Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 28 (2025), No. 7, pp. 2687–2706
DOI : 10.47974/JIM-2327

Separation axioms in q∗-rung orthopair neutrosophic topological spaces

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Abstract

In the framework of q*-rung orthopair neutrosophic topological space(q*-RONTS), this study focuses on establishing and analyzing separation axioms, such as N^*_{qn} T_0, N^*_{qn} T_1 and $N^*_{qn}T_2$. The paper explores the manifestation of these axioms within the broader framework of q*-rung orthopair neutrosophic (q*-RON) theory, taking into account the complex role of degrees of truth, falsehood, and indeterminacy. Additionally demonstrate how the unique structure of q*-RONTS influences the separation of sets and points using in-depth definitions, theorems, and examples. The analysis of the impact of separation qualities in q*-rung orthopair neutrosophic topoligical subspaces and how these properties are maintained or changed under continuous mappings is also included in this paper.

Subject Classification: *54A05, 94D05, 54A40.*

Keywords: *q*-rung orthopair neutrosophic set (q*-RONS), q*-RONTS, q*-rung orthopair neutrosophic point (q*-RONP), q*-rung orthopair neutrosophic continuous (q*-RONC), q*-RON N^*_{qn} T_i-spaces (i = 0, 1, 2).*

1. Introduction

In the field of topology, the idea of separation axioms has unquestionably always been essential. Numerous worthwhile studies in general topology were founded on this idea. Additionally, these studies had a significant impact on many aspects of daily life, and their conclusions were put into practice in several

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applications. However, as the industry and technology develop, people's needs have evolved, and general topology is no longer sufficient in practical applications. As a result, these findings' practical implications have lessened. Scientists then discovered that there were several kinds of topological spaces, and that separation axioms played a significant role in these topological spaces.Smarandache introduced the idea of a neutrophilic set in [1]. In several investigations, this concept emerged as the main player. Additionally, in [2], Salma and Alblowi presented the theory of neutrosophic topological space employing this novel idea. As a new tool for practical applications and as new terminology that we believe will be useful in future research, we describe many kinds of separation axioms in q^*-RONTS in this paper.

In the field of mathematics known as topology, the characteristics of things that resist constant change are examined. The study of spaces, forms, and their properties is what it is all about. Applications for topology can be found in many domains, including biology, chemistry, computer science, engineering, and physics [3], [4], [5], [6], [7]. The isomorphisms that define the category of topological spaces are referred as homeomorphisms, and they are essential to the theory.

In this paper introduces new formulations of separation axioms in q^*-RONTS, examining various aspects and interrelationships of these definitions. Properties of "good extension" for each type of space are demonstrated. The definition criteria aim to preserve the relationships between separation features in q^*-RONTS. The goal is to enhance the understanding of q^*-RON $N^*_{qn}T_0$-spaces, $N^*_{qn}T_1$-spaces, $N^*_{qn}T_2$-spaces, and related separation axioms.

2. Literature review

Various studies have been done in q-rung orthopair fuzzy sets(q-ROFS). Zadeh [8] established the concept of a fuzzy set(FS). A FS is one in which items can have membership degrees ranging from 0 to 1, as opposed to having a binary membership. The FS theory has many extensions and variants. The concept of an intuitionistic fuzzy set(IFS) [9] is one of these extensions, which includes a second degree of belongingness to denote nonbelongingness. In 2013, Revathi [10] introduced the concept of Ordered (r, s) intuitionistic fuzzy quasi-uniform regular $G\delta$ extremally disconnected spaces. In a subsequent study, Revathi [11] presented the notion of regular $G\delta$ -continuous mappings.The idea of the Pythagorean fuzzy set (PFS) is another extension [12]. On the other hand, the idea of q-ROFS generalizes PFS [13]. Another form, Picture fuzzy set, explains the neutral belongingness degree of the elements using a third function [14].

Proposed in 2019, spherical fuzzy set [15], [16] expressed the degrees of belongingness of elements using the idea of spherical belongingness functions. Finally, q-ROPFS [17] combines the ideas of q-ROFS and picture FS. Multiple applications in areas including pattern recognition, image processing, and decision-making have been made possible by these multiple expansions and modifications of FS theory [18], [19]. After Zadeh, a variety of FS extensions were introduced and examined, such as IFS [20], picture fuzzy sets [21],

neutrosophic sets (NS) [1], Pythagorean neutrosophic sets (PNS) [22], q-ROFS [13] and Fermatean neutrosophic sets [23]

Cheng-Ming [24] introduced the concept of fuzzy topological spaces (FTS), while Lowen [25] later proposed an alternative definition that modified a fundamental aspect of topology. Çoker [26] and Turanlı and Çoker [27] further advanced this concept by introducing intuitionistic fuzzy topological spaces (IFTS) and exploring key topological properties such as continuity and compactness. Olgun [28] refined Pythagorean fuzzy continuity for functions between picture fuzzy topological spaces, while Türkarslan et al. [29] developed the topology of q-ROFS. Razaq et al. [30] examined continuity within these spaces and picture fuzzy topology. Neutrosophic topological space can be applied to many engineering problems. This led to the development of neutrosophic sets by Smarandache [31] in 1998. In 2024, Shyamaladevi [32] conducted an extensive review of numerous papers exploring the practical applications of neutrosophic topology, a remarkable extension of classical topology.

Shen [33] introduced separation axioms namely T0, T1, T2, T3, and T4 within the framework of fuzzifying topology. Subsequently, Bayhan [34] and Lupianez [35] extended these fuzzy separation axioms to Intuitionistic Fuzzy Topological Spaces (IFTS). Later developments by Jasim [36], Ali [37], and Ozturk [38] introduced various forms of separation axioms in fuzzy topological spaces. Neutrosophic topological spaces, as a generalization of classical topology, have found numerous applications in engineering and other fields . In 2024, Shyamaladevi [39] provided a comprehensive review of literature highlighting the practical applications of neutrosophic topology, emphasizing its utility in addressing problems characterized by uncertainty and indeterminacy. More recently, Dey [40] formulated the T0, T1, and T2 separation axioms within the context of neutrosophic topological spaces. Building on these foundational studies, the present work proposes an extended formulation of separation axioms within the framework of q^*-rung orthopair neutrosophic spaces.

3. Motivation of the study

This paper aims to develop the theories of q^*-RON $N_{qn}^* T_0$-spaces, q^*-RON $N_{qn}^* T_1$-spaces, q^*-RON $N_{qn}^* T_2$-spaces, q^*-RON separation axioms. Before that, we will look into a few characteristics of q^*-RON sets. Because many features of topological spaces are related to subspaces, the importance of subspace topology and subspaces cannot be understated, much as in the study of topological spaces in classical, fuzzy, or other contexts.

The following are the primary contributions of this paper:

(i) The innovative ideas, q^*-RONT, q^*-RONP, q^*-RONC and q^*-RON subspaces are introduced.

(ii) Define the q^*-rung orthopair neutrosophic $N_{qn}^* T_i$-spaces $(i = 0,1,2)$ and examined various properties associated with these separation axioms.

4. Structure of the paper

The study is divided into distinct sections, each addressing specific aspects of FS theory and q*-RONS. Section 5 establishes the foundation by introducing fundamental concepts in FS theory and examining set-theoretic approaches related to q*-RONS , q*-RONTS and q*-RONTC functions, providing a basis for further exploration. Section 6 establishes the q* -RONP and q* -RON subspace topology. Section 7 delves into various separation axioms, analyzing their distinct properties. The paper concludes in Section 8 by emphasizing the significance of these separation axioms within the broader context of topological research and detailing their

5. Preliminaries

This section introduces the necessary preliminary definitions to provide a clear understanding of the subsequent outcomes.

Definition 5.1: [8] Consider X as a non-empty set. A *FS* A'_f is defined as $A'_f = \{(x, \mu_{A'_f}(x)) : x \in X\}$, where $\mu_{A'_f}(x) : X \to [0,1]$ specifies the membership degree of each element x in the *FS* A'_f. A fuzzy set represents a collection of elements, each associated with a membership value indicating its degree of inclusion.

Definition 5.2: [41] A q-ROFS $\acute{A}$ in the universal set X is defined as $\acute{A} = \{(x, \mu_{\acute{A}}(x), \nu_{\acute{A}}(x)), x \in X\}$ where the condition $0 \leq \mu_{\acute{A}}^q(x) + \nu_{\acute{A}}^q(x) \leq 1$ holds. Here $\mu_{\acute{A}} : X \to [0,1]$ represents the degree of belongingness and $\nu_{\acute{A}} : X \to [0,1]$ represents the degree of non-belongingness of an element $x \in X$ in the set A'.

Definition 5.3: [1] Assuming that $X \neq \emptyset$, then the set $\acute{A} = \{(x, \mu_{\acute{A}}(x), \nu_{\acute{A}}(x)), \pi_{\acute{A}}(x), x \in X\}$ is called NS on X, where $0 \leq \mu_{\acute{A}}(x) + \nu_{\acute{A}}(x) + \pi_{\acute{A}}(x) \leq 3$ for all $x \in X$, $\nu_{\acute{A}} : X \to [0,1]$ is the degree of non-belongingness and $\pi_{\acute{A}} : X \to [0,1]$ is the degree of indeterminacy and $\mu_{\acute{A}} : X \to [0,1]$ is the degree of belongingness of every $x \in X$ to the set $\acute{A}$ as well.

Definition 5.4: [42] Assume that the set X is not an empty, then the set $\acute{A}_{qn} = \{(x, \mu_{\acute{A}_{qn}}(x), \nu_{\acute{A}_{qn}}(x)), \pi_{\acute{A}_{qn}}(x), x \in X\}$ where $0 \leq \mu_{\acute{A}_{qn}}^q(x) + \nu_{\acute{A}_{qn}}^q(x) \leq 1$ and for all $x \in X$ such that $0 \leq \mu_{\acute{A}_{qn}}^q(x) + \nu_{\acute{A}_{qn}}^q(x) + \pi_{\acute{A}_{qn}}^q(x) \leq 2$ is called q-RONS where $q \geqslant 1$, $\mu_{\acute{A}_{qn}} : X \to [0,1]$ is the degree of belongingness, $\nu_{\acute{A}_{qn}} : X \to [0,1]$ is the degree of non-belongingness and $\pi_{\acute{A}_{qn}} : X \to [0,1]$ is the degree of indeterminacy of the element $x \in X$ to the set $\acute{A}_{qn}$. The symbol for the set of all q-RONS over X is $\acute{N}_{qn}(X)$.

Definition 5.5: [39] For a q-RONS

$\acute{A}_{qn} = \{(x, \mu_{\acute{A}_{qn}}(x), \pi_{\acute{A}_{qn}}(x), \nu_{\acute{A}_{qn}}(x), x) \in X\}$ of the non-empty fixed set X, the q*-RONS $\acute{A}_{qn}^*$, is defined to be the following triple structure:

$\acute{A}^*_{qn} = \{(\mu^*_{\acute{A}_{qn}}, \pi^*_{\acute{A}_{qn}}, \nu^*_{\acute{A}_{qn}})\}$ where $\mu^*_{\acute{A}_{qn}} = \min(\mu_{\acute{A}_{qn}}, 1 - \max(\pi_{\acute{A}_{qn}}, \nu_{\acute{A}_{qn}}))$, $\pi^*_{\acute{A}_{qn}} = \min(\pi_{\acute{A}_{qn}}, 1 - \max(\mu_{\acute{A}_{qn}}, \nu_{\acute{A}_{qn}}))$ $\nu^*_{\acute{A}_{qn}} = \min(\nu_{\acute{A}_{qn}}, 1 - \max(\mu_{\acute{A}_{qn}}, \pi_{\acute{A}_{qn}}))$ and $0 \leq \mu^{*q}_{\acute{A}^*_{qn}}(x) + \nu^{*q}_{\acute{A}^*_{qn}}(x) \leq 1$ and for all $x \in X$ such that $0 \leq \mu^{*q}_{\acute{A}^*_{qn}}(x) + \nu^{*q}_{\acute{A}^*_{qn}}(x) + \pi^{*q}_{\acute{A}^*_{qn}}(x) \leq 2$ is called q*-RONS where $q \geqslant 1$, $\mu^*_{\acute{A}^*_{qn}}: X \to [0,1]$ is the degree of belongingness, $\nu^*_{\acute{A}^*_{qn}}: X \to [0,1]$ is the degree of non-belongingness and $\pi^*_{\acute{A}^*_{qn}}: X \to [0,1]$ is the degree of indeterminacy of the element $x \in X$ to the set $\acute{A}^*_{qn}$. The symbol for the set of all q*-RONS over X is $\acute{N}^*_{qn}(X)$

Definition 5.6: [39] Let $X \neq \phi$, $\acute{A}^*_{qn}$ and $\acute{B}^*_{qn}$ be the q*-RON subsets in X with the notation $\acute{A}^*_{qn} = \{(x, \mu^*_{\acute{A}_{qn}}(x), \nu^*_{\acute{A}_{qn}}(x), \pi^*_{\acute{A}_{qn}}(x)): x \in X\}$ and $\acute{B}^*_{qn} = \{(x, \mu^*_{\acute{B}_{qn}}(x), \nu^*_{\acute{B}_{qn}}(x), \pi^*_{\acute{B}_{qn}}(x)): x \in X\}$. If $\mu_{\acute{A}^*_{qn}}(x) \leqslant \mu^*_{\acute{B}_{qn}}(x), \pi^*_{\acute{A}_{qn}}(x) \leqslant \pi^*_{\acute{B}_{qn}}(x)$ and $\nu^*_{\acute{B}_{qn}}(x) \geqslant \nu^*_{\acute{B}_{qn}}(x)$ then it is denoted as $\acute{A}^*_{qn} \subseteq \acute{B}^*_{qn}$.

Definition 5.7: [39] For q*-RONS $\acute{A}^*_{qn}$ and $\acute{B}^*_{qn}$, $\acute{A}^*_{qn} \cup \acute{B}^*_{qn}, \acute{A}^*_{qn} \cap \acute{B}^*_{qn}$ and $\acute{A}^{*c}_{qn}$ will be defined as

(i) $\acute{A}^*_{qn} \cup \acute{B}^*_{qn} = \{(x, max\{\mu^*_{\acute{A}_{qn}(x)}, \mu^*_{\acute{B}_{qn}(x)}\}, max\{\pi^*_{\acute{A}_{qn}}(x), \pi^*_{\acute{B}_{qn}}(x)\}, min\{\nu^*_{\acute{A}_{qn}}(x), \nu^*_{\acute{B}_{qn}}(x)\})\}$

(ii) $\acute{A}^*_{qn} \cap \acute{B}^*_{qn} = \{(x, min\{\mu^*_{\acute{A}_{qn}}(x), \mu^*_{\acute{B}_{qn}}(x)\}, min\{\pi^*_{\acute{A}_{qn}}(x), \pi^*_{\acute{B}_{qn}}(x)\}, max\{\nu^*_{\acute{A}_{qn}}(x), \nu^*_{\acute{B}_{qn}}(x)\})\}$

(iii) $\acute{A}^{*c}_{qn} = \{(x, \nu^*_{\acute{A}_{qn}}(x), 1 - \pi^*_{\acute{A}_{qn}}(x), \mu^*_{\acute{A}_{qn}}(x): x \in X)\}$.

Remark 1:

(i) All types of $\emptyset^*_{qn}$ and $\emptyset_{qn}$ are conceded , that is $\emptyset_{qn} = (0,0,1) = \emptyset^*_{qn}$.

(ii) All types of X^*_{qn} and X_{qn} are conceded, that is $X_{qn} = (1,1,0) = X^*_{qn}$.

Definition 5.8: [39] Let $X \neq \phi$ and τ^*_{qn} be a collection of q*-RON subsets of X. If τ^*_{qn} satisfies the following properties then it is called a s q*-RONT.

(i) $\emptyset^*_{qn}, X^*_{qn} \in \tau^*_{qn}$, where $\emptyset^*_{qn} = (0,0,1)$ and $X^*_{qn} = (1,1,0)$.

(ii) If $\acute{A}^*_{qn_i}, \acute{B}^*_{qn_i} \in \tau^*_{qn}$ then $\acute{A}^*_{qn_i} \cap \acute{B}^*_{qn_i} \in \tau^*_{qn}$

(iii) For all $i \in J$, if $\{\acute{A}^*_{qn_i}\} \in \tau^*_{qn}$ then $(\bigcup_{i=1}^{n} \acute{A}^*_{qn_i}) \in \tau^*_{qn}$.

Then (X, τ^*_{qn}) is called a q*-RONTS.

Every member of q* -RONT is called as open q* -RONS, and its complement is closed q*-RONS.

Definition 5.9: [39] Let $(\acute{A}^*_{qn}, \tau^*_{qn_1}), (\acute{B}^*_{qn}, \tau^*_{qn_2})$ be any two q*-RONTS and $\tau^*_{qn_1} \subset \tau^*_{qn_2}$. Then, the $\tau^*_{qn_2}$ q*-RONT is said to be finer than the q*-RONT $\tau^*_{qn_1}$.

Definition 5.10: [39] Let (X, τ^*_{qn}) be a q*-RONTS, and let $\acute{A}^*_{qn} \subseteq X$ be a q*-RONS in X. If there is an open q*-RON subset $\acute{G}^*_{qn}$ such that $\acute{A}^*_{qn} \subset \acute{A}^*_{qn} \subset X$, then X is said to be a neighbourhood of $\acute{A}^*_{qn}$.

Definition 5.11: [39] Let (X^*, τ^*_{qn}) be q*-RONTS and let $\acute{A}^*_{qn} = \{(x, \mu^*_{\acute{A}^*_{qn}}(x), \nu^*_{\acute{A}^*_{qn}}(x)), \pi^*_{\acute{A}^*_{qn}}(x): x \in X\}$ be a q*-RONS. In this case q*-rung orthopair interior and closure for $\acute{A}^*_{qn}$ are defined as

(i) $Int_{qn}(\acute{A}^*_{qn}) = \bigcup \{\acute{O}^*_{qn_i}: \acute{O}^*_{qn_i} \subset A^*_{qn_i}, \acute{O}^*_{qn_i}$ is open q*-RONS$\}$,

(ii) $Cl_{qn}(\acute{A}^*_{qn}) = \bigcap \{\acute{C}^*_{qn_i}: \acute{A}^*_{qn_i} \subset \acute{C}^*_{qn_i}, \acute{C}^*_{qn_i}$ is closed q*-RONS $\}$.

Definition 5.12: [39] Consider any two q*-RONTS, $(X, \tau^*_{qn_1})$ and $(Y, \tau^*_{qn_2})$. Then let $f: X \to Y$ be a function. If q*-RONS is open $f^{-1}(B^*_{qn})$ is an open q*-RONS of X, and B^*_{qn} of Y , then f is q*-RONC.

Definition 5.13: [39] Let $(X, \tau^*_{qn_1})$ and $(Y, \tau^*_{qn_2})$ be any two q*-RONTS and let $f: X_{qn} \to Y$ be a function. Then f is said to be q*-RON open iff the image of each q*-RON in $\tau^*_{qn_1}$ is a q*-RONS in $\tau^*_{qn_2}$.

6. q*-rung orthopair neutrosophic subspace topology

Definition 6.1: Let $\alpha_1, \alpha_2, \alpha_3 \in [0,1]$ and $\alpha_1 + \alpha_2 + \alpha_3 \leqslant 2$. A q*-RONP $\acute{P}^*_{qn}x(\alpha_1, \alpha_2, \alpha_3)$ of X defined by $\acute{P}^*_{qn}x(\alpha_1, \alpha_2, \alpha_3) = \{x, \mu^*_{qn_{\acute{P}_{qn}}}(x), \pi^*_{qn_{\acute{P}_{qn}}}(x), \nu^*_{qn_{\acute{P}_{qn}}}(x) : x \in X\}$ for $x_1 \in X$

$$\mu^*_{qn_{(\acute{P}_{qn})}}(x) = \begin{cases} \alpha_1, & \text{if } x_1 = x \\ 0, & \text{if } x_1 \neq x \end{cases}$$

$$\pi^*_{qn_{(\acute{P}_{qn})}}(x) = \begin{cases} \alpha_2, & \text{if } x_1 = x \\ 0, & \text{if } x_1 \neq x \end{cases}$$

$$\nu^*_{qn_{(\acute{P}_{qn})}}(x) = \begin{cases} \alpha_3, & \text{if } x_1 = x \\ 1, & \text{if } x_1 \neq x \end{cases}.$$

The collection of all q*-RONP (X, τ^*_{qn}) is denoted by $\acute{P}^*_{qn}$.

Definition 6.2: Let $(X, \tau^*_{qn_1})$ and $(Y, \tau^*_{qn_2})$ be any two q*-RONTS and let $f: X \to Y$ be a function. The function f is called a q* -RON homeomorphism if it satisfies the following conditions:

(i) f is a bijective(both injective and surjective).

(ii) f is a q*-RONC.

(iii) The inverse function f^{-1} is also q*-RONC.

Definition 6.3: Let X and Y be any two crisp sets such that $Y \neq \emptyset$ and $Y \subseteq X$. Define $Y^*_{qn} = \{(x, \mu^*_{qn}, \pi^*_{qn}, \nu^*_{qn}): x \in X\}$, where $\mu^*_{qn} = 1, \pi^*_{qn} = 1, \nu^*_{qn} = 0$ if

$x \in Y$ and $\mu^*_{qn} = 0, \pi^*_{qn} = 0, \nu^*_{qn} = 1$ if $x \in X|Y$. The set of all q*-RON sets over Y will be denoted by $N^*_{qn}(Y)$

Definition 6.4: Let X and Y be any two crisp sets such that $Y \neq \emptyset$ and $Y \subseteq X$, then for a q* -RON set $A^*_{qn} \in N^*_{qn}(X)$, we define $A^*_{qn}|Y = \{(x, \mu^*_{A^*_{qn}|Y}, \pi^*_{A^*_{qn}|Y}, \nu^*_{A^*_{qn}|Y}): x \in X\}$, where $\mu^*_{A^*_{qn}|Y}(x) = \mu^*_{A^*_{qn}}, \pi^*_{A^*_{qn}|Y}(x) = \pi^*_{A^*_{qn}}, \nu^*_{A^*_{qn}|Y}(x) = \nu^*_{A^*_{qn}}$ if $x \in Y$ and $\mu^*_{A^*_{qn}|Y}(x) = 0, \pi^*_{A^*_{qn}|Y}(x) = 0, \nu^*_{A^*_{qn}|Y}(x) = 1$ if $x \in X|_Y$.

Remark 2: From the Definition 6.3 and 6.4

(i) $A^*_{qn}|_Y \in N^*_{qn}(Y)$ for every $A^*_{qn} \in N^*_{qn}(X)$.

(ii) Every q*-RON set A^*_{qn} over Y can be considered as an q*-RON set over X by taking $\mu^*_{A^*_{qn}} = 0, \pi^*_{A^*_{qn}} = 0, \nu^*_{A^*_{qn}} = 1$ for all $x \in X|_Y$

(ii) $X^*_{qn}|_Y = Y^*_{qn}$ and $\emptyset^*_{qn}|_Y = \emptyset^*_{qn}$

Definition 6.5: Let (X, τ^*_{qn}) be a q* -RONTS. Let $\emptyset \neq Y \subseteq X$ and $\tau^*_{qn}|_Y = \{G^*_{qn}|_Y: G^*_{qn} \in \tau^*_{qn}\}$ .Then $(Y, \tau^*_{qn}|_Y)$ is a q* -RONTS.The topology $\tau^*_{qn}|_Y$ is called q*-RON relative topology of τ^*_{qn} on Y or the q*-RON subspace topology of Y and the q* -RONTS $(Y, \tau^*_{qn}|_Y)$ is called q* -RON subspace of the q*-RONTS (X, τ^*_{qn}).

Members of $\tau^*_{qn}|_Y$ are called $\tau^*_{qn}|_Y$-open sets in Y. An q* -RON set $A_{qn} \in N_{qn}(Y)$ such that $A^c_{qn} \in \tau^*_{qn}|_Y$ is called a $\tau_{qn}|_Y$-closed set in Y. Then $(Y, \tau^*_{qn}|_Y)$ is called a q*-RON open subspace or q*-RON closed subspace of (X, τ^*_{qn}) according as $Y_{qn} \in \tau^*_{qn}$ or $Y_{qn} \in \tau^{*c}_{qn}$.

Example 6.6: Let $X = \{x_1, x_2\}$ and $\tau^*_{qn} = \{\emptyset^*_{qn}, X^*_{qn}, A^*_{qn}, B^*_{qn}, A^*_{qn} \cup B^*_{qn}, A^*_{qn} \cap B^*_{qn}\}$, where $A^*_{qn} = \{(x_1, 0.40, 0.60, 0.40), (x_2, 0.40, 0.50, 0.40)\}$ and

$B^*_{qn} = \{(x_1, 0.30, 0.40, 0.50), (x_2, 0.40, 0.50, 0.50)\}$. Clearly (X, τ^*_{qn}) is a q* -RONTS. Let $Y = \{x_1\}$. Then $X^*_{qn}|_Y = \{(x_1, 1,1,0), (x_2, 0,0,1)\} = Y^*_{qn}$, $\emptyset^*_{qn}|_Y = \{(x_1, 0,0,1), (x_2, 0,0,1)\} = \emptyset^*_{qn}$, $A^*_{qn}|_Y = \{(x_1, 0.4, 0.6, 0.4), (x_2, 0,0,1)\}$, $B^*_{qn}|_Y = \{(x_1, 0.3, 0.4, 0.5), (x_2, 0,0,1)\}$, $A^*_{qn} \cup B^*_{qn}|_Y = \{(x_1, 0.4, 0.6, 0.4), (x_2, 0,0,1)\}$, $A^*_{qn} \cap B^*_{qn}|_Y = \{(x_1, 0.3, 0.4, 0.5), (x_2, 0,0,1)\}$. Clearly $\tau^*_{qn}|_Y = \{\emptyset^*_{qn}, Y^*_{qn}, A^*_{qn}|_Y, B^*_{qn}|_Y, (A^*_{qn} \cup B^*_{qn})|_Y, (A^*_{qn} \cap B^*_{qn})|_Y\}$ is a q* -RON subspace of Y, that is $(Y, \tau^*_{qn}|_Y)$ is a q*-RON subspace of (X, τ^*_{qn}).

7. q*-rung orthopair neutrosophic $T_i -$ Spaces $(i = 0, 1, 2)$

In this section, discuss the many kinds of separation axioms and look into their characteristics in this part. Additionally, a diagram and counterexamples are provided to illustrate the links between them.

Definition 7.1: If every subspace of an q*-RONTS $((X,\tau^*_{qn})$ possesses a particular attribute whenever the space X does, then that property is said to be hereditary.

Definition 7.2: A property of an q*-RONTS (X_{qn},τ^*_{qn}) is said to be a topological property if and only if it is preserved under homeomorphism.

Definition 7.3: An q*-RONT (X_{qn},τ^*_{qn}) is called q*-RON T_0-space or $N_{qn}T_0$-space iff for any two q*-RONPs $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ and $y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}$, $x\neq y$, $\exists$ a $U^*_{qn}\in\tau^*_{qn}$ $\ni$ $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}\in U^*_{qn}, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}\notin U^*_{qn}$ or $\exists$ a $V^*_{qn}\in\tau^*_{qn}$ $\ni$ $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}\notin V^*_{qn}, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}\in V^*_{qn}$.

Example 7.4: Let $X=\{a,b\}$ and $\tau^*_{qn}=\{\emptyset^*_{qn},X^*_{qn},A^*_{qn},B^*_{qn}\}$, where $A^*_{qn}=\{(a,1,1,0),(b,0,0,1)\}$ and $B^*_{qn}=\{(a,0,0,1),(b,1,1,0)\}$. Clearly $((X,\tau^*_{qn})$ is a q^*-RONTS and it is a $N^*_{qn}T_0$-space.

Example 7.5: Let $X=\{a,b\}$ and $\tau^*_{qn}=\{\emptyset_{qn},X_{qn}\}$, Clearly $((X,\tau^*_{qn})$ is a q^*-RONTS but it is not a $N^*_{qn}T_0$-space.

Proposition 7.6: Let τ^*_{qn} and $\tau^{*\prime}_{qn}$ be two q*-RONT on a set X $\ni$ $\tau^{*\prime}_{qn}$ is finer than τ^*_{qn}. If (X_{qn},τ^*_{qn}) is a $N^*_{qn}T_0$-space then $(X_{qn},\tau^{*\prime}_{qn})$ is also a $N^*_{qn}T_0$-space.

Proof: Let $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ and $y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}$, $x\neq y$ be any two q*-RONPs in X. Since (X,τ^*_{qn}) is a $N_{qn}T_0$-space, so $\exists$ a $G^*_{qn}\in\tau^*_{qn}$ $\ni$ $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}\in G^*_{qn}, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}\notin G^*_{qn}$ or $\exists$ a $H^*_{qn}\in\tau^*_{qn}$ $\ni$ $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}\notin H^*_{qn}, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}\in H^*_{qn}$. Since $\tau^{*\prime}_{qn}$ is finer than τ^*_{qn}, So $G^*_{qn},H^*_{qn}\in\tau^*_{qn}\Rightarrow G^*_{qn},H^*_{qn}\in\tau^{*\prime}_{qn}$. Thus for any two q*-RONPs $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ and $y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}$, $x\neq y$, $\exists$ a $G^*_{qn}\in\tau^{*\prime}_{qn}$ $\ni$ $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}\in G^*_{qn}, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}\notin G^*_{qn}$ or $\exists$ a $H^*_{qn}\in\tau^{*\prime}_{qn}$ $\ni$ $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}\notin H^*_{qn}, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}\in H^*_{qn}$. Hence $(X_{qn},\tau^{*\prime}_{qn})$ is also a $N^*_{qn}T_0$-space.

Proposition 7.7: Let (X,τ^*_{qn}) be a $N^*_{qn}T_0$-space. Then every q^*-RON subspace of X is a $N^*_{qn}T_0$-space and hence the property is hereditary.

Proof: Let $(Y,\tau^*_{qn}|_Y)$ be a q^*-RON subspace of (X,τ^*_{qn}), where $\tau^*_{qn}|_Y=\{G^*_{qn}|_Y: G^*_{qn}\in\tau^*_{qn}\}$.To prove that $(Y,\tau^*_{qn}|_Y)$ is a $N^*_{qn}T_0$-space. Let $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ and $y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}$, be two q*-RONPs in Y $\ni$ $x\neq y$. Then $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$, $y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}\in X, x\neq y$. Since (X,τ^*_{qn}) is a $N^*_{qn}T_0$-space, so $\exists$ a $U^*_{qn}\in\tau^*_{qn}$ $\ni$ $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}\in U^*_{qn}, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}\notin U^*_{qn}$ or $\exists$ a $V_{qn}\in\tau^*_{qn}$ $\ni$ $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}\notin$

$V^*_{qn}, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \in V^*_{qn}$. Then $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in U^*_{qn}|_Y, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \notin U^*_{qn}|_Y$ or $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \notin V^*_{qn}|_Y, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \in V^*_{qn}|_Y$ .Also $U^*_{qn}|_Y, V^*_{qn}|_Y \in \tau^*_{qn}|_Y$. Thus for any two q^*-RONPs $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \in Y \ni x \neq y$, $\exists$ a $\tau^*_{qn}|_Y$- open q^*-RON set $U^*_{qn}|_Y \ni x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in U^*_{qn}|_Y, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \notin U^*_{qn}|_Y$ or $\exists$ a $\tau^*_{qn}|_Y$- open q^* -RON set $V^*_{qn}|_Y \ni x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \notin V^*_{qn}, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \in V^*_{qn}$. Therfore $(Y, \tau^*_{qn}|_Y)$ is $N^*_{qn}T_0$-space and hence the property is hereditary.

Proposition 7.8: Let f_{qn} be a one-to-one q*-RON continuous function from a q*-RONTS (X, τ^*_{qn}) to the q*-RONTS (Y, σ_{qn}). If (Y, σ^*_{qn}) is $N^*_{qn}T_0$ then (X, τ^*_{qn}) is also a $N^*_{qn}T_0$-space.

Proof: Let $x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ and $x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}$, be any two q*-RONPs in X such that $x^1 \neq x^2$. Since f_{qn} is one-to-one, so $\exists$ two q* -RONPs $y^1_{l_{qn},m_{qn},n_{qn}}$ and $y^2_{l'_{qn},m'_{qn},n'_{qn}}, y^1 \neq y^2$ in $Y \ni f_{qn}(x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}}) = y^1_{l_{qn},m_{qn},n_{qn}}$ and $f_{qn}(x^2_{\aleph_{qn},\beta_{qn},\gamma_{qn}}) = y^2_{l'_{qn},m'_{qn},n'_{qn}}$, that is $x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}} = f^{-1}_{qn}(y^1_{l_{qn},m_{qn},n_{qn}})$ and $x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} = f^{-1}_{qn}(y^2_{l'_{qn},m'_{qn},n'_{qn}})$. Since Y is $N^*_{qn}T_0$, so $\exists$ a σ_{qn}-open q*-RON set $G_{qn} \ni y^1_{l_{qn},m_{qn},n_{qn}} \in G^*_{qn}, y^2_{l'_{qn},m'_{qn},n'_{qn}} \notin G^*_{qn}$ or $\exists$ a σ_{qn}-open q*-RON set $H^*_{qn} \ni y^1_{l_{qn},m_{qn},n_{qn}} \notin H^*_{qn}, y^2_{l'_{qn},m'_{qn},n'_{qn}} \in H^*_{qn}$. Again, since f_{qn} is a q* -RON continuous, so $f^{-1}_{qn}(G^*_{qn})$ is a τ_{qn}-open q*-RON set. Also $y^1_{l_{qn},m_{qn},n_{qn}} \in G^*_{qn} \Rightarrow f^{-1}_{qn}(y^1_{l_{qn},m_{qn},n_{qn}}) \in f^{-1}_{qn}(G^*_{qn}) \Rightarrow x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in f^{-1}_{qn}(G^*_{qn})$ and $y^2_{l'_{qn},m'_{qn},n'_{qn}} \notin G^*_{qn} \Rightarrow f^{-1}_{qn}(y^2_{l'_{qn},m'_{qn},n'_{qn}}) \notin f^{-1}_{qn}(G^*_{qn}) \Rightarrow x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \notin f^{-1}_{qn}(G^*_{qn})$. Similarly $f^{-1}_{qn}(H^*_{qn})$ is a τ_{qn}-open q*-RON set $\ni x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \in f^{-1}_{qn}(H^*_{qn}), x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \notin f^{-1}_{qn}(H^*_{qn})$. Thus for any two q*-RONPs $x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ and $x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}$ in $X \ni x^1 \neq x^2$, $\exists$ a τ_{qn}-open q* -RON set $f^{-1}_{qn}(G^*_{qn}) \ni x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in f^{-1}_{qn}(G^*_{qn}), x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \notin f^{-1}_{qn}(G^*_{qn})$ or $\exists$ a τ_{qn} -open q* -RON set $f^{-1}_{qn}(G^*_{qn}) \ni x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \notin f^{-1}_{qn}(H^*_{qn}), x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \in f^{-1}_{qn}(H^*_{qn})$. Therefore (X, τ^*_{qn}) is a $N^*_{qn}T_0$-space.

Proposition 7.9: The property of being $N^*_{qn}T_0$ -space is preserved under a bijective q*-RON open function.

Proof: Let (X, τ^*_{qn}) and (Y, σ^*_{qn}) be any two q*-RON sets. Also let (X, τ^*_{qn}) be a $N^*_{qn}T_0$-space and $f_{qn}: X \to Y$ be a bijective q*-RON open function. To prove that (Y, σ^*_{qn}) is a $N^*_{qn}T_0$-space. Let $y^1_{l_{qn},m_{qn},n_{qn}}$ and $y^2_{l'_{qn},m'_{qn},n'_{qn}}$ be any two q* -RONPs in $Y \ni y^1 \neq y^2$. Since f_{qn} is bijective, so $\exists$ two q* -RONPs $x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ and $x^2_{\aleph_{qn},\beta_{qn},\gamma_{qn}}, x^1 = x^2$ in $X \ni f_{qn}(x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}}) = y^1_{l_{qn},m_{qn},n_{qn}}$

and $f_{qn}(x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}) = y^2_{l'_{qn},m'_{qn},n'_{qn}}$. Since X is $N^*_{qn}T_0$, so $\exists$ a τ_{qn}-open q*-RON set G_{qn} $\ni$ $x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in G^*_{qn}, x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \notin G_{qn}$ or $\exists$ a τ_{qn}-open q*-RON set H_{qn} $\ni$ $x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \notin H^*_{qn}, x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \in H_{qn}$. Suppose G^*_{qn} exists $\ni$ $x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in G^*_{qn}$, and $x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \notin G^*_{qn}$. Since f_{qn} is a q*-RON open function, so $f_{qn}(G^*_{qn})$ is a σ^*_{qn}-open q*-RON set $\ni$ $y^1_{l_{qn},m_{qn},n_{qn}} = f_{qn}(x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}}) \in f_{qn}(G^*_{qn})$ and $y^2_{l'_{qn},m'_{qn},n'_{qn}} = f_{qn}(x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}) \notin f_{qn}(G^*_{qn})$. Similarly if H^*_{qn} exists $\ni$ $x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \notin H^*_{qn}$, and $x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \in H^*_{qn}$ then $f_{qn}(H^*_{qn})$ is a σ^*_{qn}- open q*-RON set $\ni$ $y^1_{l_{qn},m_{qn},n_{qn}} = f_{qn}(x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}}) \notin f_{qn}(H^*_{qn})$ and $y^2_{l'_{qn},m'_{qn},n'_{qn}} = f_{qn}(x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}) \in f_{qn}(H^*_{qn})$. Thus for any two q*-RONPs $y^1_{l_{qn},m_{qn},n_{qn}}$ and $y^2_{l'_{qn},m'_{qn},n'_{qn}}$ in Y $\ni$ $y^1 \neq y^2$, $\exists$ a σ^*_{qn})-open q*-RON set$f_{qn}(G^*_{qn})$ $\ni$ $y^1_{l_{qn},m_{qn},n_{qn}} \in f_{qn}(G^*_{qn}), y^2_{l'_{qn},m'_{qn},n'_{qn}} \notin f_{qn}(G^*_{qn})$ or $\exists$ a σ^*_{qn}-open q*-RON set $f_{qn}(H^*_{qn})$ $\ni$ $y^1_{l_{qn},m_{qn},n_{qn}} \notin f_{qn}(H^*_{qn}), y^2_{l'_{qn},m'_{qn},n'_{qn}} \in f_{qn}(H^*_{qn})$. Therefore (Y, σ^*_{qn}) is a $N^*_{qn}T_0$-space.

Proposition 7.10: The property of being $N^*_{qn}T_0$-space is a topological property.

Proof: Let (X, τ^*_{qn}) and (Y, σ^*_{qn}) be two q*-RON sets. Also let (X, τ^*_{qn}) be a $N^*_{qn}T_0$-space and $f_{qn}: X \to Y$ be a q*-RON homeomorphism.Since f_{qn} is a q*-RON homeomorphism, so f_{qn} is a bijective q*-RON open function. By Proposition 7.8, (Y, σ^*_{qn}) is a $N^*_{qn}T_0$-space. Hence proved.

Definition 7.11: An q*-RONT (X_{qn}, τ^*_{qn}) is called q*-RON T_1-space or $N^*_{qn}T_1$-space iff for any two q*-RONPs $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ and $y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}$, $x \neq y$, $\exists$ a $U^*_{qn} \in \tau^*_{qn}$ $\ni$ $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in U^*_{qn}, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \notin U^*_{qn}$ and $\exists$ a $V^*_{qn} \in \tau^*_{qn}$ $\ni$ $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \notin V^*_{qn}, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \in V^*_{qn}$.

Example 7.12: Let $X = \{a, b\}$ and $\tau^*_{qn} = \{\emptyset^*_{qn}, X^*_{qn}, A^*_{qn}, B^*_{qn}\}$, where

$A^*_{qn} = \{(a, 1,1,0), (b, 0,0,1)\}$ and $B^*_{qn} = \{(a, 0,0,1), (b, 1,1,0)\}$ .Clearly $((X, \tau^*_{qn})$ is a q^*-RONTS and it is a $N^*_{qn}T_1$-space.

Example 7.13: Let $X = \{a, b\}$ and $\tau^*_{qn} = \{\emptyset_{qn}, X_{qn}\}$, Clearly $((X, \tau^*_{qn})$ is a q^*-RONTS but it is not a $N^*_{qn}T_1$-space.

Proposition 7.14: Let τ^*_{qn} and $\tau^{*\prime}_{qn}$ be any two q*-RONT on a set X $\ni$ $\tau^{*\prime}_{qn}$ is finer than τ^*_{qn}. If (X_{qn}, τ^*_{qn}) is a $N^*_{qn}T_1$-space then $(X_{qn}, \tau^{*\prime}_{qn})$ is also a $N^*_{qn}T_1$-space.

Proof: Let $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ and $y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}$, $x \neq y$ be two q*-RONPs in X. Since $((X, \tau^*_{qn})$ is a $N^*_{qn}T_1$-space, so $\exists$ a $G^*_{qn} \in \tau^*_{qn}$ $\ni$ $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in G^*_{qn}, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}$

$\notin G^*_{qn}$ and $\exists$ a $H^*_{qn} \in \tau^*_{qn} \ni x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \notin H^*_{qn}, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \in H_{qn}$. Since $\tau^{*\prime}_{qn}$ is finer than τ^*_{qn}, so $G_{qn}, H^*_{qn} \in \tau^*_{qn} \Rightarrow G^*_{qn}, H^*_{qn} \in \tau^{*\prime}_{qn}$. Thus for any two q* -RONPs $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ and $y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}$ in $X \ni x \neq y$, $\exists$ a $G^*_{qn} \in \tau^{*\prime}_{qn} \ni x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in G^*_{qn}, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \notin G^*_{qn}$ and $\exists$ a $H^*_{qn} \in \tau^{*\prime}_{qn} \ni x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \notin H^*_{qn}, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \in H^*_{qn}$. Hence $(X_{qn}, \tau^{*\prime}_{qn})$ is also a $N^*_{qn}T_1$-space.

Proposition 7.15: Let (X_{qn}, τ^*_{qn}) be a q* -RONTS. If (X_{qn}, τ^*_{qn}) is a $N^*_{qn}T_1$-space then it is a $N^*_{qn}T_0$-space.

Proof: Let Let $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ and $y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}$, $x \neq y$ be two q*-RONPs in X. Since X is $N^*_{qn}T_1$-space, so $\exists$ a $U^*_{qn} \in \tau^*_{qn} \ni x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in U^*_{qn}, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \notin U^*_{qn}$ and $\exists$ a $V^*_{qn} \in \tau^*_{qn} \ni x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \notin V_{qn}, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \in V^*_{qn}$. Hence (X_{qn}, τ^*_{qn}) is a $N^*_{qn}T_0$-space.

Remark 3: As demonstrated by Example 7.16, the opposite of Proposition 7.15 need not be true.

Example 7.16: Let $X = \{a, b\}$ and $\tau^*_{qn} = \{\emptyset^*_{qn}, X^*_{qn}, A^*_{qn}\}$, where $A^*_{qn} = \{(a, 1,1,0), (b, 0,0,1)\}$. Clearly (X, τ^*_{qn}) is a $N^*_{qn}T_0$ -space but not a $N^*_{qn}T_1$-space.

Proposition 7.17: Let (X, τ^*_{qn}) be a $N^*_{qn}T_1$ -space. Then every q^* -RON subspace of X is a $N^*_{qn}T_1$-space and hence the property is hereditary.

Proof: Let $(Y, \tau^*_{qn}|_Y)$ be a q^*-RON subspace of (X, τ^*_{qn}), where $\tau^*_{qn}|_Y = \{G^*_{qn}|_Y : G_{qn} \in \tau^*_{qn}\}$. Let $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ and $y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}$, be any two q*-RONPs in $Y \ni x \neq y$.

Then $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \in X$, and $x \neq y$. Given (X, τ^*_{qn}) is a $N^*_{qn}T_1$-space, so $\exists$ a $U^*_{qn} \in \tau^*_{qn} \ni x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in U^*_{qn}, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \notin U^*_{qn}$ and $\exists$ a $V^*_{qn} \in \tau^*_{qn} \ni x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \notin V^*_{qn}, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \in V^*_{qn}$. Then $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in U^*_{qn}|_Y, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \notin U^*_{qn}|_Y$ and $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \notin V^*_{qn}|_Y, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \in V^*_{qn}|_Y$. Also $U^*_{qn}|_Y, V^*_{qn}|_Y \in \tau^*_{qn}|_Y$. Thus for any two q^*-RONPs $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \in Y \ni x \neq y$, $\exists$ a $\tau^*_{qn}|_Y$ - open q^*-RON set $U^*_{qn}|_Y \ni x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in U^*_{qn}|_Y$, $y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \notin U^*_{qn}|_Y$ and $\exists$ a $\tau^*_{qn}|_Y$- open q^*-RON set $V^*_{qn}|_Y \ni x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \notin V^*_{qn}, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \in V^*_{qn}$. Therfore $(Y, \tau^*_{qn}|_Y)$ is $N^*_{qn}T_1$ -space and hence the property is hereditary.

Proposition 7.18: Let (X_{qn}, τ^*_{qn}) be a q*-RONTS. Then every q*-RON crisp point in X is τ^*_{qn}-closed q*-RON set if and only if X is a $N^*_{qn}T_1$-space.

Proof: Necessary part: Let $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ and $y_{p_{qn},q_{qn},r_{qn}}$ be two q*-RONPs in X $\ni$ $x \neq y$. Since $x \neq y$, so $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in (y_{1,1,0})^c$. By assumption $y_{1,1,0}$ is a τ^*_{qn}-closed q*-RON set. Therefore $(y_{1,1,0})^c$ is a τ^*_{qn}-open q*-RON set. Thus $\exists$ a τ^*_{qn}-open q*-RON set $(y_{1,1,0})^c$ $\ni$ $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in (y_{1,1,0})^c$ but $y_{p_{qn},q_{qn},r_{qn}} \notin (y_{1,1,0})^c$. Similarly $x_{1,1,0}$ is a τ^*_{qn}-open q*-RON set $\ni$ $y_{p_{qn},q_{qn},r_{qn}} \in (x_{1,1,0})^c$ but $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \notin (x_{1,1,0})^c$. Therefore X is a $N_{qn}T_1$-space.

Sufficient part: Let $x_{1,1,0}$ be a q*-RON crisp point in X. Also let $y_{p_{qn},q_{qn},r_{qn}}$ be a q*-RONP in $X \ni x \neq y$.Then $y_{p_{qn},q_{qn},r_{qn}} \in (x_{1,1,0})^c$. Let us consider a q*-RONP $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ with support x. Since X is a $N^*_{qn}T_1$-space, so for $y_{p_{qn},q_{qn},r_{qn}}$ and $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$, $\exists$ a τ^*_{qn} -open q* -RON set G^*_{qn} $\ni$ $y_{p_{qn},q_{qn},r_{qn}} \in G^*_{qn}$ and $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \notin G^*_{qn}$. Since for all $\aleph_{qn}, \beta_{qn}, \gamma_{qn}$ with $0 < \aleph_{qn} \leq 1, 0 < \beta_{qn} \leq 1, 0 \leq \gamma_{qn} < 1$, one such G^*_{qn} exists, therefore we must have a τ^*_{qn}-open q*-RON set H^*_{qn} $\ni$ $y_{p_{qn},q_{qn},r_{qn}} \in H^*_{qn}$ and $x_{1,1,0} \cap H^*_{qn} = \emptyset^*_{qn}$, that is $y_{p_{qn},q_{qn},r_{qn}} \in H_{qn} \subseteq (x_{1,1,0})^c$. Therefore $(x_{1,1,0})^c$ is a τ^*_{qn}-open q*-RON set and hence $x_{1,1,0}$ is a τ^*_{qn}-closed q*-RON set. Hence proved.

Proposition 7.19: Let f_{qn} be a one-to-one q* -RONC function from a q* -RONTS (X, τ^*_{qn}) to the q* -RONTS (Y, σ_{qn}). If (Y, σ_{qn}) is $N^*_{qn}T_1$ then (X, τ^*_{qn}) is also a $N^*_{qn}T_1$-space.

Proof: Let $x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ and $x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}$, be any two q* -RONPs in X $\ni$ $x^1 \neq x^2$. Since f_{qn} is one-to-one, so $\exists$ two q* -RONPs $y^1_{l_{qn},m_{qn},n_{qn}}$ and $y^2_{l'_{qn},m'_{qn},n'_{qn}}, y^1 \neq y^2$ in $Y \ni f_{qn}(x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}}) = y^1_{l_{qn},m_{qn},n_{qn}}$ and $f_{qn}(x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}) = y^2_{l'_{qn},m'_{qn},n'_{qn}}$, that is $x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}} = f^{-1}_{qn}(y^1_{l_{qn},m_{qn},n_{qn}})$ and $x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} = f^{-1}_{qn}(y^2_{l'_{qn},m'_{qn},n'_{qn}})$. Since Y is $N^*_{qn}T_0$, so $\exists$ a σ_{qn}-open q*-RON set G^*_{qn} $\ni$ $y^1_{l_{qn},m_{qn},n_{qn}} \in G^*_{qn}, y^2_{l'_{qn},m'_{qn},n'_{qn}} \notin G^*_{qn}$ and $\exists$ a σ_{qn}-open q*-RON set H^*_{qn} $\ni$ $y^1_{l_{qn},m_{qn},n_{qn}} \notin H^*_{qn}, y^2_{l'_{qn},m'_{qn},n'_{qn}} \in H^*_{qn}$. Since f_{qn} is a q*-RON continuous, so $f^{-1}_{qn}(G^*_{qn})$ is a τ_{qn} -open q* -RON set. Also $y^1_{l_{qn},m_{qn},n_{qn}} \in G_{qn} \Rightarrow f^{-1}_{qn}(y^1_{l_{qn},m_{qn},n_{qn}}) \in f^{-1}_{qn}(G_{qn}) \Rightarrow (x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in f^{-1}_{qn}(G_{qn})$ and $y^2_{l'_{qn},m'_{qn},n'_{qn}} \notin G_{qn} \Rightarrow f^{-1}_{qn}(y^2_{l'_{qn},m'_{qn},n'_{qn}}) \notin f^{-1}_{qn}(G^*_{qn}) \Rightarrow x^2_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \notin f^{-1}_{qn}(G^*_{qn})$. Similarly $f^{-1}_{qn}(H^*_{qn})$ is a τ_{qn}-open q*-RON set $\ni$ $x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \in f^{-1}_{qn}(H^*_{qn}), x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \notin f^{-1}_{qn}(H^*_{qn})$. Thus for any two q*-RONPs $x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ and $x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}$ in X $\ni$ $x^1 \neq x^2$, $\exists$ a τ^*_{qn}-open q* -RON set $f^{-1}_{qn}(G^*_{qn})$ $\ni$ $x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in f^{-1}_{qn}(G_{qn})$, $x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \notin f^{-1}_{qn}(G^*_{qn})$ and $\exists$ a τ_{qn} -open q* -RON set $f^{-1}_{qn}(G^*_{qn})$ $\ni$ $x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \notin f^{-1}_{qn}(H^*_{qn}), x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \in f^{-1}_{qn}(H^*_{qn})$. Therefore (X, τ^*_{qn}) is a $N^*_{qn}T_1$-space.

Proposition 7.20: The property of being $N_{qn}^{*}T_1$-space is preserved under a bijective q*-RON open function.

Proof: Let (X,τ_{qn}^{*}) and (Y,σ_{qn}^{*}) be two q*-RON sets. Also let (X,τ_{qn}^{*}) be a $N_{qn}^{*}T_1$-space and $f_{qn}: X \to Y$ be a bijective q*-RON open function. To prove that (Y,σ_{qn}^{*}) is a $N_{qn}^{*}T_1$-space. Let $y^1_{l_{qn},m_{qn},n_{qn}}$ and $y^2_{l'_{qn},m'_{qn},n'_{qn}}$ be two q*-RONPs in $Y \ni y^1 \neq y^2$. Since f_{qn} is bijective, so $\exists$ two q*-RONPs $x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ and $x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}, x^1 = x^2$ in $X \ni f_{qn}(x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}}) = y^1_{l_{qn},m_{qn},n_{qn}})$ and $f_{qn}(x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}) = y^2_{l'_{qn},m'_{qn},n'_{qn}})$. Since X is $N_{qn}^{*}T_1$, so $\exists$ a τ_{qn}^{*}-open q*-RON set $G_{qn}^{*} \ni x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in G_{qn}^{*}, x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \notin G_{qn}^{*}$ and $\exists$ a τ_{qn}-open q*-RON set $H_{qn}^{*} \ni x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \notin H_{qn}^{*}, x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \in H_{qn}^{*}$. Suppose G_{qn}^{*} exists $\ni x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in G_{qn}^{*}$, and $x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \notin G_{qn}^{*}$. Since f_{qn} is a q* -RON open function, so $f_{qn}(G_{qn}^{*})$ is a σ_{qn}^{*} -open q* -RON set $\ni y^1_{l_{qn},m_{qn},n_{qn}} = f_{qn}(x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}}) \in f_{qn}(G_{qn}^{*})$ and $y^2_{l'_{qn},m'_{qn},n'_{qn}} = f_{qn}(x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}) \notin f_{qn}(G_{qn}^{*})$. Similarly if H_{qn}^{*} exists $\ni x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \notin H_{qn}^{*}$, and $x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \in H_{qn}^{*}$ then $f_{qn}(H_{qn}^{*})$ is a σ_{qn}^{*} -open q* -RON set $\ni y^1_{l_{qn},m_{qn},n_{qn}} = f_{qn}(x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}}) \notin f_{qn}(H_{qn}^{*})$ and $y^2_{l'_{qn},m'_{qn},n'_{qn}} = f_{qn}(x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}) \in f_{qn}(H_{qn}^{*})$. Thus for any two q* -RONPs $y^1_{l_{qn},m_{qn},n_{qn}}$ and $y^2_{l'_{qn},m'_{qn},n'_{qn}}$ in $Y \ni y^1 \neq y^2$, $\exists$ a σ_{qn}^{*} -open q*-RON set $f_{qn}(G_{qn}^{*}) \ni y^1_{l_{qn},m_{qn},n_{qn}} \in f_{qn}(G_{qn}^{*}), y^2_{l'_{qn},m'_{qn},n'_{qn}} \notin f_{qn}(G_{qn}^{*})$ and $\exists$ a σ_{qn}^{*} -open q* -RON set $f_{qn}(H_{qn}^{*}) \ni y^1_{l_{qn},m_{qn},n_{qn}} \notin f_{qn}(H_{qn}^{*}), y^2_{l'_{qn},m'_{qn},n'_{qn}} \in f_{qn}(H_{qn}^{*})$. Therefore (Y,σ_{qn}^{*}) is a $N_{qn}^{*}T_1$-space.

Proposition 7.21: The property of being $N_{qn}^{*}T_1$-space is a topological property.

Proof: Let (X,τ_{qn}^{*}) and (Y,σ_{qn}^{*}) be two q*-RON sets.Also let (X,τ_{qn}^{*}) be a $N_{qn}^{*}T_1$-space and $f_{qn}: X \to Y$ be a q*-RON homeomorphism. Since f_{qn} is a q*-RON homeomorphism, so f_{qn} is a bijective q*-RON open function. By proposition 7.20, (Y,σ_{qn}^{*}) is a $N_{qn}^{*}T_1$-space. Hence proved.

Proposition 7.22: Let (X,τ_{qn}^{*}) be a q*-RON set. Then X is $N_{qn}^{*}T_1$ if and only if the intersection of all the q*-RON neighbourhoods of an arbitrary q*-RONP of X is an q*-RONP.

Proof: Necessary part: Let $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ be an arbitrary q*-RONP in X and N_{qn}^{*} be the intersection of all the q*-RON neighbourhoods of $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$. Also let $y_{p_{qn},q_{qn},r_{qn}}$ be any q*-RONP in $X \ni x \neq y$. Since X is $N_{qn}^{*}T_1$, so $\exists$ a q*-RON neighbourhoods G_{qn}^{*} of $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \ni y_{p_{qn},q_{qn},r_{qn}} \notin G_{qn}^{*}$ and consequently $y_{p_{qn},q_{qn},r_{qn}} \notin N_{qn}^{*}$. Since $y_{p_{qn},q_{qn},r_{qn}}$ is arbitrary, so $N_{qn}^{*} = x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$.

Sufficient part: Let $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ and $y_{p_{qn},q_{qn},r_{qn}}$ be two q*-RONPs in X such that $x \neq y$. By assumption, the intersection of all the q* -RON neighbourhoods of $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ is $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$. So, there must exist a q*-RON neighbourhood of $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ which dose not contain $y_{p_{qn},q_{qn},r_{qn}}$. Similarly, there must exist a q*-RON neighbourhood of $y_{p_{qn},q_{qn},r_{qn}}$ which dose not contain $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$. Therefore X is a $N_{qn}^{*}T_1$. Hence proved

Definition 7.23: An q*-RONT (X_{qn}, τ_{qn}^{*}) is called $N_{qn}^{*}T_2$-space or q*-RON Hausdorff space if and only if for any two q* -RONPs $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ and $y_{\aleph_{qn'},\beta_{qn'},\gamma_{qn'}}, x \neq y$, there exist $U_{qn}^{*}, V_{qn}^{*} \in \tau_{qn}^{*}$ such that $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in U_{qn}^{*}, y_{\aleph_{qn'},\beta_{qn'},\gamma_{qn'}} \in V_{qn}^{*}$ and $U_{qn}^{*} \cap \ V_{qn}^{*} = \emptyset_{qn}^{*}$.

Example 7.24: Let $X = \{l, m\}$ and $\tau_{qn}^{*} = \{\emptyset_{qn}^{*}, X_{qn}^{*}, L_{qn}^{*}, M_{qn}^{*}\}$, where $L_{qn}^{*} = \{(l, 1,1,0), (m, 0,0,1)\}$ and $M_{qn}^{*} = \{(l, 0,0,1), (m, 1,1,0)\}$. Clearly $((X, \tau_{qn}^{*})$ is a q^{*}-RONTS and it is a $N_{qn}^{*}T_2$-space.

Example 7.25: Let $X = \{l, m\}$ and $\tau_{qn}^{*} = \{\emptyset_{qn}^{*}, X_{qn}^{*}\}$, Clearly $((X, \tau_{qn}^{*})$ is a q^{*}-RONTS but it is not a $N_{qn}^{*}T_2$-space.

Proposition 7.26: Let τ_{qn}^{*} and $\tau_{qn}^{*\prime}$ be two q*-RONT on a set $X \ni \tau_{qn}^{*\prime}$ is finer than τ_{qn}^{*}. If (X, τ_{qn}^{*}) is a $N_{qn}^{*}T_2$-space then $(X, \tau_{qn}^{*\prime})$ is also a $N_{qn}^{*}T_2$-space.

Proof: Let $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ and $y_{\aleph_{qn}',\beta_{qn}',\gamma_{qn}'}$, $x \neq y$ be two q*-RONPs in X. Since $((X, \tau_{qn}^{*})$ is a $N_{qn}^{*}T_2$ -space, so $\exists$ a $G_{qn}^{*}, H_{qn}^{*} \in \tau_{qn}^{*} \ \ni \ x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in G_{qn}^{*}, y_{\aleph_{qn}',\beta_{qn}',\gamma_{qn}'} \in H_{qn}^{*}$ and $G_{qn}^{*} \cap \ H_{qn}^{*} = \emptyset_{qn}^{*}$. Since $\tau_{qn}^{*\prime}$ is finer than τ_{qn}^{*}, so $G_{qn}^{*}, H_{qn}^{*} \in \tau_{qn}^{*} \Rightarrow G_{qn}^{*}, H_{qn}^{*} \in \tau_{qn}^{*\prime}$. Thus for any two q*-RONPs $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ and $y_{\aleph_{qn}',\beta_{qn}',\gamma_{qn}'}$ in $X \ \ni \ x \neq y$, $\exists$ a $G_{qn}^{*}, H_{qn}^{*} \in \tau_{qn}^{*\prime}$ such that $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in G_{qn}^{*}, y_{\aleph_{qn}',\beta_{qn}',\gamma_{qn}'} \in H_{qn}^{*}$ and $G_{qn}^{*} \cap \ H_{qn}^{*} = \emptyset_{qn}^{*}$. Hence $(X, \tau_{qn}^{*\prime})$ is also a $N_{qn}^{*}T_2$-space.

Proposition 7.27: Let (X, τ_{qn}^{*}) be a q*-RONTS.If (X, τ_{qn}^{*}) is a $N_{qn}^{*}T_2$-space then it is a $N_{qn}^{*}T_1$-space.

Proof: Let Let $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ and $y_{\aleph_{qn}',\beta_{qn}',\gamma_{qn}'}$, $x \neq y$ be two q*-RONPs in X. Since (X, τ_{qn}^{*}) is a $N_{qn}^{*}T_2$-space, so $\exists$ τ_{qn}^{*}-open q*-RON sets H_{qn}^{*} and $K_{qn}^{*} \ni x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in H_{qn}^{*}, y_{\aleph_{qn}',\beta_{qn}',\gamma_{qn}'} \in K_{qn}^{*}$ and $H_{qn}^{*} \cap \ K_{qn}^{*} = \emptyset_{qn}$. Since $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in H_{qn}^{*}$ and $H_{qn}^{*} \cap \ K_{qn}^{*} = \emptyset_{qn}^{*}$, so $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \notin K_{qn}^{*}$. Similarly, $y_{\aleph_{qn}',\beta_{qn}',\gamma_{qn}'} \notin H_{qn}^{*}$. Thus $\exists$ a $H_{qn}^{*} \in \tau_{qn}^{*} \ \ni \ x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in H_{qn}^{*}, y_{\aleph_{qn}',\beta_{qn}',\gamma_{qn}'} \notin H_{qn}^{*}$ and $\exists$ a $K_{qn}^{*} \in \tau_{qn}^{*} \ \ni \ x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \notin K_{qn}^{*}, y_{\aleph_{qn}',\beta_{qn}',\gamma_{qn}'} \in K_{qn}^{*}$. Hence (X, τ_{qn}^{*}) is a $N_{qn}^{*}T_1$-space.

Remark 4: As demonstrated by Example 7.28, the opposite of Proposition 7.27 need not be true.

Example 7.28: Let $X = \{a, b\}$ and $\tau^*_{qn} = \{\emptyset^*_{qn}, X^*_{qn}, A^*_{qn},\}$, where $A^*_{qn} = \{(a, 1,1,0), (b, 0,0,1)\}$. Clearly (X, τ^*_{qn}) is a $N^*_{qn}T_0$ -space but not a $N^*_{qn}T_1$-space.

Proposition 7.29: Let (X, τ^*_{qn}) be a $N^*_{qn}T_2$ -space. Then every q^* -RON subspace of X is a $N^*_{qn}T_2$-space and hence the property is hereditary.

Proof: Let $(Y, \tau^*_{qn}|_Y)$ be a q^*-RON subspace of (X, τ^*_{qn}), where $\tau^*_{qn}|_Y = \{G^*_{qn}|_Y : G^*_{qn} \in \tau^*_{qn}\}$. To prove that $(Y, \tau^*_{qn}|_Y)$ is a $N^*_{qn}T_2$-space. Let $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ and $y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}$, be two q* -RONPs in $Y \ni x \neq y$. Then $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$, $y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \in X, x \neq y$. Since (X, τ^*_{qn}) is a $N^*_{qn}T_2$ -space, so $\exists$ τ^*_{qn} -open q* -RON sets U^*_{qn} and $V^*_{qn} \ni x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in U^*_{qn}, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \in V^*_{qn}$ and $U^*_{qn} \cap V^*_{qn} = \emptyset_{qn}$. Then $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in U^*_{qn}|_Y, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \in V^*_{qn}|_Y$ and $(U^*_{qn}|_Y \cap V^*_{qn}|_Y) = (U^*_{qn} \cap V^*_{qn})|_Y = \emptyset_{qn}$. Thus for any two q^*-RONPs $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \in Y \ni x \neq y$, $\exists$ a $\tau^*_{qn}|_Y$ - open q^*-RON sets $U^*_{qn}|_Y, V^*_{qn}|_Y \ni x_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in U^*_{qn}|_Y, y_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \in V^*_{qn}|_Y$ and $(U^*_{qn}|_Y \cap V^*_{qn}|_Y) = \emptyset_{qn}$. Therfore $(Y, \tau^*_{qn}|_Y)$ is $N^*_{qn}T_2$-space and hence the property is hereditary.

Proposition 7.30: Let f_{qn} be a one-to-one q* -RONC function from a q*-RONTS (X, τ^*_{qn}) to the q*-RON Hausdorff space (Y, σ^*_{qn}). Then (X, τ^*_{qn}) is also a q*-RON Hausdorff space.

Proof: Let $x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ and $x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}$, be any two q* -RONPs in $X \ni x^1 \neq x^2$. Since f_{qn} is one-to-one, so $\exists$ two q* -RONPs $y^1_{l_{qn},m_{qn},n_{qn}}$ and $y^2_{l'_{qn},m'_{qn},n'_{qn}}, y^1 \neq y^2$ in $Y \ni f_{qn}(x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}}) = y^1_{l_{qn},m_{qn},n_{qn}}$ and $f_{qn}(x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}) = y^2_{l'_{qn},m'_{qn},n'_{qn}}$, that is $x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}} = f^{-1}_{qn}(y^1_{l_{qn},m_{qn},n_{qn}})$ and $x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} = f^{-1}_{qn}(y^2_{l'_{qn},m'_{qn},n'_{qn}})$. Since (Y, σ_{qn}) is q*-RON Hausdorff space, so $\exists$ a σ_{qn}-open q*-RON sets $H^*_{qn_1}, H^*_{qn_2} \ni y^1_{l_{qn},m_{qn},n_{qn}} \in H^*_{qn_1}, y^2_{l'_{qn},m'_{qn},n'_{qn}} \in H^*_{qn_2}$ and $H^*_{qn_1} \cap H^*_{qn_2} = \emptyset^*_{qn}$. Since f_{qn} is a q* -RON continuous, so $f^{-1}_{qn}(H^*_{qn_1})$ and $f^{-1}_{qn}(H^*_{qn_2})$ are τ_{qn} -open q* -RON sets. Now $f^{-1}_{qn}(H^*_{qn_1}) \cap f^{-1}_{qn}(H^*_{qn_2}) = f^{-1}_{qn}(H^*_{qn_1} \cap H^*_{qn_2}) = f^{-1}_{qn}(\emptyset^*_{qn})$. Also $y^1_{l_{qn},m_{qn},n_{qn}} \in H^*_{qn_1} \Rightarrow f^{-1}_{qn}(y^1_{l_{qn},m_{qn},n_{qn}}) \in f^{-1}_{qn}(H^*_{qn_1}) \Rightarrow x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}} \in f^{-1}_{qn}(H^*_{qn_1})$. Similarly $x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}} \in f^{-1}_{qn}(H^*_{qn_2})$. Thus for any two q* -RONPs $x^1_{\aleph_{qn},\beta_{qn},\gamma_{qn}}$ and $x^2_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}$ in X such that $x^1 \neq x^2$, $\exists$ τ^*_{qn}-open q*-RON sets $f^{-1}_{qn}(H^*_{qn_1})$,

$f_{qn}^{-1}(H_{qn_2}^*) \ni x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}^1 \in f_{qn}^{-1}(H_{qn_1}^*), x_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}^2 \in f_{qn}^{-1}(H_{qn_2}^*)$ and $f_{qn}^{-1}(H_{qn_1}^*) \cap f_{qn}^{-1}(H_{qn_2}^*) = \emptyset_{qn}^*$. Therefore (X, τ_{qn}^*) is a q*-RON Hausdorff space.

Proposition 7.31: The property of being $N_{qn}^* T_2$-space is preserved under a bijective q*-RON open function.

Proof: Let (X, τ_{qn}^*) and (Y, σ_{qn}^*) be two q*-RON sets. Also let (X, τ_{qn}^*) be a $N_{qn}^* T_2$-space and $f_{qn}: X \to Y$ be a bijective q*-RON open function. To prove that (Y, σ_{qn}^*) is a $N_{qn}^* T_2$-space. Let $y_{l_{qn},m_{qn},n_{qn}}^1$ and $y_{l'_{qn},m'_{qn},n'_{qn}}^2$ be two q*-RONPs in $Y \ni y^1 \neq y^2$. Since f_{qn} is bijective, so $\exists$ two q*-RONPs $x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}^1$ and $x_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}^2, x^1 = x^2$ in $X \ni f_{qn}(x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}^1) = y_{l_{qn},m_{qn},n_{qn}}^1$ and $f_{qn}(x_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}^2) = y_{l'_{qn},m'_{qn},n'_{qn}}^2$. Since X is $N_{qn}^* T_2$, so $\exists$ a τ_{qn}-open q*-RON set $G_{qn}^*, H_{qn}^* \ni x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}^1 \in G_{qn}^*, x_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}^2 \in H_{qn}^*$ and $G_{qn}^* \cap H_{qn}^* = \emptyset_{qn}^*$. Since f_{qn} is a q*-RON open function, so $f_{qn}(G_{qn}^*), f_{qn}(H_{qn}^*)$ are a σ_{qn}^*-open q*-RON set $\ni y_{l_{qn},m_{qn},n_{qn}}^1 = f_{qn}(x_{\aleph_{qn},\beta_{qn},\gamma_{qn}}^1) \in f_{qn}(G_{qn}^*)$, $y_{l'_{qn},m'_{qn},n'_{qn}}^2 = f_{qn}(x_{\aleph'_{qn},\beta'_{qn},\gamma'_{qn}}^2) \in f_{qn}(H_{qn}^*)$. Again f_{qn} is bijective, so $f_{qn}(G_{qn}^*) \cap f_{qn}(H_{qn}^*) = f_{qn}(G_{qn}^* \cap H_{qn}^*) = f_{qn}(\emptyset_{qn}^*) = \emptyset_{qn}^*$. Thus for any two q* -RONPs $y_{l_{qn},m_{qn},n_{qn}}^1$ and $y_{l'_{qn},m'_{qn},n'_{qn}}^2$ in $Y \ni y^1 \neq y^2$, $\exists$ a σ_{qn}^*-open q*-RON sets $f_{qn}(G_{qn}^*), f_{qn}(H_{qn}^*) \ni y_{l_{qn},m_{qn},n_{qn}}^1 \in f_{qn}(G_{qn}^*), y_{l'_{qn},m'_{qn},n'_{qn}}^2 \in f_{qn}(H_{qn}^*)$ and $f_{qn}(G_{qn}^*) \cap f_{qn}(H_{qn}^*) = \emptyset_{qn}^*$. Therefore (Y, σ_{qn}^*) is a $N_{qn}^* T_2$-space.

Proposition 7.32: The property of being $N_{qn}^* T_2$-space is a topological property.

Proof: Let (X, τ_{qn}^*) and (Y, σ_{qn}^*) be two q*-RON sets.Also let (X, τ_{qn}^*) be a $N_{qn}^* T_2$-space and $f_{qn}: X \to Y$ be a q*-RON homeomorphism. Since f_{qn} is a q*-RON homeomorphism, so f_{qn} is a bijective q*-RON open function. By proposition 7.31, (Y, σ_{qn}^*) is a $N_{qn}^* T_2$-space. Hence proved.

8. Conclusion and Future Directions

Our goal in this paper is to explore the important separation axioms in q*-RONTS . Since many features of topological spaces are intrinsically related to their subspaces, the role of subspace topology and subspaces is crucial, just as it is in classical and fuzzy topological studies. In this paper we define q*-RON ,$N_{qn}^* T_0$,$N_{qn}^* T_1$ and$N_{qn}^* T_2$ spaces and clarify the different features associated with these fundamental separation axioms, therefore highlighting their importance in the larger topological research environment.Future studies will concentrate on examining other ideas pertaining to q*-RONTS. The results presented in this article are expected to be useful to the community of researchers and to help further the development of different aspects of q*-RONT.

Data Availability

No publicly stored datasets were created throughout the study, and no underlying data were developed or examined to support the findings.

Acknowledgement

The authors express their gratitude to the Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology Chennai, for their unwavering support and encouragement in conducting this research.

Statement of Declaration of Interest

The authors confirm that they have no known financial conflicts of interest.

Author contributions statement

Shyamaladevi Vaiyanan: Conceptualization, methodology, writing-original draft, writing-review and editing.

Revathi Govindasamy Krishnasamy: Conceptualization, methodology, supervision.

Funding

No external funding agency has provided support to the authors.

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Received January, 2025