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Numerical solution of random ordinary differential equations by using modified decomposition method

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Abstract

In the following work, we employ the modified decomposition method to obtain approximate solutions to random ordinary linear and nonlinear differential equations. We will introduce a fast technique for finding approximate solutions and demonstrate the effectiveness, speed, and accuracy of this method through examples, the obtained results, and the impact of the stochastic process (generation of a Wiener process on a computer) on the shape of the solution to differential equations.

Subject Classification: *65C20, 65L05.*

Keywords: *Modified decomposition method, Random ordinary differential equations, Stochastic process (Wiener process).*

1. Introduction

The topic of Random ordinary differential equations (RODEs) is essential in several modeling applications of real-world phenomena where randomness has a significant effect, such as in financial fields, engineering, and biology. These equations are differential equations (DEs) that contain a random process (Wiener process). There are a vast number of articles that have appeared on RDEs released in recent years [1-9].

In (1994), George A. and Rach used a modified decomposition solution for a boundary (linear and nonlinear) value problem. Also, in (2000), Wazwaz applies the modified decomposition method to a higher-order problem to find

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approximate solutions to boundary value problems [10-11]. Convergence is important to prove the validity and quality of using this method [12-14], but by using it in the form of the equation, Random ordinary nonlinear differential equations. Anyway, the modified decomposition method (MDM) provides a solution in at most one or two iterations. In this study, MDM was applied to the formula of the equation shown as follows:

$$L(\mathrm{D}(\mathrm{q},\omega)) + G(\mathrm{D}(\mathrm{q},\omega)) + K\big((D(\mathrm{q},\omega)\big) = Q(\mathrm{q},\omega) \quad (1)$$

with initial conditions:

$$D(\mathrm{q}_0,\omega) = D_0\,, D'\,(\mathrm{q}_0,\omega) = D_0', D''(\mathrm{q}_0,\omega) = \; D_0''\ldots, D^{(n-1)}(\mathrm{q}_0,\omega) = D_0^{(n-1)} \quad (3)$$

Where $L = \frac{d^n}{d\mathrm{q}^n}$ and G is the operator of a linear differential operator of order less than $L, K(D(\mathrm{q},\omega))$ is a linear or nonlinear function, it determines the type of general function, and $Q(\mathrm{q},\omega)$ is any function, $\mathrm{q} \in [0,1]$.

Section 2 provides a review of fundamental concepts. Section 3the previously provided equations were formulated and analyzed (linear and nonlinear) according to MDM. In Section 4, we derived the conclusions and future work.

2. Preliminaries

Definition 1,[15]: A sample space Ω includes a collection of finite or infinite possible outcomes for a random experiment.

Definition 2,[16]: The random variable **D** ($\boldsymbol{\omega}$), $\boldsymbol{\omega} \in \Omega$, is a real-valued function, and it’s a measurable function with respect to the probability measure *P*.

Definition 3,[16]: Let $t \in [\mathrm{q}_0, T]$ on a probability space (Ω,F,*P*), the family for random variables $\mathrm{D}_\mathrm{q}(\omega)$ (or D_q) for two variables is called a stochastic process.

Definition 4, [16]: Any stochastic process M_q, every $\mathrm{q} \in [0, \infty)$, that satisfies the following conditions is said to be a Wiener process:

1. $P(\{\omega \in \Omega \mid \mathrm{M}_\mathrm{q} = 0\})$ is equal to 1.
2. $\forall\, 0 < \mathrm{q}_0 < \mathrm{q}_1 < \mathrm{q}_2 < \mathrm{q}_3 < \ldots\ldots < \mathrm{q}_n$ the addition $\mathrm{M}_{\mathrm{q}_1} - \mathrm{M}_{\mathrm{q}_0}, \mathrm{M}_{\mathrm{q}_2} - \mathrm{M}_{\mathrm{q}_1}, \mathrm{M}_{\mathrm{q}_3} - \mathrm{M}_{\mathrm{q}_2}, \mathrm{M}_{\mathrm{q}_4} - \mathrm{M}_{\mathrm{q}_3}, \ldots, \mathrm{M}_{\mathrm{q}_n} - \mathrm{M}_{\mathrm{q}_{n-1}}$ are disjoint.
3. If $h > 0$ and let ɋ be an arbitrary, $\mathrm{M}_{\mathrm{q}+h} - \mathrm{M}_\mathrm{q}$ had a (Normal distribution) or (Gaussian distribution), s.t mean *0* and variance *h*.

3. Modified Decomposition Method

The random ordinary differential equation (1) with the initial conditions (2), and by being made

$$L\big(D(\mathrm{q},\omega)\big) = Q(\mathrm{q},\omega) - G(D(\mathrm{q},\omega)) - K\big(D(\mathrm{q},\omega)\big) \quad (3)$$

Take integral operator L^{-1} is n[th] -fold integration operator (from *0* to *t*) both side for eq. (3) so, we getting :

$$D(q,\omega)=\sum_{j=0}^{n-1}\frac{D_j}{j!}q^j+L^{-1}\left(Q(q,\omega)\right)-L^{-1}\left(G(D(q,\omega))-\ L^{-1}\left(K\left(D(q,\omega)\right)\right)\right. \tag{4}$$

Where the solution $D(q,\omega)$ comes from infinite sum of components by form:

$$D(q,\omega)\ =\ \sum_{n=0}^{\infty}D_n(q,\omega)\,,n\geq 0 \tag{5}$$

Let $f(q,\omega)=f_1(q,\omega)+f_2(q,\omega)=\ \sum_{j=0}^{n-1}\frac{D_j}{j!}q^j+L^{-1}\left(Q(q,\omega)\right)$ it represents the partition on which the method depends

3.1 *Linear Random ordinary differential equations*

If $K\left(D(q,\omega)\right)$ is linear in equation *(4)* then this situation is summarized by assuming that

$$D_0\,(q,\omega)=f_1(q,\omega) \tag{6}$$

$$D_1\,(q,\omega)=f_2(q,\omega)-L^{-1}\,(G(D_0\,(q,\omega)))-L^{-1}\,(K\left(D_0\,(q,\omega)\right)) \tag{7}$$

$$D_2(q,\omega)=-L^{-1}\,(G(D_1\,(q,\omega)))-L^{-1}\,(K\left(D_1\,(q,\omega)\right)) \tag{8}$$

And after continuing with the same arrangement, we get

$$D_{n+1}\,(q,\omega)=-L^{-1}\,(G(D_n\,(q,\omega)))-L^{-1}\,(K\left(D_n\,(q,\omega)\right))\,,n\geq 0 \tag{9}$$

Hence, we continued with the solution following equation (5)

3.2 *Nonlinear Random Ordinary Differential Equation*

If $K\left(D(q,\omega)\right)$ is nonlinear in equation (4), be separated by an infinite series of Adomian Polynomials $B_n\,(q,\omega)$ given by

$$B_n\,(q,\omega)\ =\ \frac{1}{n!}\,\frac{d^n}{d\delta^n}\left[K(\textstyle\sum_{i=0}^{n}\delta^i\,D_i\,(q,\omega))\right]_{\delta=0}\,n=0,1,2\,\ldots\ldots \tag{10}$$

$$K\left(D(q,\omega)\right)=\sum_{n=0}^{\infty}B_n(q,\omega) \tag{11}$$

Where the Adomian Polynomials $B_n\,(q,\omega)$ be estimated for all forms of nonlinearity. The general formula (3) can be used as follow: $D_0\,(q,\omega)=\sum_{j=0}^{n-1}\frac{D_j}{j!}q^j\ +L^{-1}\,(Q(q,\omega)),\,B_0\,(q,\omega)=K(D_0\,(q,\omega))$

$$D_1\,(q,\omega)=f_2(q,\omega)-L^{-1}\,(G(D_0\,(q,\omega)))-L^{-1}\left(B_0\,(q,\omega)\right)$$

$$B_1\,(q,\omega)=\ D_1(q,\omega)\,K'(D_0\,(q,\omega))$$

$$D_2\,(q,\omega)=-L^{-1}\,(G(D_1\,(q,\omega)))-L^{-1}\left(B_1\,(q,\omega)\right)$$

$$B_2(q,\omega)\ =\ D_2(t,\omega)+\frac{1}{2!}D_1^2\,(K''\left(D_0\,(q,\omega)\right))$$

$$D_3\,(q,\omega)=-L^{-1}\,(G(D_n\,(q,\omega)))-L^{-1}\left(B_2\,(q,\omega)\right)$$

$$B_3(q,\omega)\ =D_3\,(q,\omega)K'(D_0\,(q,\omega))+\ D_1(q,\omega)\,D_2\,(q,\omega)\,K''(D_0\,(q,\omega)) \\ +\frac{1}{3!}\,D_1^3\,(q,\omega)\,K'''(D_0\,(q,\omega))$$

$D_4\,(\mathrm{q},\omega) = -L^{-1}\,(G(D_3\,(\mathrm{q},\omega))) - L^{-1}\,(B_3\,(\mathrm{q},\omega))$

$$\vdots$$

$$D_{n+1}\,(\mathrm{q},\omega) = -L^{-1}\,\Big(G\big(D_n\,(\mathrm{q},\omega)\big)\Big) - L^{-1}\,\big(B_n\,(\mathrm{q},\omega)\big)$$

In the following, some numerical examples (the first is linear and the second is nonlinear) will be simulated and solved using the above methods, and it is important to note that the generation of a distinct discretized Wiener process can be achieved by using the computer. These generations are given with 100 and 500.

Example 1: Consider the equation $\frac{d^3}{d\mathrm{q}^3}D(\mathrm{q},\omega) - \frac{d^2}{d\mathrm{q}^2}D(\mathrm{q},\omega) - \frac{d}{d\mathrm{q}}D(\mathrm{q},\omega) + D(\mathrm{q},\omega) = W_{\mathrm{q}}(\omega)$ with the initials value $D(0,\omega) = \frac{1}{2}$,$D'(0,\omega) = 0$ *and* $D''(0,\omega) = \frac{1}{2}$ by taking the triple integral from *0 to* q both sides it is obtained and by compensating the initial values: $D(\mathrm{q},\omega) = \frac{1}{2} - \frac{1}{2}\mathrm{q} + \frac{\mathrm{t}^3}{6}W_{\mathrm{q}}(\omega) + \int_0^{\mathrm{q}}\left(1 + \mathrm{q} - s - \frac{\mathrm{q}^2}{2} + \mathrm{q}s - \frac{s^2}{2}\right)D\,(s,\omega)ds$. *Let* $f(\mathrm{q},\omega) = \frac{1}{2} - \frac{1}{2}\mathrm{q} + \frac{\mathrm{t}^3}{6}W_{\mathrm{q}}(\omega)$ *where* $f_1(\mathrm{q},\omega) = \frac{1}{2} - \frac{1}{2}\mathrm{q}$ *and* $f_2(\mathrm{q},\omega) = \frac{\mathrm{q}^3}{6}W_{\mathrm{q}}(\omega)$, $D_0(\mathrm{q},\omega) = f_1(\mathrm{q},\omega) = \frac{1}{2} - \frac{1}{2}\mathrm{q}$

$$D_1(\mathrm{q},\omega) = f_2(\mathrm{q},\omega) + \int_0^{\mathrm{q}}\left(1 + \mathrm{q} - s - \frac{\mathrm{q}^2}{2} + \mathrm{q}s - \frac{s^2}{2}\right)D_0(s,\omega)\,ds$$

$$D_1(\mathrm{q},\omega) = \frac{\mathrm{q}^3}{6}W_{\mathrm{q}}(\omega) + \frac{(\mathrm{q}^3 + 18\mathrm{q}^2 - 12\mathrm{q})(\mathrm{q} - 2)}{48}$$

$$D_2(\mathrm{q},\omega) = \int_0^{\mathrm{q}}\left(1 + \mathrm{q} - s - \frac{\mathrm{q}^2}{2} + \mathrm{q}s - \frac{s^2}{2}\right)D_1(s,\omega)\,ds$$

$$D_2(\mathrm{q},\omega) = \frac{1}{4}\mathrm{q}^2 - \frac{3}{40}\mathrm{q}^5W_{\mathrm{q}}(\omega) - \frac{3}{4}\mathrm{q}^3 + \frac{31}{48}\mathrm{q}^4 - \frac{1}{720}\mathrm{q}^6W_{\mathrm{q}}(\omega) - \frac{31}{240}\mathrm{q}^5 - \frac{1}{96}\mathrm{q}^6 - \frac{1}{10080}\mathrm{q}^7 + \frac{1}{24}\mathrm{q}^4W_{\mathrm{q}}(\omega)$$

$$D_3(\mathrm{q},\omega) = \int_0^{\mathrm{q}}\left(1 + \mathrm{q} - s - \frac{\mathrm{q}^2}{2} + \mathrm{q}s - \frac{s^2}{2}\right)D_2(s,\omega)\,ds$$

$$D_3(\mathrm{q},\omega) = \frac{1}{120}\mathrm{q}^5W_{\mathrm{q}}(\omega) - \frac{1}{1680}\mathrm{q}^8W_{\mathrm{q}}(\omega) - \frac{121}{480}\mathrm{q}^6 + \frac{1}{362880}\mathrm{q}^9W_{\mathrm{q}}(\omega) + \frac{1}{12}\mathrm{q}^3 + \frac{1}{7257600}\mathrm{q}^{10} + \frac{37}{80}\mathrm{q}^5 - \frac{1}{36}\mathrm{q}^6W_{\mathrm{q}}(\omega) + \frac{17}{480}\mathrm{q}^7 + \frac{17}{5376}\mathrm{q}^8 + \frac{1}{22680}\mathrm{q}^9 + \frac{23}{1008}\mathrm{q}^7W_{\mathrm{q}}(\omega) - \frac{1}{3}\mathrm{q}^4$$

$$\vdots$$

As shown in the example solution in the figures (1), (2), (3), and (4):

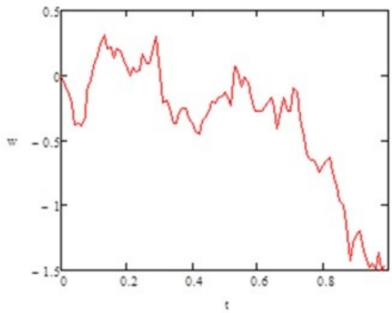

Figure 1
Discretized Wiener process with 100 generations

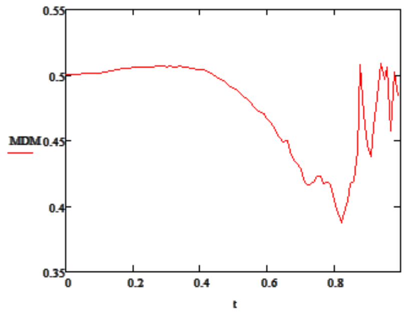

Figure 2
approximate solutions example (1) using MDM with 100 generations

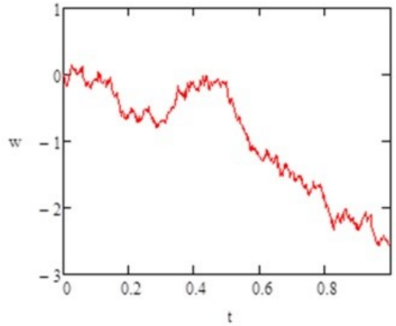

Figure 3
Discretized Wiener process with 500 generations

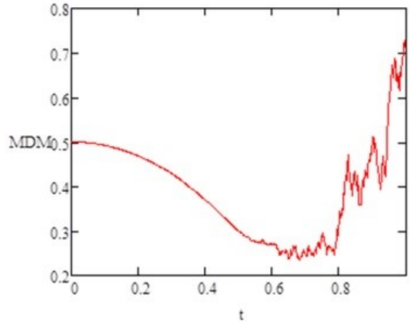

Figure 4
approximate solutions example (1) using MDM with 500 generations

Example 2: Take into consideration the nonlinear random ordinary differential equation $D'(\mathrm{q},\omega) = -2 + \frac{3+2\mathrm{q}}{1+\mathrm{q}} - Ln^2(1+\mathrm{q}) + W_{\mathrm{q}}(\omega) + D^2(\mathrm{q},\omega)$ With initial value $D(0,\omega) = 0$, and by taking integral both sides from 0 to q

$$D(\mathrm{q},\omega) = -2\mathrm{q} + (Ln(1+\mathrm{q}))(3+2\mathrm{q}) - (1+\mathrm{q})(Ln(1+\mathrm{q}))^2 + \mathrm{q}W_{\mathrm{q}}(\omega) + \int_0^{\mathrm{q}} D^2(s,\omega)ds$$

Let $f_1(\mathrm{q},\omega) = -2\mathrm{q} + (2\mathrm{q}\,\mathrm{Ln}\,(1+\mathrm{q}) + 3\,\mathrm{Ln}\,(1+\mathrm{q}))$

$$f_2(\mathrm{q},\omega) = -(\mathrm{Ln}\,(1+\mathrm{q}))^2(1+\mathrm{q}) + \mathrm{q}W_{\mathrm{q}}(\omega)$$

$$D_0\,(\mathrm{q},\omega) = -2\mathrm{q} + 3\,\mathrm{Ln}\,(1+\mathrm{q}) + 2\mathrm{q}\,\mathrm{Ln}\,(1+\mathrm{q})$$

$$B_0(\mathrm{q},\omega) = (D_0\,(\mathrm{q},\omega))^2 \,\&\, D_1\,(\mathrm{q},\omega) = f_2(\mathrm{q},\omega) + \int_0^{\mathrm{q}} B_0(s,\omega)ds$$

$$D_1\,(q,\omega)=\frac{32}{9}q^2-\frac{14}{9}(1+q)\,\mathrm{Ln}\,(1+q)^2-l\,\mathrm{n}\,(1+q)-\frac{26}{3}\mathrm{Ln}(1+q)+\frac{68}{27}q^3+\frac{14}{9}q+\mathrm{Ln}(1+q)^2\left(6q^2+9q+\frac{13}{3}+\frac{4}{3}q^3\right)-\frac{32}{3}\mathrm{Ln}(1+q)-\frac{32}{9}q^3\,\mathrm{Ln}(1+q)\ +qW_q(\omega)$$

$$B_1(q,\omega)=2D_0\,(q,\omega)D_1\,(q,\omega)\&D_2(q,\omega)=\int_0^q B_1(s,\omega)ds$$

$$D_2(q,\omega)=\Big[\frac{902}{1125}q-\frac{902}{1125}Ln(1+q)-q^2\left(\frac{2618}{45}Ln(1+q)^2-\frac{92}{3}Ln(1+q)^3\right)-q^3\left(\frac{2668}{45}Ln(1+q)^2-\frac{68}{3}Ln(1+q)^3\right)-q^4\left(\frac{136}{5}Ln(1+q)^2-8Ln(1+q)^3\right)-q^5\left(\frac{1024}{225}Ln(1+q)^2-\frac{16}{15}Ln(1+q)^3\right)+\frac{1348}{225}q\,Ln(1+q)-\frac{5}{3}\,W_q(\omega)\,Ln(1+q)-\frac{5}{6}\,W_q(\omega)q^2-\frac{16}{9}\,q^3\,W_q(\omega)-\frac{3821}{1125}\,q^2-\frac{32338}{3375}\,q^3-\frac{62213}{6750}\,q^4-\frac{18848}{5625}\,q^5-\frac{368}{15}\,q\,Ln(1+q)^2+\frac{6596}{225}\,q^2\,Ln(1+q)+20\,q\,Ln(1+q)^3+\frac{30488}{675}q^3\,Ln(1+q)+\frac{6598}{225}q\,Ln(1+q)+\frac{2254}{3375}q^5\,Ln(1+q)+\frac{5}{3}qW_q(\omega)-\frac{674}{225}Ln(1+q)^2+\frac{76}{15}Ln(1+q)^3+3q^2W_q(\omega)\,Ln(1+q)+\frac{4}{3}q^3W_q(\omega)\,Ln(1+q)\Big]$$

$$\vdots$$

As shown in the example solution in the figures (5) and (6):

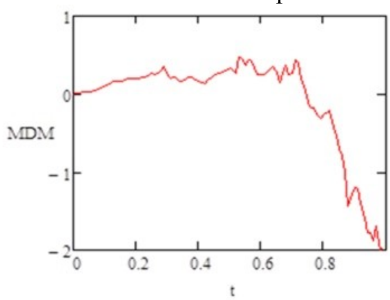

Figure 5
approximate solutions example (2) using MDM with 100 generations

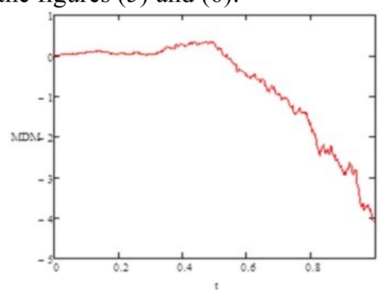

Figure 6
approximate solutions example (2) using MDM with 500 generations

4. Discussion and Conclusion

The proposed MDM in this study demonstrates high precision and efficiency in solving random ordinary differential equations, yielding reliable results for problems involving Brownian motion (Wiener process). Also, it's important that a solution of the (RODE) be based on the number of generations to the specific Wiener process. In addition, MDM can be applied to Random fractional differential equations or Random partial differential equations as future work.

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