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Generalization of FFW-transform

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Abstract

Smart grids have made power systems to be efficient, reliable and sustainable thus the importance of integrating smart grids into F-differential equation to F-boundary problem necessitates the use of important techniques of which fuzzy integral transforms is an imporcott method. The study extends the formula of third-order fuzzy derivative of FFWT based on which it can determine its generalized nth-order fuzzy derivative capability. The manipulation is done over a research structure based on highly generalized Hukuhara

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differentiability. The drug distribution measurement reveals drug body level that appears as the ratio of plasma or blood drug quantity to entire body drug quantity and by the same token, the addressed fuzzy FFW-transform solves drug concentration in organ equations.

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Keywords: *FFW-transform, Fuzzy FFW-transform about 3-order, Fuzzy FFWT generalization order.*

1. Introduction

Different methods exist today which address the solution of fuzzy initial and boundary value problems. The most popular approach to resolve differential equations together with integral equations utilizes fuzzy transforms. The conversion of conventional L-Transform [1-5] and Sumudu transform [7-10] into fuzzy transforms serves to solve fuzzy differential equations. FWWymaa maki and Alkiffai A.N published a new work about fuzzy complex integral transform which they named FFW-transform in [6, 11-15]. This paper builds its structure according to the following organizing scheme. The necessary foundational information appears in Section 2. The third part of this work contains essential characteristics of FFW-transform. Section 4 establishes FFW-transform for third order derivative calculation. Section 5 contained the general expression for the fuzzy FFW-transform. Section 6 provides an illustration of the generalized fuzzy FFW-transform technique (nth-order fuzzy derivative) which includes an example for drug concentration measurement in body organs. Drug amounts in these areas can be evaluated through blood (plasma) and urine along with saliva and other sampled fluids.

2. Main Results

Theorem 1: Let $\vartheta(\xi)$ by fuzzy function $\xi \geq 0$, $\eta(\mu) = \mu, \mu \neq 0$ be positive real function and $\beta(\mu) = i^{5}\sqrt[a]{\mu} + \mu, \mu \neq 0$ be $\mathbb{C}^{+}$ −function then $\vartheta(\xi)\,'$for n^th^ - order will be as following:

1. $FFW\{\xi\ \vartheta(\xi)\} = -\frac{\mu}{i^{5}\sqrt[a]{\mu}+\mu}\left(\frac{FFW(\vartheta(\xi),\mu)}{\mu}\right)'$

2. $FFW\{\xi^{2}\vartheta(\xi)\} = (-1)^{2}\frac{\mu}{i^{5}\sqrt[a]{\mu}+\mu}\left(\frac{1}{i^{5}\sqrt[a]{\mu}+\mu}\left(\frac{FFW(\vartheta((\xi),\mu)}{\mu}\right)'\right)3.$

3. $FFW\{\xi^{n}\vartheta(\xi)\} = (-1)^{n}\frac{\mu}{i^{5}\sqrt[a]{\mu}+\mu}\left(\frac{1}{i^{5}\sqrt[a]{\mu}+\mu}\left(\frac{1}{i^{5}\sqrt[a]{\mu}+\mu}\left(\cdots\left(\frac{1}{i^{5}\sqrt[a]{\mu}+\mu}\left(\frac{FFW(\vartheta(\xi),\cot s)}{\mu}\right)'\right)'\right)'\right)\cdots\right)'$

Proof: Since

$$FFW\left\{\vartheta(\xi),\mu\right\}\ \mu\int_0^\infty \underline{\vartheta}(\xi;\mu)e^{-(i^{5a}\sqrt{\mu}+\mu)\xi}d\xi,\mu\int_0^\infty \overline{\vartheta}(\xi;\mu)e^{-(i^{5a}\sqrt{\mu}+\mu)\xi}d\xi \Rightarrow$$

$$\frac{FFW\{\vartheta(\xi),\mu\}}{\mu}\int_0^\infty \underline{\vartheta}(\xi;\mu)e^{-(i^{5a}\sqrt{\mu}+\mu)\xi}d\xi,\mu\int_0^\infty \overline{\vartheta}(\xi;\mu)e^{-(i^{5a}\sqrt{\mu}+\mu)\xi}d\xi$$

Derivative

$$\left(\frac{FFW\{\vartheta(\xi),\mu\}}{\mu}\right)' = \frac{d}{d\mu}\left[\int_0^\infty \underline{\vartheta}(\xi;\mu)e^{-(i^{5a}\sqrt{\mu}+\mu)\xi}d\xi,\int_0^\infty \overline{\vartheta}(\xi;\mu)e^{-(i^{5a}\sqrt{\mu}+\mu)\xi}d\xi\right]$$

$$\left(\frac{FFW\{\vartheta(\xi),\mu\}}{\mu}\right)' = -(i^{5a}\sqrt{\mu}+\mu)\int_0^\infty \underline{\vartheta}(\xi;\mu)e^{-(i^{5a}\sqrt{\mu}+\mu)\xi}d\xi - (i^{5a}\sqrt{\mu}+\mu)\int_0^\infty \overline{\vartheta}(\xi;\mu)e^{-(i^{5a}\sqrt{\mu}+\mu)\xi}d\xi,$$

$$\left(\frac{FFW\{\vartheta(\xi),\mu\}}{\mu}\right)' = -(i^{5a}\sqrt{\mu}+\mu)\frac{FFW\{\underline{\vartheta}(\xi;\mu),\mu\}}{\mu} - (i^{5a}\sqrt{\mu}+\mu)\frac{FFW\{\overline{\vartheta}(\xi;\mu),\mu\}}{\mu},$$

$$\left(\frac{FFW(\vartheta(\xi),\mu)}{\mu}\right)' = -(i^{5a}\sqrt{\mu}+\mu)\frac{FFW\{\xi\ \vartheta(\xi),\mu\}}{\mu}$$

Then, to get: $FFW\{\xi\ \vartheta(\xi)\} = -\frac{\mu}{i^{5a}\sqrt{\mu}+\mu}\left(\frac{FFW(\vartheta(\xi),\mu)}{\mu}\right)'$
since from the first part, we have and assume that $\lambda = i^{5a}\sqrt{\mu}+\mu$

$$FFW\{\xi\ \vartheta(\xi),\mu\} = -\frac{\mu}{i\lambda}\left(\frac{FFW(\vartheta(\xi),\mu)}{\mu}\right)' - (i\lambda)\frac{1}{\mu}\int_0^\infty \xi^2\,\underline{\vartheta}(\xi;\cot s)e^{-(\lambda)\xi}d\xi,$$

$$-(i\lambda)\int_0^\infty \xi^2\,\overline{\vartheta}(\xi;\mu)e^{-(i\lambda)\xi}d\xi = \left(-\frac{\mu}{(i\lambda)}\left(\frac{FFW\{\vartheta(\xi),\mu\}}{\mu}\right)'\right)'$$

Thus: $FFW\{\xi^2\vartheta(\xi)\} = (-1)^2\frac{\mu}{i\lambda}\left(-\frac{1}{i\lambda}\left(\frac{FFW(\vartheta(\xi),\mu)}{\mu}\right)'\right)'$

3. we can prove : $FFW\{\xi^2\vartheta(\xi)\} = (-1)^2\frac{\mu}{i\lambda}\left(-\frac{1}{i\lambda}\left(\frac{FFW(\vartheta(\xi),\mu)}{\mu}\right)'\right)'$
we get:

$$FFW\{\xi^n\vartheta(\xi)\} = (-1)^n\frac{\mu}{i\lambda}\left(\frac{1}{i\lambda}\left(\frac{1}{i\lambda}\left(\cdots\left(\frac{1}{i\lambda}\left(\frac{FFW(\vartheta(\xi),\mu)}{\mu}\right)'\right)'\right)'\right)'\cdots\right)'$$

Theorem 2: *Let* $\eta(\mu) = \mu$ *and* $\beta(\mu) = i^{5a}\sqrt{\mu}+\mu$ *are differe-functions S.T* $\vartheta(\xi)$*be F-function, then:* $FFW\{\xi\vartheta^{(n)}(\xi)\} = -\frac{\mu}{i^{5a}\sqrt{\mu}+\mu}\frac{d}{d\mu}\left(\frac{FFW(\vartheta^{(n)}(\xi))}{\mu}\right)$

Proof: Since

$$FFW\{\vartheta^{(n)}(\xi),\mu\} = \mu\int_0^\infty \underline{\vartheta}^{(n)}(\xi;\vartheta)e^{-(i^{5a}\sqrt{\mu}+\mu)\xi}d\xi,\mu\int_0^\infty \overline{\vartheta}^{(n)}(\xi;\vartheta)e^{-(i^{5a}\sqrt{\mu}+\mu)\xi}d\xi$$

$$\frac{FFW\{\vartheta^{(n)}(\xi),\mu\}}{\mu} = \int_0^\infty \underline{\vartheta}^{(n)}(\xi;\vartheta)e^{-(i^{5a}\sqrt{\mu}+\mu)\xi}d\xi,\int_0^\infty \overline{\vartheta}^{(n)}(\xi;\vartheta)e^{-(i^{5a}\sqrt{\mu}+\mu)\xi}d\xi$$

then : $\frac{d}{d\mu}\left[\frac{FFW\{\vartheta^{(n)}(\xi),\mu\}}{\mu}\right]=\left[\int_0^\infty \underline{\vartheta}^{(n)}(\xi;\vartheta)e^{-(i\sqrt[5a]{\mu}+\mu)\xi}d\xi,\int_0^\infty \overline{\vartheta}^{(n)}(\xi;\vartheta)e^{-(i\sqrt[5a]{\mu}+\mu)\xi}d\xi\right]$

$$\frac{d}{d\mu}\left[\frac{FFW\{\vartheta^{(n)}(\xi),\mu\}}{\mu}\right]=-(i\sqrt[5a]{\mu}+\mu)\int_0^\infty \underline{\vartheta}^{(n)}(\xi;\vartheta)e^{-(i\sqrt[5a]{\mu}+\mu)\xi}d\xi,-(i\sqrt[5a]{\mu}+\mu)\int_0^\infty \overline{\vartheta}^{(n)}(\xi;\vartheta)e^{-(i\sqrt[5a]{\mu}+\mu)\xi}d\xi$$

From equation 2:

$$\frac{d}{d\mu}\left[\frac{FFW\left\{\vartheta^{(n)}(\xi),\mu\right\}}{\mu}\right]=-(i\sqrt[5a]{\mu}+\mu)\frac{FFW\left\{\xi\underline{\vartheta}^{(n)}(\xi;\vartheta),\mu\right\}}{\mu},-(i\sqrt[5a]{\mu}+\mu)\frac{FFW\left\{\xi\overline{\vartheta}^{(n)}(\xi;\vartheta),\mu\right\}}{\mu}$$

$$\frac{d}{d\mu}\left[\frac{FFW\left\{\vartheta^{(n)}(\xi),\mu\right\}}{\mu}\right]=-(i\sqrt[5a]{\mu}+\mu)\frac{FFW\left\{\xi\vartheta^{(n)}(\xi;\vartheta),\mu\right\}}{\mu}.$$

$$Then: FFW\left\{\xi\vartheta^{(n)}(\xi)\right\}=-\frac{\mu}{i\sqrt[5a]{\mu}+\mu}\frac{d}{d\mu}\left(\frac{FFW\left(\vartheta^{(n)}(\xi)\right)}{\mu}\right)$$

Theorem 3: *Assume that* $\vartheta(\xi),\vartheta^{\backslash}(\xi),\dots,\vartheta^{n-1}(\xi),\vartheta^{(n)}(\xi)$ *are CF-valued functions on* $[0,\infty)$ *and Let*$\vartheta^{(i_1)}(\xi),\vartheta^{(i_2)}(\xi),\dots,\vartheta^{(i_\varphi)}(\xi)$*are the 2nd form DF for* $0\le i_1\le i_2\ldots\le i_\varphi\le n-1$*and* $\vartheta^{(p)}$*be IST for* $p\ne i_j, j=1,2,\dots,\varphi$*, then:*

(1) If $\varphi$is an even number, we have

$$M(\vartheta^{(n)}(\xi))=(\sqrt[5a]{\mu}+\mu)^n M(\vartheta(\xi))\mu(\sqrt[5a]{\mu}+\mu)\vartheta(0)®\sum_{\kappa=1}^{n-1}\mu^{n-\kappa+1}\vartheta^{(\kappa)}(0),$$

$$M(\vartheta^{(n)}(\xi))=-\mu^{n+1}\vartheta(0)(-\mu^n)M(\vartheta(\xi))®\sum_{\kappa=1}^{n-1}\mu^{n-\kappa+1}\vartheta^{(\kappa)}(0),$$

Proof: $FFW(\mu)=FFW[\vartheta(\mu)]F(p)=L[\vartheta(\aleph)]$ $FFW(\mu)=FFW[\vartheta^{(n)}(\xi)]$ and $F_n(p)=L[\vartheta^{(n)}(\aleph)]$ assume that $\lambda=\sqrt[5a]{\mu}+\mu$

$$FFW(\xi)=FFW[\vartheta^{(n)}(\xi)]=\mu F_n(\lambda)$$

$$FFW(\xi)=(\lambda)^n[\mu F(\lambda)]\mu(\lambda)^{(n-1)}\vartheta(0)®\mu\sum_{\kappa=1}^{n-1}(\lambda)^{n-(\kappa+1)}\vartheta^{(\kappa)}(0)$$

$$=(\lambda)^n FFW[\vartheta(\xi)]\mu(\lambda)^{(n-1)}\vartheta(0)®\mu\sum_{\kappa=1}^{n-1}(\lambda)^{n-(\kappa+1)}\vartheta^{(\kappa)}(0)$$

$$FFW(\mu)=-\mu(\lambda)^{(n-1)}\vartheta(0)(-(\lambda)^n)[\mu F(\lambda)]®\mu\sum_{\kappa=1}^{n-1}(\lambda)^{n-(\kappa+1)}\vartheta^{(\kappa)}(0)$$

$$=-\mu(\lambda)^{(n-1)}\vartheta(0)(-(\lambda)^n)FFW[\vartheta(\xi)]®\mu\sum_{\kappa=1}^{n-1}(\lambda)^{n-(\kappa+1)}\vartheta^{(\kappa)}(0)$$

Example 4: $\vartheta^{\backslash}(t)=\Psi\Upsilon-\hbar\vartheta(t),\vartheta(0)=(\vartheta(0,r),\vartheta(r,0))$
ST $\Psi=\Upsilon=c(t)=\hbar=v(t)=1$

Case 1: By using FFFW-Transform

$$(\sqrt[5a]{\mu}+\mu)FFW[\vartheta(\xi)]-\mu\vartheta(0)=-FFW[\vartheta(\xi)]$$

$$(\sqrt[5a]{\mu}+\mu)FFW[\underline{\vartheta}(\xi)]-\mu\underline{\vartheta}(0)=-FFW[\underline{\vartheta}(\xi)],(\sqrt[5a]{\mu}+\mu)FFW[\overline{\vartheta}(\xi)]-\mu\overline{\vartheta}(0)$$
$$=-FFW[\overline{\vartheta}(\xi)]$$
$$FFW[\underline{\vartheta}(\xi;\vartheta)]=\frac{\mu}{(\sqrt[5a]{\mu}+\mu)+1}\underline{\vartheta}(0;\xi),FFW[\overline{\vartheta}(\xi;\vartheta)]$$
$$=\frac{\mu}{(\sqrt[5a]{\mu}+\mu)+1}\overline{\vartheta}(0;\xi)$$
$$\underline{\vartheta}(\xi;\vartheta)=e^{-\xi}\underline{\vartheta}(0;\xi),\overline{\vartheta}(\xi;\vartheta)=e^{-\xi}\overline{\vartheta}(0;\xi)$$

Case 2. By using fuzzy FFW-Transform

$$-\mu\vartheta(0)\,\square\left(-(\sqrt[5a]{\mu}+\mu)\right)FFW[\vartheta(\xi)]=-FFW[\vartheta(\xi)](5)$$
$$\left(\sqrt[5a]{\mu}+\mu\right)FFW\left[\overline{\vartheta}(\xi)\right]-\mu\overline{\vartheta}(0)=-FFW\left[\underline{\vartheta}(\xi)\right],$$
$$\left(\sqrt[5a]{\mu}+\mu\right)FFW\left[\underline{\vartheta}(\xi)\right]-\mu\underline{\vartheta}(0)=-FFW\left[\overline{\vartheta}(\xi)\right]$$
$$FFW\left[\underline{\vartheta}(\xi;\vartheta)\right]=\frac{\mu(\sqrt[5a]{\mu}+\mu)}{(\sqrt[5a]{\mu}+\mu)^2-1}\overline{\vartheta}(0;\xi)-\frac{\mu}{(\sqrt[5a]{\mu}+\mu)^2-1}\underline{\vartheta}(0;\xi)$$
$$FFW\left[\overline{\vartheta}(\xi;\vartheta)\right]=\frac{\mu(\sqrt[5a]{\mu}+\mu)}{(\sqrt[5a]{\mu}+\mu)^2-1}\underline{\vartheta}(0;\xi)-\frac{\mu}{(\sqrt[5a]{\mu}+\mu)^2-1}\overline{\vartheta}(0;\xi)$$

Now we use inverse fuzzy FFW-transform

$$\underline{\vartheta}(\xi;\vartheta)=\cosh\varpi\overline{\vartheta}(0;\xi)-\sinh\varpi\underline{\vartheta}(0;\xi),$$
$$\overline{\vartheta}(\xi;\vartheta)=\cosh\varpi\underline{\vartheta}(0;\xi)-\sinh\varpi\overline{\vartheta}(0;\xi)$$

3. Conclusion

The research work presents multiple fuzzy FFW-transform properties while also providing a third-order fuzzy FFW-transform and a generalized fuzzy FFW-transform equation. The given formulas help solve systems that require drug concentration analysis or determine medication quantity within body organs based on measuring blood (plasma), urine, saliva, or different testing fluids.

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