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LRS Bianchi type II stiff fluid inflationary model with flat potential in general relativity

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Abstract

In the current research, I have used the framework of Bianchi Type II LRS type metric using the condition of the stiff fluid inflationary model with flat potential. For a relativistic solution of the field equation, we have assumed an auxiliary condition $R = S^m$ between the metric potential R & S., where m is a constant, which can take any value apart from one. For a stiff fluid, apply the equation of state p=ρ . Some dynamic feature of the model is also discussed. The physical parameters are also illustrated through graphical representations.

Subject Classification: *83C15.*

Keywords: *Stiff Fluid, B-Type II, Inflation, General relativity.*

1. Introduction

An accelerating expansion of the universe is called inflation. The idea of the inflationary model was initially presented in the framework of a grand unified theory by Guth [1]. This theory was explained the prior expansion of the universe and accepted as a model of the early universe. Many considerable problems Flatness of the known universe, homogeneity, isotropy, and the solution of the monopole issue by inflation in the grand unified field theory are

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examples of contemporary cosmology. There are two main inflationary cosmological models which are based on immersion of flat potential and scalar curvature-squared term respectively. In general relativity, several authors [2-6] have proposed different aspects of the inflationary universe.

Stiff fluid models play a more significant role in an inflationary universe. In terms of speed, sound travels at the same rate as light, and equations provided by this model have the same nature as the gravitation field [7]. Barrow has suggested the importance of the "stiff equation of state p=ρ concerning the universe's matter content" during its early evolution to uphold the relevance of stiff fluid models. The stiff fluid model is also investigated by Mak and Harlo [8], and Bali et al. [9-11]. In electromagnetic field theory, the stiff fluid model of Bianchi Type ii is studied by Venkatewarlu and Reddy [12]. Forming a cosmological model requires using Bianchi Type space-time, representing the primordial phases of cosmic evolution. Reddy [13] has studied the Inflationary Universe with Massive String. Several authors have already considered inflationary Bianchi space-time. [14-20].

To arrive at a meaningful conclusion, we explore the Bianchi- II LRS type cosmological model using the condition of the stiff fluid in this study. We have applied the condition $R = S^m$ within the matric potential, with m denoting a non-negative integer and R and S being dependent on t. The model's structural elements are also covered.

2. Field equation with solution

The Bianchi Type II geometry is defined by the metric given as

$$ds^2 = -dt^2 + R^2dx^2 + S^2(dy - xdz)^2 + R^2dz^2 \tag{1}$$

The quantities R and S are purely functions of t.

With effective potential, the lagrangian of gravity minimally connected to φ is

$$L = \int \sqrt{-g}\left(R - V(\varphi) - \frac{1}{2}g_{ij}\partial_i\varphi\partial^j\varphi\right)d^4x \tag{2}$$

Einstein Field equation are formulated as

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi T_{ij} \tag{3}$$

We can assume $8\pi G = C = 1$

Energy momentum tensor which are given as

$$T_{ij} = (p + \rho)v_iv_j + pg_{ij} - \partial_i\varphi\partial_j\varphi - \left[\frac{1}{2}\partial_p\varphi\partial^p\varphi + V(\varphi)\right]g_{ij} \tag{4}$$

Using matric (1) and field equation (3), we get

$$\left(\frac{R_4}{R}\right)^2 - \frac{3}{4}\frac{S^2}{R^4} + 2\frac{R_{44}}{R} = -\frac{\varphi_4^2}{2} + V(\varphi) - P \tag{5}$$

$$\frac{R_{44}}{R} + \frac{S_{44}}{S} + \frac{R_4S_4}{RS} + \frac{1}{4}\frac{S^2}{R^4} = -P - \frac{\varphi_4^2}{2} + V(\varphi) \tag{6}$$

$$\left(\frac{R_4}{R}\right)^2 + 2\frac{R_4 S_4}{RS} - \frac{1}{4}\frac{S^2}{R^2} = \rho - \frac{\varphi_4^2}{2} + V(\varphi) \tag{7}$$

And

$$\varphi_{44} + \varphi_4\left(\frac{S_4}{S} + 2\frac{R_4}{R}\right) = -\frac{dV}{d\varphi} \tag{8}$$

Where indices 4 denote the differential relative to time t
For the Bianchi Type II LRS type, the spatial volume is determine by

$$V = R^2 S \tag{9}$$

We define

$$a = (R^2 S)^{\frac{1}{3}} \tag{10}$$

For the Bianchi Type II LRS type, the Hubble parameter (H) given as,

$$H = \frac{1}{3}\left(2\frac{R_4}{R} + \frac{S_4}{S}\right) \tag{11}$$

One way to represent the universal mean H is as

$$3H = H_1 + H_2 + H_3 \tag{12}$$

Where $H_1 = \frac{S_4}{S}$ and $H_2 = H_3 = \frac{R_4}{R}$

$$q = \frac{-aa_{44}}{a_4^2} \tag{13}$$

Scalar expansion (θ), the shear scalar (σ^2) are defined by

$$\theta = \frac{S_4}{S} + 2\frac{R_4}{R} \tag{14}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - \frac{1}{3}\theta^2\right) \tag{15}$$

3. Field equation solutions

We consider that effective potential V (φ) is flat and V (φ) = constant to get the inflationary scenario

$$\varphi_{44} + \varphi_4\left(\frac{S_4}{S} + 2\frac{R_4}{R}\right) = 0 \tag{16}$$

$$\varphi_4 = \frac{l}{R^2 S} \tag{17}$$

Applying stiff fluid condition p=ρ we have Solution of Field Equation

$$\frac{R_{44}}{R} - \frac{S_{44}}{S} + \frac{R_4^2}{R^2} - \frac{R_4 S_4}{RS} - \frac{S^2}{R^4} = 0 \tag{18}$$

Assuming a proportional relationship between the expansion θ and σ, we obtain

$$R = S^m \tag{19}$$

Where m is constant
From equation (18) and (19)

$$2S_{44} + 4m\frac{S_4^2}{S} = \frac{2}{m-1}S^{3-4m} \tag{20}$$

Let

$$S_4 = \eta \tag{21}$$

$S_{44} = \eta\frac{d\eta}{ds}$ Where $\eta' = \frac{d\eta}{ds}$

From equation (20) and (21)

$$\frac{d\eta^2}{ds} + 4m\frac{\eta^2}{S} = \frac{2}{m-1}S^{3-4m} \tag{22}$$

From Equation (22)

$$\eta^2 = \frac{1}{2(m-1)}S^{4-4m} + kS^{-4m} \tag{23}$$

Where k is integration constant

$$\left(\frac{ds}{dt}\right)^2 = \eta^2 = \frac{1}{2(m-1)}S^{(1-4m)} + kS^{-4m} \tag{24}$$

And the usin these transformation

$$S = T, x = X, y = Y, z = Z$$

$$(ds)^2 = -\left[\frac{1}{2(m-1)}T^{4(1-m)} + KT^{-4m}\right]^{-1} dT^2 + T^{2m}[dX^2 + dZ^2] + T^2[dY - XdZ]^2 \tag{25}$$

4. Physical and Geometrical Aspects

Volume is obtained by

$$V = R^2S = S^{2m+1} = T^{2m+1} \tag{26}$$

σ^2 Is obtain by

$$\sigma^2 = \frac{1}{3}(m-1)^2\left[\frac{1}{2(m-1)T^{(2m-1)}} + kT^{-2(2m+1)}\right] \tag{27}$$

θ is given by

$$\theta = (2m+1)\sqrt{\frac{1}{2(m-1)T^{2(2m-1)}} + kT^{-2(2m+1)}} \tag{28}$$

Hubble parameter is given by

$$H = \frac{1}{3}(2m+1)\sqrt{\frac{1}{2(m-1)T^{2(2m-1)}}} + kT^{-2(2m+1)} \tag{29}$$

$$\frac{\sigma}{\theta} = constant \tag{30}$$

Deceleration parameter is define as

$$q = -1 + \frac{3}{2m+1} \tag{31}$$

And

The energy density ρ is

$$\rho = (m^2 + 2m)\left(\frac{1}{2(m-1)}T^{4(1-m)} + KT^{-4m}\right) - \frac{1}{4}T^{2-2m} + \frac{1}{2}\frac{l^2}{T^{4m+2}} + \alpha \qquad (32)$$

Graphical representation

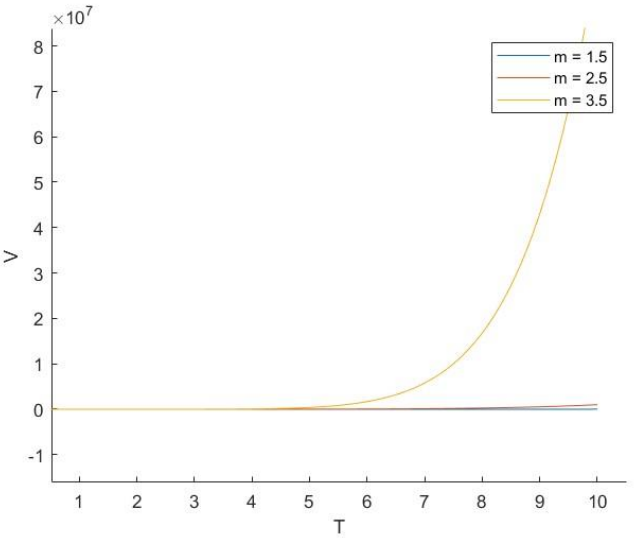


Figure 1
Graph between volume verses time

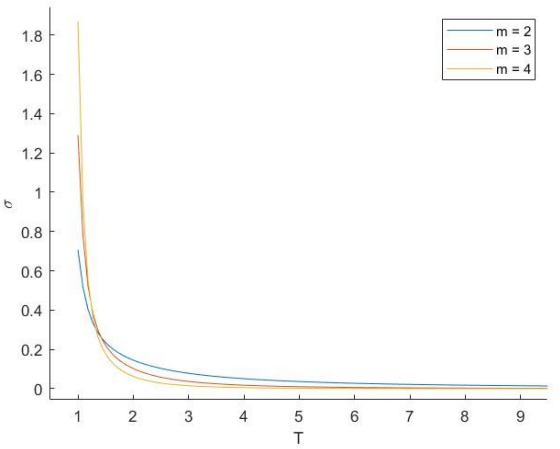


Figure 2
Graph between shear scalar verses time

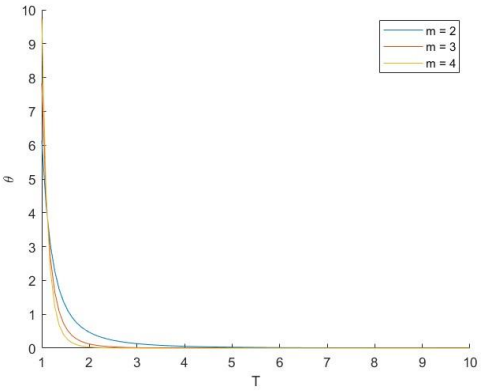


Figure 3
Graph between scalar expansion verses time

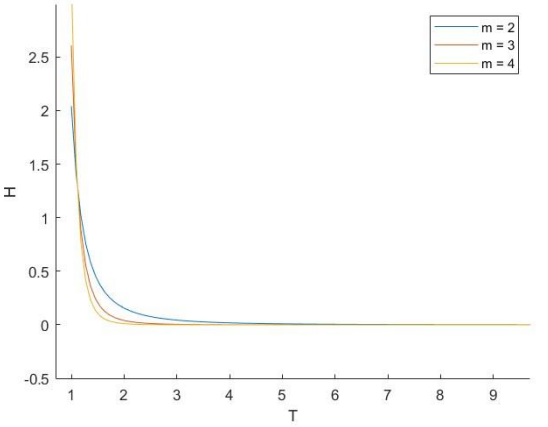


Figure 4
Graph between Hubble parameter verses time

Figure 1: its represent equation (26). This shows that the overall size of the universe increases continuously as time progresses.

Figure 2: its represent equation (27). In all three curves σ decreases monotonically as T increases, indicating that the anisotropy of the model fades with cosmic expansion.

Figure 3: its represent equation (28). In all three cases θ starts with a very high value when T is small (early epoch) and decreases rapidly as T increases.

Figure 4: its represent equation (29). For all three curves, H is very large at the early epoch (small T) and decreases sharply as T increases.

5. Conclusion

It is investigated a new cosmological model of LRS Bianchi Type II incorporating a stiff fluid. As cosmic time t rises, the model's suitable volume increases as well. $\frac{\sigma}{\theta}$ is continuous for this model. The model mentioned above is not line isotropic. At T = 0, a singularity of the point type is detected. The Shear scalar goes to zero for infinity and is decreasing with respect to time.

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