

A novel approach of digital parametric metric space

K. Dinesh *

Department of Mathematics

K. Ramakrishnan College of Engineering (Autonomous)

Tiruchirappalli 621112

Tamil Nadu

India

Eriola Sila [†]

Department of Mathematics

Faculty of Natural Sciences

University of Tirana

Tirana

Albania

Kastriot Zoto [§]

Department of Mathematics, Informatics and Physics

Faculty of Natural Sciences

University of Gjirokastra

Gjirokastra 6001

Albania

B. Shoba [‡]

Department of Mathematics

St Joseph's College of Engineering

OMR Chennai 600119

Tamil Nadu

India

* E-mail: dinesh.skksv93@gmail.com (Corresponding Author)

[†] E-mail: eriola.sila@unitir.edu.al

[§] E-mail: kzoto@uogj.edu.al

[‡] E-mail: shobabalasubramaniam@gmail.com

Doaa Rizk^{*}

*Department of Mathematics
College of Science
Qassim University
Buraydah 51452
Saudi Arabia*

Abstract

This work broadens fixed point (FP) theory by introducing results within the setting of digital parametric metric spaces (DPMS). Motivated by the discrete nature of digital data and the flexibility offered by parametric distance measures, we treat a DPMS as a structure in which separation between points is evaluated through a digitally defined metric that depends on a positive parameter. Several new FP results are established aimed at self-mappings on such spaces, and their applicability is illustrated in contexts such as computational geometry, discrete dynamical systems, and digital image analysis. The convergence patterns of iteratively generated sequences are explored under suitable contractive conditions, supporting the theoretical developments. In addition to enriching the theory of fixed points in discrete spaces, these results create potential avenues for use in problems requiring digital computation and structured data processing.

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Introduction

For demonstrating the being and ultimately the uniqueness of results to a variety of mathematical models (variational inequalities & integral and partial differential equations), FP theorems are a crucial tool. Various kinds of generalized metric spaces have been created in current times by various authors using various methodologies [2-5]. Cone metric space, D-metric space, and others are examples of generalized metric spaces [8-21]. Hussain et al. [1] recently proposed and investigated the idea that parametric metric (PMM) spaces are a natural extension of MS. In this study, we introduce a few FP theorems in DPMS under different expanding circumstances.

Definition 2.1.[7] Let q be a positive integer and let Z is the collection of all integers. Define the following for the set Z^q :

$$Z^q = \{(a_1, a_2, a_3, \dots, a_n)/a_i \in Z, 1 \leq i \leq q\}$$

The symbol Z^q refers the collection of all points with integer coordinates in q -dimensional Euclidean space.

^{*} E-mail: d.hussien@qu.edu.sa

Definition 2.2: [6,7] Let $q_* = (q_{*1}, q_{*2}, q_{*3} \dots q_{*q})$ and $q_{\natural} = (q_{\natural 1}, q_{\natural 2}, q_{\natural 3} \dots q_{\natural q})$ be distinct points in Z_q . Let m be a positive integer such with $1 \leq m < q$. The points q_* and $q_{\natural}$ are said to be k_m -nearby in Z_q if there are at most m indices i the difference satisfies $|q_{*i} - q_{\natural i}| = 1$ is at most m , for all other indices j , the condition $|q_{*i} - q_{\natural i}| \neq 1$

If $|q_* - q_{\natural}| = 1$, then two points in Z , q_* and $q_{\natural}$, are 2-adjacent (Refer Figure 2.1).

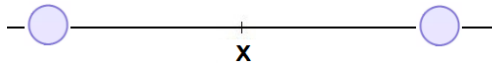


Figure 2.1
2-adjacent

If two points q_* and $q_{\natural}$ in Z^2 are 8 adjacent and differ in precisely one coordinate, they are 4-adjacent (Refer Figure 2.2).

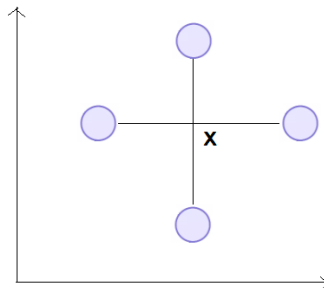


Figure 2.2
4-adjacent

If two unique points q_* and $q_{\natural}$ in Z^2 differ by no more than 1 in each coordinate, they are 8-adjacent (Refer Figure 2.3).

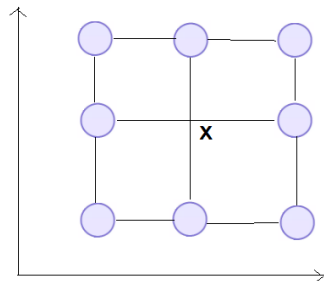


Figure 2.3
8-adjacent

Definition 2.3: [6,7] A digital image can be represented by an ordered pair (P, κ) , where P is a finite subset of Z_q and κ denotes an adjacency relation defined on the elements of P , for some positive integer n .

Definition 2.4: [6,7] If $Dq_* = q_*$, then $q_* \in (P, \rho_D, \kappa)$ is named a FP of the mapping $D: P \rightarrow P$. Consider the digital image (P, κ) . The FP property is present in the digital image (P, κ) [6]. If there is a FP in each (κ, κ) continuous map $D: (P, \kappa) \rightarrow (P, \kappa)$.

The Banach contraction mapping theory of digital images was created and illustrated by Ege & Karaca [6] in 2015, as shown below.

Theorem 2.5: [6] Assume that the digital metric space (P, ρ_D, κ) is complete. Let T be a digital contraction map $T: (P, \rho_D, \kappa) \rightarrow (P, \rho_D, \kappa)$. There is a unique point $p \in P$ such that $Tp = p$, indicating that T has a unique FP.

Definition 2.6: Let the digital image $(P, \kappa) \subset Z^q$. Let P represent a nonempty (NE) set with $\rho_D: P \times P \times (0, +\infty) \rightarrow [0, +\infty)$

Let $(P, \kappa) \subset Z^q$ be a digital image. Let P be a NE set and a function $\rho_D: P \times P \times (0, +\infty) \rightarrow [0, +\infty)$ is called a PMM on P if it fulfills the following circumstances for all $t > 0$ and all $q_*, q_\Delta, q_\nabla \in P$

- (i) $\rho_D(q_*, q_\nabla, t) = 0 \Leftrightarrow q_* = q_\nabla$
 - (ii) $\rho_D(q_*, q_\nabla, t) = \rho_D(q_\nabla, q_*, t)$
 - (iii) $\rho_D(q_*, q_\nabla, t) \leq \rho_D(q_*, q_\Delta, t) + \rho_D(q_\Delta, q_\nabla, t)$
- (P, ρ_D, κ) is called digital PMS.

The digital image and the function ρ_D together form a DMS with κ -adjacency.

Definition 2.7: In a digital PMS (P, ρ_D, κ) , $\{q_{*q}\}_{q=1}^\infty$ be a sequence

- (1) $\{q_{*q}\}_{q=1}^\infty$ is claimed to be converge to $q_* \in P$, if $\lim_{q \rightarrow \infty} \rho_D(q_{*q}, q_*, t) = 0$

$$\lim_{n \rightarrow \infty} q_{*q} = q_* \text{ for all } t > 0$$

- (2) $\{q_{*q}\}_{q=1}^\infty \subset P$ is called a Cauchy sequence (C-Seq) in P if, for every $t > 0$, if

$$\lim_{q, m \rightarrow \infty} \rho_D(q_{*q}, q_{*m}, t) = 0$$

- (3) The digital PMS (P, ρ_D) is named complete if every C-seq in P converges to some point in P .

Definition 2.8: Let (P, ρ_D, κ) be a DPMS and a function $D: P \rightarrow P$ stands continuous on $q_* \in P$, if for every sequence $\{q_{*q}\}_{q=1}^\infty$ in P the kind that $\lim_{n \rightarrow \infty} q_{*q} = q_*$ then $\lim_{q \rightarrow \infty} Dq_{*q} = Tq_*$

Lemma 2.9: Let $\{\varrho_{*q}\}_{q=1}^{\infty}$ be a sequence in a DPMS (P, ρ_D, κ) the kind that

$$\rho_D(\varrho_{*q}, \varrho_{*q+1}, t) \leq \lambda \rho_D(\varrho_{*q-1}, \varrho_{*q}, t)$$

Wherever $\lambda \in [0, 1)$, $q = 1, 2, \dots$ then $\{\varrho_{*q}\}_{q=1}^{\infty}$ is a C-seq in (P, ρ_D)

Lemma 2.10: Let (P, ρ_D, κ) be a digital PMS. Let $\{\varrho_{*q}\}_{q=1}^{\infty}$ be a sequence of points of P such that

$$\rho_D(\varrho_{*q}, \varrho_{*q+1}, t) \leq \lambda \rho_D(\varrho_{*q-1}, \varrho_{*q}, t)$$

Wherever $\lambda \in \left[0, \frac{1}{s}\right)$, $n = 1, 2, \dots$ Now $\{\varrho_{*q}\}_{q=1}^{\infty}$ is a C-Seq in (P, ρ_D, κ)

Main Results

Theorem 3.1: Consider the triple (P, ρ_D, κ) be a Complete DPMS and D is a continuous mapping satisfying the following condition:

$$\rho_D(D\varrho_*, D\varrho_{\sup}, t) \leq \delta_1 \max \left[\rho_D(\varrho_*, \varrho_{\sup}, t), \rho_D(\varrho_*, D(\varrho_*), t), \right. \\ \left. \rho_D(\varrho_{\sup}, D(\varrho_{\sup}), t), \rho_D(\varrho_*, D(\varrho_{\sup}), t), \rho_D(D(\varrho_*), \varrho_{\sup}, t) \right]$$

For all $\varrho_*, \varrho_{\sup} \in P$, $\varrho_* \neq \varrho_{\sup}$ and $\forall t > 0$ with $\delta_1 \in [0, 1)$. The mapping D possesses a unique FP in P .

Proof: Take $\varrho_0 \in P$, $\varrho_{q+1} = D\varrho_q$, $q \geq 0$

$$\text{Set } \mathfrak{K}_q(t) = \rho_D(\varrho_q, \varrho_{q+1}, t) \quad (q \geq 0, t > 0)$$

Apply the hypothesis with $\varrho_* = \varrho_q$, $\varrho_{\sup} = \varrho_{q+1}$

Then for each $q \in \mathbb{N}$ and $t > 0$

$$\rho_D(\varrho_{q+1}, \varrho_{q+2}, t) \\ \leq \delta_1 \max \left[\rho_D(\varrho_q, \varrho_{q+1}, t), \rho_D(\varrho_q, D\varrho_q, t), \right. \\ \left. \rho_D(\varrho_{q+1}, D\varrho_{q+1}, t), \rho_D(\varrho_q, D\varrho_{q+1}, t), \rho_D(D\varrho_q, \varrho_{q+1}, t) \right]$$

$D\varrho_q = \varrho_{q+1}$ and $D\varrho_{q+1} = \varrho_{q+2}$ this inequality reduces to

$$\mathfrak{K}_{q+1}(t) \leq \delta_1 \max[\rho_D(\varrho_q, \varrho_{q+2}, t), \mathfrak{K}_q(t), \mathfrak{K}_q(t), \mathfrak{K}_{q+1}(t), \mathfrak{K}_q(t)]$$

Applying the triangle property of

$$\mathfrak{K}_{q+1}(t) \leq \delta_1 \max[\rho_D(\varrho_q, \varrho_{q+2}, t), \mathfrak{K}_q(t), \mathfrak{K}_{q+1}(t)] \quad (3.1)$$

From triangle inequality ρ_D

$$\rho_D(\varrho_q, \varrho_{q+2}, t) \leq \mathfrak{K}_{q+1}(t) + \mathfrak{K}_q(t)$$

Substituting into the previous inequality yields three possible situations for $\mathfrak{K}_{q+1}(t)$.

There exists a constant $\mathfrak{Z} \in [0, 1)$,

$$\text{Choosing } \mathfrak{Z} = \max \left\{ \delta_1, \frac{\delta_1}{1-\delta_1} \right\},$$

ensures the recurrence inequality $3\mathfrak{K}_q(t) \geq \mathfrak{K}_{q+1}(t)$ holds

$$t > 0 \text{ \& \& } \forall q \geq 0$$

Fix $t > 0$ for $m > n$

$$\rho_D(q_n, q_m, t) \leq \frac{3^n}{1-3} \mathfrak{K}_0(t)$$

It tends to 0 as $n \rightarrow \infty$

Thus $\{q_q\}$ is Cauchy in P (uniformly in the parameter $t > 0$ in the sense used here). Completeness of (P, ρ_D) gives $q_q \rightarrow q^* \in P$

Using continuity of D

$$Dq^* = D\left(\lim_{q \rightarrow \infty} q_q\right) = \lim_{q \rightarrow \infty} Dq_q = \lim_{q \rightarrow \infty} q_{q+1} = q^*$$

So q^* is a fixed point.

Uniqueness: Suppose $u, v \in P$ are both FPs. Invoking the contractive inequality for the pair (u, v) we obtain

$$\rho_D(u, v, t) = \rho_D(Du, Dv, t) \leq \delta_1 \max\{\rho_D(u, v, t), 0, 0, \rho_D(u, v, t), \rho_D(u, v, t)\}$$

The RHS simplifies to $\delta_1 \rho_D(u, v, t)$. Because $\delta_1 < 1$, the only possibility is $\rho_D(u, v, t) = 0$ which implies $u = v$

Theorem 3.2: Let D be continuous mapping that fulfils the succeeding condition, and let (P, ρ_D, κ) be a CDPMS.

$$\begin{aligned} & \rho_D(Dq_*, Dq_2, t) \\ & \leq \delta_1 [\rho_D(q_*, D(q_*), t), \rho_D(q_2, D(q_2), t)] \\ & \quad + \delta_2 [\rho_D(q_*, D(q_2), t), \rho_D(D(q_*), q_2, t)] \end{aligned} \quad (3.2)$$

for all $q_*, q_2 \in P$

And $\delta_1 + \delta_2 < \frac{1}{2}$, $\delta_1, \delta_2 \in \left[0, \frac{1}{2}\right)$. Then T has a FP in P .

Proof: Let $q_0 \in P$ be arbitrary and define the sequence $q_{q+1} = Dq_q$ for

$$q \geq 0$$

$$\text{Set } \mathfrak{L}_q(t) = \rho_D(q_q, q_{q+1}, t) \quad (q \geq 0, t > 0)$$

$$\mathfrak{L}_{q+1}(t) = \rho_D(q_{q+1}, q_{q+2}, t)$$

Applying the assumption with $q_* = q_q$, we have $q_2 = q_{q+1}$

Thus, for each $q \in \mathbb{N}$ and $t > 0$,

$$\begin{aligned} \mathfrak{L}_{q+1}(t) & \leq \delta_1 [\rho_D(q_q, Dq_q, t), \rho_D(q_{q+1}, Dq_{q+1}, t)] \\ & \quad + \delta_2 [\rho_D(q_q, Dq_{q+1}, t), \rho_D(Dq_q, q_{q+1}, t)] \end{aligned}$$

Since $Dq_q = q_{q+1}$ and $Dq_{q+1} = q_{q+2}$ and using the triangle inequality of

ρ_D

$$\mathfrak{L}_{q+1}(t) \leq \delta_1 (\mathfrak{L}_{q+1}(t) + \mathfrak{L}_q(t)) + \delta_2 (\mathfrak{L}_{q+1}(t) + \mathfrak{L}_q(t))$$

Rearranging terms leads to

$$(1 - (\delta_1 + \delta_2))\mathfrak{L}_{q+1}(t) \leq (\delta_1 + \delta_2)\mathfrak{L}_q(t)$$

$\{\mathfrak{L}_q(t)\}$ satisfies a geometric-type inequality

$$\mathfrak{L}_{q+1}(t) \leq \mathfrak{Y}\mathfrak{L}_q(t), \mathfrak{Y} = \frac{\delta_1 + \delta_2}{1 - (\delta_1 + \delta_2)}$$

By the assumption $\delta_1 + \delta_2 < \frac{1}{2}$ we have $\mathfrak{Y} \in [0, 1)$

Iteration gives $b_q(t) \leq \mathfrak{Y}^q b_0(t)$

Consequently, for $m > n$

$$\begin{aligned} \rho_D(\varrho_n, \varrho_m, t) &\leq \mathfrak{L}_0(t) \sum_{k=n}^{\infty} \mathfrak{Y}^k \\ &= \frac{\mathfrak{Y}^n}{1 - \mathfrak{Y}} \mathfrak{L}_0(t) \rightarrow 0 (n \rightarrow \infty) \end{aligned}$$

So $\{\varrho_q\}$ is Cauchy and converges to some $\varrho^* \in P$.

Continuity of D yields $D\varrho^* = \varrho^*$. Uniqueness follows by substituting two FPs into the equation (3.2), the same calculation forces their distance to be zero.

Theorem 3.3: Let (P, ρ_D) be a CDPMS and D is a continuous mapping filling the succeeding condition:

$$\rho_D(D\varrho_*, D\varrho_{\beth}, t) + \delta_1 \rho_D(\varrho_{\beth}, D\varrho_*, t) \geq \delta_2 \frac{\rho_D(\varrho_*, D\varrho_*, t) \rho_D(\varrho_*, D\varrho_{\beth}, t)}{\rho_D(\varrho_*, \varrho_{\beth}, t)} + \delta_3 \rho_D(\varrho_*, \varrho_{\beth}, t)$$

For all $\varrho_*, \varrho_{\beth} \in P, \varrho_* \neq \varrho_{\beth}$ and $\forall t > 0$ where $\delta_1, \delta_2, \delta_3 \geq 0$ are real constants and $\delta_2 + \delta_3 - 2\delta_1 > 1, \delta_3 - \delta_1 > 1$. Now D has unique FP in P .

Proof: Let $\varrho_0 \in P$ be arbitrary and define the sequence $\varrho_{q+1} = D\varrho_q$ for $q \geq 0$

Set $\mathfrak{M}_q(t) = \rho_D(\varrho_q, \varrho_{q+1}, t)$ ($q \geq 0, t > 0$)

Applying the inequality in the theorem to the pair $(\varrho_q, \varrho_{q+1})$

Since $\rho_D(\varrho_q, D\varrho_q, t) = \mathfrak{M}_q(t)$ and $\rho_D(\varrho_{q+1}, D\varrho_{q+1}, t) = \mathfrak{M}_{q+1}(t)$

From assumed inequality, one obtains

$$\rho_D(\varrho_{q+1}, \varrho_{q+2}, t) + \delta_1 \rho_D(\varrho_{q+1}, \varrho_{q+2}, t) \geq \delta_2 \frac{\mathfrak{M}_{q+1}(t) \mathfrak{M}_q(t)}{\mathfrak{M}_q(t)} + \delta_3 \mathfrak{M}_q(t)$$

Assuming $\mathfrak{M}_q(t) > 0$, this reduces to

$$(1 + \delta_1 - \delta_2) \mathfrak{M}_{q+1}(t) \geq \delta_3 \mathfrak{M}_q(t)$$

Thus,

$$\mathfrak{M}_{q+1}(t) \geq \frac{\delta_3}{1 + \delta_1 - \delta_2} \mathfrak{M}_q(t) \quad (3.3.1)$$

On the other hand, by reapplying the inequality at the subsequent index and using the triangular property of the metric, we also arrive at the upper estimate

$$\mathfrak{M}_{q+1}(t) \leq \frac{1 + \delta_1 - \delta_2}{\delta_3 - \delta_1} \mathfrak{M}_q(t) = k \mathfrak{M}_q(t) \quad (3.3.2)$$

Where the algebraic manipulations are valid under the stated parameter constraints, the condition $\delta_3 - \delta_1 > 1$ guarantees $k < 1$

Combining (3.3.1) and (3.3.2) show that $\mathfrak{M}_{q+1}(t)$ is squeezed and in particular satisfies the geometric upper bound

$$\mathfrak{M}_q(t) \leq k^p \mathfrak{M}_0(t) \quad (q \geq 0)$$

With $k \in [0,1)$. Therefore the series, $\sum_q \mathfrak{M}_q(t)$ converges and $\{\varrho_q\}$ is Cauchy. Completeness yields convergence $\varrho_q \rightarrow \varrho^* \in P$

Continuity of D gives $D\varrho^* = \varrho^*$ so a FP exists.

Uniqueness: Suppose $\tilde{\varrho}$ is another FP.

$$\rho_D(\varrho^*, \tilde{\varrho}, t) + \delta_1 \rho_D(\tilde{\varrho}, \varrho^*, t) \geq \delta_3 \rho_D(\varrho^*, \tilde{\varrho}, t)$$

But ρ_D is symmetric, so this reads

$$(1 + \delta_1 - \delta_3) \rho_D(\varrho^*, \tilde{\varrho}, t) \leq 0$$

Because of the hypothesis ensures $\delta_3 - \delta_1 > 1$ we have

$1 + \delta_1 - \delta_3 < 0$ forcing $\rho_D(\varrho^*, \tilde{\varrho}, t) = 0$ and hence $\varrho^* = \tilde{\varrho}$. This proves uniqueness.

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