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A new variant of secant technique for simulation of nonlinear equations

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Abstract

In this article, we report a new iterative technique for solving nonlinear equations. We derive the technique by using traditional Secant method. It is established that the new technique has 2.732 order of convergence. Some numerical examples show that newly technique is better than well-known existing techniques. The modified technique is a promising numerical tool for tackling nonlinear equations in various applications. This gives reason to be confident in the suggested approach and offers opportunities for future work to improve it even further.

Subject Classification: *65Hxx, 65H04.*

Keywords: *Convergence analysis, Number of iterations, Nonlinear problems, Secant method.*

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Introduction

Like in many other domains, nonlinear equations arise when modeling real-world issues in the domains of science, engineering, and applied mathematics [1,2]. These problems are always nontrivial, for which analytical solutions are not feasible, necessitating the use of numerical methods. [7]. In general, numerical methods are iterative in nature. They are characterized by estimating approximate solutions for the equations in an organized manner, starting with an initial approximate solution, from which a better approximate solution is expected to be obtained in the subsequent step, and so onUsing the weight function and composition techniques, Ogbereyivwe and Ojo-Orobosa [29] created a new two-step optimal fourth order family of iterative algorithms for solving nonlinear equations. The developed techniques require evaluation of three distinct function per iterations. For any iterative process, convergence is therefore essential. Furthermore, the rate of convergence is crucial since it might have economic or practical repercussions, particularly when it comes to real-life problems [1]. Inderjeet and Bhardwaj [31-32] presented modified Newton Raphson approaches to tackle the nonlinear equations with improved accuracy and robustness of the technique, and the proposed approaches are better than the existing techniques like Secant technique, Fixed–Point iteration etc. In technical terms, the obtained solution is referred to as the equation's root. The type of equation will determine whether there is a single real unique root, numerous real distinct or recurring roots, or even some imaginary roots. [10]. The focus of this research is in finding real and distinct (or simple) roots [3-21].

Researchers have created several variations of accelerated approaches that have proven useful in predicting the roots of nonlinear equations. Several techniques that exhibit comparable or superior performance are the Bisection, Regula-Falsi, and Newton approaches [12]. Many derivative-based iterative approaches for numerical solutions have been created, but there have also been many derivative-free methods with higher rates of convergence [1]. To strengthen the literature review, we can integrate recent studies that address the shortcomings of the classical Secant method and highlight the relevance of the proposed Secant technique. Mehdi Hosseini, M., Bidabadi, N., and R. Dehghani introduced two novel quasi-methods for unconstrained minimization. By establishing the theory of global convergence for one of the modified techniques and demonstrating that the suggested algorithms outperform the conventional BFGS method, the newly developed methods are appropriate for large-scale problems since they are derivative-free. [31].

Secant technique is preferred over Newton-Raphson technique in cases where the function is not differentiable. However, the classical Secant method has limitations, such as slower convergence rates and potential divergence, especially for functions with flat regions or multiple roots close to each other and Zhong et al. modified the Secant like method for nonlinear equations. [22, 23]. Laylani, Abdullah, and Hassan [30] discuss the classical Newtonian method is the most often used in single-variable scenarios. The second-order Taylor expansion, which the new secant technique is built on, does away with the need to compute the second derivative. Comparing the New Secant strategy to the current Newton method, numerical testing has demonstrated that it is both numerical and effective.

Recent studies have explored these shortcomings in details. For instance, Dutta and Bhattacharya [24] highlighted the instability of the classical Secant method in the presence of closely spaced roots and demonstrated that modifications are necessary to improve robustness and convergence. Additionally, Kheirandish and Ebrahimi [25] discussed the sensitivity of the classical Secant method to initial guesses, noting that poor initial estimates can significantly affect convergence.

To overcome the limitations of the traditional secant method, various modifications and enhancements have been proposed. These modifications aim to improve the convergence rate, stability, and efficiency of secant technique [26, 27]. One promising approach is to exploit higher-order derivative information to accelerate convergence and enhance robustness [28].

Proposed Modified Secant Method:

Let $y = f(x)$ be the graph of an arbitrary function f. Let $(x_0, f(x_0))$ and $(x_1, f(x_1))$ be two points on curve and a tangent is drawn by joining these two points x_0, x_1 and x_2 is x – axis intersection.

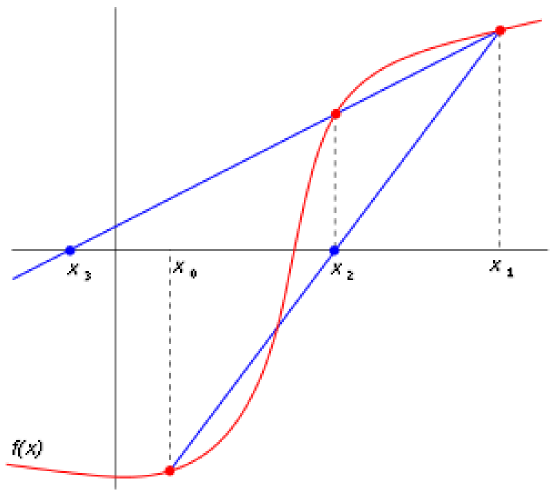

Figure 1
The starting point of the Secant technique showing first two points of the iterations.

Equation of line in slope – intercept form from figure 1 joining $(x_0, f(x_0))$ and $(x_1, f(x_1))$ is

$$y = \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_1) + f(x_1) \tag{1}$$

Equation (1) has root when $y = 0$

$$0 = \frac{f(x_1)-f(x_0)}{x_1-x_0}(x - x_1) + f(x_1)$$

We solve this equation with respect to x

$$x = x_1 - f(x_1)\,\frac{x_1-x_0}{f(x_1)-f(x_0)} \tag{2}$$

This x is used for the next iteration as x_2 and x_1, x_2 are used in place of x_0, $x_1$respectively. This procedure is carried out repeatedly until a high level of precision is attained.

$$x_n = x_{n-1} - f(x_{n-1})\,\frac{x_{n-1}-x_{n-2}}{f(x_{n-1})-f(x_{n-2})} \tag{3}$$

Equation (3) is Secant formula.

The Secant method's convergence is not always guaranteed and hence when using a computer, the number of iterations must be restricted.

Now we derived modified Secant method

The classical Secant technique is

$$x_n = x_{n-1} - f(x_{n-1})\,\frac{x_{n-1}-x_{n-2}}{f(x_{n-1})-f(x_{n-2})} \tag{4}$$

The traditional Secant method does not require any derivative, but it has 1.618 convergence order.

But Zhang at el. (22) modified the traditional Secant method and proposed a new formula as follows

$$x_{n+1} = x_n - \frac{x_n-y_n}{f(x_n)-f(y_n)}f(x_n) \tag{5}$$

Where $y_{n+1} = x_{n+1} - \frac{x_n-y_n}{f(x_n)-f(y_n)}f(x_{n+1})$

This method has 2.618 convergence order.

Now we derive the modified Secant technique by using the polynomial function

$$P(x) = f(x_n) + {w_n}^{-1}(x - x_n) + \frac{({w_{n-1}}^{-1}-{w_n}^{-1})(x-x_n)(x-y_n)}{\gamma_1 x_{n-1}+\gamma_2 y_{n-1}+(2-\gamma_{n-1}-\gamma_{n-2})z_{n-1}-\delta_1 x_n-\delta_2 y_n-(2-\delta_1-\delta_2)z_n} \tag{6}$$

Where $\gamma_1, \gamma_2, \delta_1,$ and $\delta_2 \in R$, $y_n = x_n - w_{n-1}f(x_n)$, $z_n = x_n - w_n f(x_n)$, and $w_n = \frac{y_n-x_n}{f(y_n)-f(x_n)}$

To remove nonlinearity, we replace x_{n+1} by $(x_{n+1} - x_n)$ and $(x_{n+1} - y_n)$ of $P(x_{n+1})$ with y_n and z_n respectively.

Hence, proposed modified technique is

$$y_n = x_n - w_{n-1}f(x_n)$$

$$w_n = \frac{y_n-x_n}{f(y_n)-f(x_n)}$$

$$z_n = x_n - w_n f(x_n)$$

$$x_{n+1} = z_n - \frac{(y_n-z_n)^2}{\gamma_1 x_{n-1}+\gamma_2 y_{n-1}+(2-\gamma_{n-1}-\gamma_{n-2})z_{n-1}-\delta_1 x_n-\delta_2 y_n-(2-\delta_1-\delta_2)z_n} \tag{7}$$

Where $\gamma_1, \gamma_2, \delta_1,$ and $\delta_2 \in R$

Convergence Analysis:

Theorem: *Suppose f be differentiable function on an interval I, $\alpha \in I$ is simple zero. If initial point x_0 is sufficiently near to α then the proposed algorithm has 2.732 orders of convergence when $\gamma_1 = \gamma_2 = 1$ and satisfying the following error equation*

$$e_{n+1} = c^{t+1} {e_{n-1}}^{t^2} - c^3 {e_{n-1}}^{2t+2} + \ldots$$

Where $c_n = \frac{f^n(\alpha)}{n! f'(\alpha)}$, $n = 2,3,4 \ldots$ and $e_n = x_n - \alpha$.

Proof: Let $R_n = y_n - \alpha$ and $S_n = z_n - \alpha$

Taylor's series expansion of $f(x_n)$ around α , we obtained

$$f(x_n) = f(\alpha + e_n) = f'(\alpha)[e_n + c_2 {e_n}^2 + c_3 {e_n}^3 + c_4 {e_n}^4 + c_5 {e_n}^5 + c_6 {e_n}^6 + c_7 {e_n}^7 + c_8 {e_n}^8 + \tag{8}$$

$$f(y_n) = f(\alpha + e_n) = f'(\alpha)[R_n + c_2 {R_n}^2 + c_3 {R_n}^3 + c_4 {R_n}^4 + c_5 {R_n}^5 + c_6 {R_n}^6 + c_7 {R_n}^7 + c_8 {R_n}^8 + \ldots] \tag{9}$$

From equation (8) and equation (9), we get

$$w_n = \frac{y_n - x_n}{f(y_n) - f(x_n)}$$

$$w_n = \frac{1}{f'(\alpha)} \frac{(y_n - \alpha) - (x_n - \alpha)}{(R_n - e_n) + c_2({R_n}^2 - {e_n}^2) + c_3 ({R_n}^3 - {e_n}^3) + \ldots} \tag{10}$$

$$w_n = \frac{1}{f'(\alpha)} \frac{1}{1 + c_2(e_n + R_n) + \frac{c_3}{2}(e_n + R_n)^2 + \ldots} \tag{11}$$

$$w_n = \frac{1}{f'(\alpha)}[1 - c_2(e_n + R_n) - \frac{c_3}{2}(e_n + R_n)^2 + \ldots] \tag{12}$$

So, from the equation (8), (9), and (12), we obtained

$$R_n = c_2 e_n(e_{n-1} + R_{n-1}) + (c_3 - {c_2}^2)e_n + (e_{n-1} + R_{n-1})^2 - c_2 {e_n}^2 + \tag{13}$$

$$S_n = x_n - \alpha - w_n f(x_n) \tag{14}$$

$$S_n = c_2 e_n R_n + (c_3 - {c_2}^2)({e_n}^2 R_n + {R_n}^2 e_n + \tag{15}$$

From equation (7), we get

$$e_{n+1} = S_n - \frac{(R_n - S_n)^2}{\gamma_1 e_{n-1} + \gamma_2 R_{n-1} + (2 - \gamma_1 - \gamma_2) S_{n-1} - \delta_1 e_n - \delta_2 R_n - (2 - \delta_1 - \delta_2) S_n} \tag{16}$$

Let us suppose that $\gamma_1 \neq 0$, so we obtained

$$e_{n+1} = c_2 e_n R_n - \frac{1}{\gamma_1} \frac{c_2 e_n\left(1 + \frac{R_{n-1}}{e_{n-1}}\right) + \ldots}{1 + \frac{\gamma_2}{\gamma_1}\frac{R_{n-1}}{e_{n-1}} + \ldots}(R_n - S_n) + \tag{17}$$

$$e_{n+1} = \left(1 - \frac{1}{\gamma_1}\right) {c_2}^2 {e_n}^2 e_{n-1} + [1 - \frac{1}{\gamma_1}\left(2 - \frac{\gamma_2}{\gamma_1}\right)] {c_3}^2 {e_n}^2 e_{n-1} e_{n-2} + \tag{18}$$

From equation (18) we observed that the convergence order can be improved by taking $\delta_1 = \delta_2 = 1$, then from equations (13), (15), and (16), we get

$$e_{n+1} = S_n - \frac{R_n - S_n}{e_{n-1} + R_{n-1} - \delta_1 e_n - \delta_2 R_n - (2 - \delta_1 - \delta_2) S_n}(R_n - S_n) \tag{19}$$

$$e_{n+1} = c_2 e_n R_n - (R_n - S_n) \frac{c_2 e_n (e_{n-1} + R_{n-1}) + (c_3 - c_2^2) e_n (e_{n-1} + R_{n-1}^2) + \dots}{e_{n-1} + R_{n-1} - \delta_1 e_n - \delta_2 R_n - (2 - \delta_1 - \delta_2) S_n} \quad (20)$$

$$e_{n+1} = (c_2^2 - c_3) e_n e_{n-1} R_n - c_2 \delta_1 \frac{e_n^2}{e_{n-1}} R_n + \quad (21)$$

$$e_{n+1} = c_2\big((c_2^2 - c_3) e_n^2 e_{n-1}^2 + c_2^2 e_n^3 \delta_1 + \quad (22)$$

$$e_{n+1} = c_2 (c_2^2 - c_3) e_n^2 e_{n-1}^2 + \quad (23)$$

$$e_{n+1} = c e_n^2 e_{n-1}^2 + \quad (24)$$

Suppose that the proposed method defined in equation (7) has convergence order t, then

$$e_n = c e_{n-1}^t + \dots \text{ and } e_{n+1} = c e_n^t + \dots = c^{t+1} e_{n-1}^{t^2} + \dots$$

$$c^{t+1} e_{n-1}^{t^2} = c^3 e_{n-1}^{2t+2} + \dots$$

$$t^2 = 2t + 2$$

$$t = 2.73$$

Hence, the proposed algorithm has 2.732 order of convergence.

Proposed Algorithm

So, by taking $\delta_1 = \delta_2 = 1$ in equation (7) so we get the proposed methodology

$$y_n = x_n - w_{n-1} f(x_n)$$

$$w_n = \frac{y_n - x_n}{f(y_n) - f(x_n)}$$

$$z_n = x_n - w_n f(x_n)$$

$$x_{n+1} = z_n - \frac{(y_n - z_n)^2}{x_{n-1} + y_{n-1} - \delta_1 x_n - \delta_2 y_n - (2 - \delta_1 - \delta_2) z_n}$$

Where δ_1, and $\delta_2 \in R$

Numerical Examples:

To evaluate performance of the modified Newton-Raphson approach, numerical simulations were conducted on several benchmark problems involving nonlinear equations. The proposed method was compared with others existing techniques likes Secant method (NRM), Zhang method (ZM), and proposed method (PM).

The stopping criterion are used in the proposed algorithm is

$$|x_{n+1} - x_n| < \epsilon = 10^{-15}.$$

Table 1

Comparison of various iterative schemes

Function	Initial Point	Actual Root	Number of Iterations		
			SM	ZM	PM
$x^3 + 4x^2 - 10$	-0.3	1.3652	54	21	11

Contd...

$x^2 - e^x - 3x + 2$	0.5	0.2575	9	5	4
$sin^2x - x^2 + 1$	1.2	1.4044	5	2	2
$\cos x - x$	1	0.7391	13	7	2
$e^{-x} + \cos x$	2.4	1.7461	12	8	5

Result Discussions and Conclusion:

In this article, we presented a modified technique for numerically simulating and approximation roots of nonlinear equations. Proposed technique enhances the efficiency and convergence rate of classical Secant technique and Zhang technique. Theoretical analysis and convergence results were established, showing that the modified method achieves a convergence rate of order 2.73, which is faster than the traditional Secant technique and Zhang technique. Numerical experiments on benchmark problems demonstrated the superior performance of the modified technique compared to others established methods like Secant technique and Zhang technique as shown in table 1. The modified technique exhibited faster convergence, higher accuracy, and required fewer number of iterations see table1.

In conclusion, the modified approach presented in this paper offers a powerful and efficient numerical algorithm for simulating and solving nonlinear equations. Its enhanced convergence properties, fewer number of iterations.

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Conflict of Interest

None.

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