](\acute{\mathcal{S}}^{\kappa} + \acute{\nu}^{\kappa}),$

We obtained that, $= (\acute{\mathcal{S}}^{*^{\mu}} + \acute{\nu}^{*^{\mu}})\left[(\acute{\mathcal{S}}^*\acute{\mathcal{S}} + \nu\ ^*\acute{\nu}\ )(\acute{\mathcal{S}}\acute{\mathcal{S}}^* + \acute{\nu}\ \acute{\nu}^*)\right](\acute{\mathcal{S}}^k + \acute{\nu}^k)$. Subsequently, following multiplication and simplification, we obtain the resulting $= [\acute{\mathcal{S}}^{*^{\mu}}\left(\acute{\mathcal{S}}^*\acute{\mathcal{S}}\acute{\mathcal{S}}\acute{\mathcal{S}}^*\right) + \acute{\nu}^{*^{\mu}}(\acute{\nu}^*\acute{\nu}\acute{\nu}\ \acute{\nu}^*)](\acute{\mathcal{S}}^{\kappa} + \acute{\nu}^{\kappa})$ and this implies that. $= \left[\acute{\mathcal{S}}^{*^{\mu}}\left(\acute{\mathcal{S}}^*\acute{\mathcal{S}}\acute{\mathcal{S}}\acute{\mathcal{S}}^*\right)\acute{\mathcal{S}}^{\kappa} + \acute{\upsilon}^{*^{\mu}}(\acute{\nu}^*\acute{\nu}\acute{\nu}\ \acute{\nu}^*)\right)\acute{\mathcal{S}}^{\kappa} + \acute{\mathcal{S}}^{*^{\mu}}\left(\acute{\mathcal{S}}^*\acute{\mathcal{S}}\acute{\mathcal{S}}\acute{\mathcal{S}}^*\right)\nu^{\cdot\kappa} + \grave{\nu}^{\cdot *^{\mu}}(\acute{\nu}^*\acute{\nu}\acute{\nu}\ \acute{\upsilon}^{\cdot *})$ $\nu^{\cdot\cdot\kappa} = [\acute{\mathcal{S}}^{*^{\mu}}\left(\acute{\mathcal{S}}^*\acute{\mathcal{S}}\acute{\mathcal{S}}\acute{\mathcal{S}}^{\cdot *}\right)\left(\acute{\mathcal{S}}^k\right) + \acute{\nu}^{\cdot *^{\mu}}(\acute{\nu}^{\cdot *}\acute{\nu}\acute{\nu}\ \acute{\nu}^*)\ \acute{\nu}^{\kappa}]$, since $\acute{\mathcal{S}}$ and $\acute{\nu}$are fuzzy k-skew ($\mu - \mathfrak{I}$) quasi binormal operator, then we obtain that, $= \mathfrak{F}\left[\left(\acute{\mathcal{S}}^{\kappa}\left(\acute{\mathcal{S}}\ \acute{\mathcal{S}}^*\acute{\mathcal{S}}^*\acute{\mathcal{S}}\right)\acute{\mathcal{S}}^{*^{\mu}}\right) + \left(\acute{\nu}^k\left(\acute{\upsilon}\acute{\nu}^*\grave{\nu}\ ^*\acute{\nu}\right)\acute{\nu}^{*^{\mu}}\right)\right.$ this is equal to $= \mathfrak{F}\left(\acute{\mathcal{S}}^{\kappa} + \acute{\nu}^{\kappa}\right)$ $\left[\left(\acute{\mathcal{S}}\ \acute{\mathcal{S}}^* + \acute{\nu}\acute{\nu}^*\right)\left(\acute{\mathcal{S}}^*\acute{\mathcal{S}} + \acute{\nu}^*\acute{\nu}\right)\right](\acute{\mathcal{S}}^{*^{\mu}} + \acute{\nu}^{*^{\mu}})$, So, we get the following result: $\mathfrak{F}(\acute{\mathcal{S}} + \acute{\nu})^{\kappa}\ [(\acute{\mathcal{S}} + \acute{\nu})(\acute{\mathcal{S}} + \acute{\nu})^*(\acute{\mathcal{S}} + \acute{\nu})^*(\acute{\mathcal{S}} + \acute{\nu}).](\acute{\mathcal{S}} + \acute{\nu})^*)^{\mu}$ Hence, $\acute{\mathcal{S}} + \acute{\nu}$, is fuzzy k-skew ($\mu - \mathfrak{F}$) quasi binormal operator

3. Discussion and Conclusion

This work combines two types of operators, fuzzy k-skew-binormal and fuzzy ($\mu - \mathfrak{F}$) quasi binormal, on the fuzzy Hilbert space. And it includes various properties and theorems of this notion, as well as additional requirements for including specific properties into this research. In addition to generalizations of several of the concepts that we used in our study.

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