Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 28 (2025), No. 6, pp. 2311–2318
DOI : 10.47974/JIM-2373

Inflationary scenario in Bianchi type VIII space time with bulk viscosity using flat potential in general relativity

Rakhi Mathur [†]
Laxmi Poonia [*]
Varsha Maheshwari [§]
Manish Khowal [‡]
Department of Mathematics and Statistics
Manipal University Jaipur
Jaipur 303007
Rajasthan
India

Abstract

This paper has been written to study an anisotropic Bianchi Type VIII cosmological model which incorporates a Higgs scalar field with a flat effective potential and a bulk viscous fluid. The model is described under the condition that the shear scalar and the scalar expansion are proportional, which leads to the relation A = Bn concerning the scale factors. It is further assumed that V=constant, which results in non-linear differential equations that are solvable. The model's solutions are showing using a scalar field and metric functions. Other crucial variables, such as scalar expansion, shear scalar, Hubble parameter, volume, and D.P, have been computed and have been shown to result in accelerated expansion in contrast with the inflationary behavior of the universe during early times.

Subject Classification: *83C15.*

Keywords: *Bianchi Type-VIII, Flat potential, General relativity, Bulk viscosity.*

1. Introduction

The term inflation refers to an exponential expansion of the cosmos at its earliest stages, a concept that gained traction during the early 1980s; however, it is more widely discussed now. Following the grand unification epoch, the

* *E-mail:* **Rakhi.221051021@muj.manipal.edu**
† *E-mail:* **laxmi.poonia@jaipur.manipal.edu (Corresponding Author)**
§ *E-mail:* **varsha.202505047@muj.manipal.edu**
‡ *E-mail:* **manish.210180015@muj.manipal.edu**

electroweak epoch, which lasted from 10^{33} to 10^{32} seconds, included the epoch of inflation as its initial phase. The phase of inflation allows for solutions to many fundamental problems of Big-Bang cosmology, such as the structure, the magnetic monopole, and the horizontal and flatness problems. Guth [1] conceived the idea of rapid expansion during the Grand Unification Field Theory, suggesting the phenomenon resulted from symmetry-breaking phase transitions occurring as the universe cooled shortly after it formed, adding that expansion was due to the false vacuum, which CMBR (Cosmic Microwave Background Radiation) later validated. It is said to resolve the horizon problems, the flatness problems, and the need for isotropy and homogeneity noted by Liddle and Lyth [5].

Rothman and Ellis [2] contended that if we start with anisotropic metrics, which can be isotropized under extremely generic conditions, we can get an isotropic space-time. Stein-Schabes [3] has demonstrated that expansion will occur only when the flat potential $V(\phi)$ has a flat region for slow evolution of the Higgs field (ϕ) while the universe undergoes exponential expansion as a result of vacuum field energy dominance. In order for the cosmos to expand slowly and become homogeneous and isotropic, it is believed that the scalar field crosses the flat zone for a sufficient amount of time. The significant implications of inflation on the isotropization of the universe were examined by Anninos et al. [4].

The Bianchi Type VIII model has been extensively researched in general relativity, as it is one of the significant anisotropic cosmological models. Anisotropy is of great importance in the initial phases of the development of the cosmos, making the study of anisotropic and homogeneous cosmological models of great significance. An inflationary universe of Bianchi Type-VIII with a scalar field was studied by Bali and Swati [17], who discovered the cosmological parameters in a hyperbolic function.

Many researchers [7, 8, 14, 16, 22, 23] have examined the impact of bulk viscosity on evolution and explored the cosmological phenomenon under investigation. In general relativity, Sharma and Poonia have examined the scenario of inflation in various Bianchi Type models with bulk viscous fluid [19-21]. An inflationary cosmic solution within the structure of general relativity was also established by Poonia and Maheshwari after they studied the Bianchi Type V space-times with bulk viscous fluid [24, 26]. Many authors, such as Bali Bali and Jain [6, 13, 15, 18], Singh and Kumar [9], and Reddy, Naidu, and Rao [10, 12], as well as Chhajed and Tyagi [25] examined several Bianchi space-time inflationary cosmological scenarios, considering a self-interacting scalar field within the framework of G.R.

Singh and Meitei, in their work, developed an evolution of a viscous fluid string universe in the Bianchi Type-III metric with the Λ term [27]. A dissipative bulk viscous cosmological model, the LRS Bianchi Type V stiff fluid, was developed by Bali and Kumawat [11] in G.R. They investigated solutions considering both changing and fixed bulk viscosity values, along with cases where the cosmological constant remains unchanged or varies with time.

We focus our study on a Bianchi Type VIII cosmological model populated with a bulk viscous fluid and a minimally coupled Higgs scalar field with a constant potential. Focusing on a constraint in which the expansion scalar (θ) is proportional to the shear scalar (σ), we calculate the Einstein field equations and achieve simplification. The purpose is to evaluate whether such a model can sustain an inflationary stage and subsequently approach isotropy during the late period. We derive exact solutions while analysing the physically meaningful parameters like the scalar expansion, shear scalar, Hubble, and deceleration parameters to comprehend the dynamical behavior of the universe.

2. The Fields and Metric Equations

In the form, the Bianchi Type VIII metric is taken into consideration as

$$ds^2 = dt^2 - B^2dx^2 - A^2dy^2 - (A^2Sinh^2y + B^2Cosh^2y)dz^2 \\ -2B^2Coshydxdz \tag{1}$$

where A and B are assumed to vary only with the cosmic time t.

With an effective potential V (ϕ), which is determined by Stein-Schabes, gravity has a minimum relationship with the Higgs field (ϕ), which the Lagrangian represents.

$$\mathrm{L} = \int \sqrt{-g}\,\left(R - \tfrac{1}{2}g^{ij}\partial_i\phi\partial_j\phi - v(\phi)\right)d^4x \tag{2}$$

According to the dynamical field, the variation of L yields the Einstein field equation.

$$R_{ij} - \tfrac{1}{2}Rg_{ij} = -T_{ij} \tag{3}$$

where $$T_{ij} = -\xi\theta\big(g_{ij} - u_iu_j\big) + \partial_i\phi\partial_j\phi - \left[\tfrac{1}{2}\partial_\rho\phi\partial^\rho\phi + v(\phi)\right]g_{ij} \tag{4}$$

Higgs field is represented by ϕ, the effective potential by V, the bulk viscosity coefficient by ξ, and the model's expansion by θ.

The law of energy conservation aligns with the equation of motion for ϕ, resulting in the expression

$$\partial_i\left[\sqrt{-g}\,\tfrac{1}{\sqrt{-g}}\partial_i\phi\right] = -\frac{dV}{d\phi} \tag{5}$$

The following non-linear differential equations can be obtained from Einstein's field equations corresponding to the space-time metric (1):

$$\frac{2A_{44}}{A} + \frac{A_4^2}{A^2} - \frac{1}{A^2} - \frac{3B^2}{4A^4} = -\xi\theta - \tfrac{1}{2}\phi_4^2 - V(\phi) \tag{6}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} + \frac{B^2}{4A^4} = -\xi\theta - \tfrac{1}{2}\phi_4^2 - V(\phi) \tag{7}$$

$$\frac{2A_4B_4}{AB} + \frac{A_4^2}{A^2} - \frac{1}{A^2} - \frac{B^2}{4A^4} = -2\xi\theta - \tfrac{1}{2}\phi_4^2 + V(\phi) \tag{8}$$

For the scalar field, the equation yields

$$\phi_{44} + \phi_4\left(\frac{2A_4}{A} + \frac{B_4}{B}\right) = -\frac{dV(\phi)}{d\phi} \tag{9}$$

3. The Field Equation Solution

Therefore, we take into consideration the dynamic requirement that the scalar shear (σ) be proportional to the scalar expansion (θ) to provide a deterministic solution. Thus, we have

$$A = B^n \tag{10}$$

For the inflationary situation, as previously stated, we assume that V (ϕ) = (k) is constant and that the effective potential V (ϕ)is flat. Thus, equation (9) yields

$$\phi_{44} + \phi_4 \left(\frac{2A_4}{A} + \frac{B_4}{B}\right) = 0 \tag{11}$$

Where, $$V(\phi) = k \tag{12}$$

From equation (11), we obtain,

$$\phi_4 = \frac{L}{A^2 B} \tag{13}$$

where L is an integration constant.

For line element (1), the scale factor (R) provided by

$$R^3 = A^2 B = B^{2n+1} \tag{14}$$

From equations (6) and (7),

$$\frac{A_{44}}{A} + \frac{A_4^2}{A^2} - \frac{1}{A^2} - \frac{B_{44}}{B} - \frac{B^2}{A^4} - \frac{A_4 B_4}{AB} = 0 \tag{15}$$

From equations (10) and (15), we obtain

$$(n-1)\frac{B_{44}}{B} + 2(n^2 - n)\frac{B_4^2}{B^2} = \frac{B^2}{B^{4n}} + \frac{1}{B^{2n}} \tag{16}$$

$$B_{44} + 2n\frac{B_4^2}{B} = \frac{1}{(n-1)}\left[\frac{1}{B^{2n-1}} + \frac{1}{B^{4n-3}}\right] \tag{17}$$

Let $f(B) = B_{44}$ then $B_{44} = f'f$ in equation (17),

$$\frac{df^2}{dB} + \frac{4nf^2}{B} = \frac{2}{(n-1)}\left[\frac{1}{B^{2n-1}} + \frac{1}{B^{4n-1}}\right] \tag{18}$$

$$f^2 = \frac{2}{(n-1)}\left[\frac{B^{2-2n}}{2n+2} + \frac{B^{4-4n}}{4n}\right] + \frac{L}{B^{4n}} \tag{19}$$

Where L is an integrating constant. From equation (19), we have

$$\frac{dB}{dt} = \sqrt{\left[\frac{B^{2-2n}}{n^2-1} + \frac{B^{4-4n}}{2(n-1)}\right] + \frac{L}{B^{4n}}} \tag{20}$$

We get,

$$\frac{dB}{\sqrt{\left[\frac{B^{2-2n}}{n^2-1} + \frac{B^{4-4n}}{2(n-1)}\right] + \frac{L}{B^{4n}}}} = t + M \tag{21}$$

Equation (1) reduces to the following form with an appropriate coordinate transformation:

$$ds^2 = \frac{dT^2}{\left[\frac{T^{2-2n}}{n^2-1} + \frac{T^{4-4n}}{2(n-1)}\right] + \frac{L}{T^{4n}}} - T^2 dX^2 - T^{2n} dY^2 - (T^{2n} Sinh^2 Y + T^2 Cosh^2 Y) dZ^2 - -2T^2 CoshY dX dZ \tag{22}$$

4. Geometrical and Physical Aspects

The scalar for the model's expansion is defined by

$$\theta = \frac{A_4}{A} + \frac{2B_4}{B} = (n+2)\frac{B_4}{B} \tag{23}$$

$$\theta = \frac{(n+2)}{T}\left[\left(\frac{T^{2-2n}}{n^2-1} + \frac{T^{4-4n}}{2(n-1)}\right) + \frac{L}{T^{4n}}\right]^{\frac{1}{2}} \tag{24}$$

And shear scalar σ can be obtained from

$$\sigma^2 = \frac{1}{3}\left(\frac{A_4}{A} - \frac{B_4}{B}\right)^2 = \frac{(n-1)^2}{3}\frac{B_4^2}{B^2}$$

$$\sigma = \frac{(n-1)}{\sqrt{3}T}\left[\left(\frac{T^{2-2n}}{n^2-1} + \frac{T^{4-4n}}{2(n-1)}\right) + \frac{L}{T^{4n}}\right]^{\frac{1}{2}} \tag{25}$$

$$\frac{\sigma}{\theta} = \frac{(n-1)}{\sqrt{3}(n+2)} \tag{26}$$

And Hubble's parameter is

$$H = \frac{(n+2)}{3T}\left[\left(\frac{T^{2-2n}}{n^2-1} + \frac{T^{4-4n}}{2(n-1)}\right) + \frac{L}{T^{4n}}\right]^{\frac{1}{2}} \tag{27}$$

Scale factor is

$$V = R^3 = A^2B = B^{2n+1} = T^{2n+1} \tag{28}$$

Deceleration Parameter

$$q = -\frac{2n-2}{2n+1} \tag{29}$$

5. Discussion and Conclusion

This research focuses on the Bianchi Type VIII anisotropic cosmic model in general relativity within a universe consisting of bulk viscosity and a Higgs-type scalar field with an effective potential. To determine precise solutions, we postulated proportionality between the expansion scalar (θ) and the shear scalar (σ), which resulted in a particular form of functional dependence between the metric potentials: $A=B^n$. The scalar field equation was solved under the assumption that the flat potential $V(\phi)=k$ represents a constant vacuum energy supporting inflationary models.

The model's description of an expanding cosmos is supported by the scalar field's evolution and the volume scale factor R^3. The value of the deceleration parameter $q=-(2n-2)/(2n+1)$ is paramount in describing the expansion's nature. For $n > 1$, the universe undergoes accelerated expansion (inflation), while for $n = 1$, the universe coasts at a constant expansion rate, and for $n < 1$, it decelerates. The model remains anisotropic, but with a constant shear-to-expansion ratio σ/θ, and shows that both σ and θ tend to decrease with time, indicating isotropization.

Figure analyses provide a more detailed examination of these behaviors. As seen in Figure 1, a larger n causes a faster expansion by increasing volume growth, whereas a smaller n causes a slower expansion. Figure 2 shows how the shear scalar σ decays with time, with higher n speeding up the decay and causing faster isotropization. Figure 3 shows that the expansion scalar θ, which represents sustained expansion, starts higher and stays larger for larger n. According to Figure 4, a larger n denotes a faster and longer-lasting expansion because the Hubble parameter H is higher.

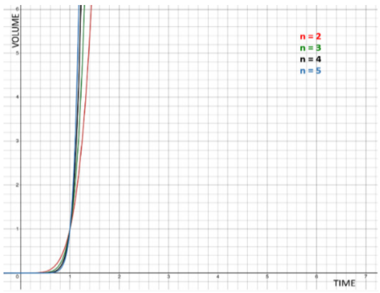

Figure 1
Graph plot between Volume with Time

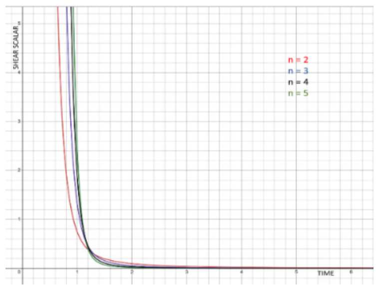

Figure 2
Graph plot between Shear Scalar with Time

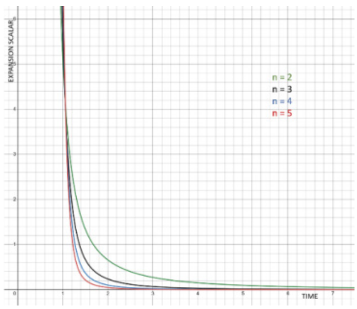

Figure 3
Graph plot between Expansion Scalar with Time

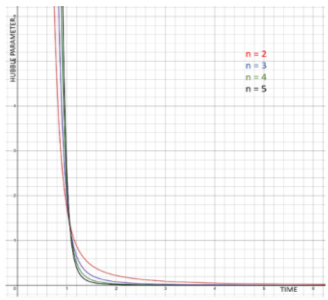

Figure 4
Graph plot between Hubble Parameter with Time

To conclude, the model offers a plausible inflationary scenario regarding the evolution of Bianchi Type VIII cosmology. It successfully describes an early anisotropic universe that evolves into a nearly isotropic, accelerated expansion phase. This work contributes to understanding anisotropic inflation better while reinforcing the notion that inflation is still possible in complex geometrical spacetimes with dissipative characteristics.

References

[1] A. H. Guth, "Inflationary universe: A possible solution to the horizon and flatness problems," *Physical Review D*, vol. 23, no.2, pp. 347 (1981).

[2] T. Rothman and G. F. R. Ellis, "Can inflation occur in anisotropic cosmologies?" *Physics Letters B,* vol. 180, no.1-2, pp.19-24(1986).

[3] J. A. Stein-Schabes, "Inflation in spherically symmetric inhomogeneous models," *Physical Review D*, vol.35, no.8, pp. 2345 (1987).

[4] P. Anninos, R. A. Matzner, T. Rothman, and M. P. Ryan Jr, "How does inflation isotropize the Universe?" *Physical Review D*, vol.43, no.12, pp. 3821 (1991).

[5] A. R. Liddle and D. H. Lyth, "COBE, gravitational waves, inflation and extended inflation," *Physics Letters B*, vol.291, no.4, pp. 391-398 (1992).

[6] R. Bali and V.C. Jain, "Bianchi type I inflationary universe in general relativity," *Pramana*, vol.59, pp.1-7 (2002).

[7] P.J.E Peebles and B. Ratra, "The cosmological constant and dark energy," *Reviews of modern physics*, vol.75, no.2, pp.559 (2003).

[8] B. Saha, "Bianchi type I universe with viscous fluid," *Modern Physics Letters A,* vol.20, no.28, pp. 2127-2143 (2005).

[9] C.P. Singh and S. Kumar, "Bianchi Type-II inflationary models with constant deceleration parameter in general relativity," *Pramana*, vol.68, pp.707-720 (2007).

[10] D.K.R. Reddy, R.L Naidu, and S.A. Rao, "Axially symmetric inflationary universe in general relativity," *International Journal of Theoretical Physics,* vol. 47, pp.1016-1020 (2008).

[11] R. Bali and P. Kumawat, "Bulk viscous LRS Bianchi type V tilted stiff fluid cosmological model in general relativity," *Physics Letters B*, vol. 665, no.5, pp.332-337 (2008).

[12] D.K.R. Reddy, S.A. Rao and R.L. Naidu, "A plane symmetric Bianchi type-I inflationary universe in general relativity," *Astrophysics and Space Science*, vol.319, pp.89-91(2009).

[13] R. Bali, "Inflationary scenario in Bianchi type I space-time," *International Journal of Theoretical Physics,* vol. 50, pp. 3043-3048 (2011).

[14] S. Kandalkar, P. Khade, and M. Gaikwad, "Bianchi-V cosmological models with viscous fluid and constant deceleration parameter in general relativity," *Turkish Journal of Physics*, vol. 36, no.1, pp. 141-154 (2012).

[15] R. Bali, "LRS Bianchi Type-II Inflationary Universe with Massless Scalar Field and Time Varying Λ," *Chinese Physics Letters*, vol. 29, no.8, pp. 080404 (2012).

[16] R. Bali and S. Singh, "LRS Bianchi type I stiff fluid inflationary universe with variable bulk viscosity," *Canadian Journal of Physics*, vol. 92, no.5, pp. 365-369 (2014).

[17] R. Bali, "Bianchi Type VIII Inflationary Universe with Massless Scalar Field in General Relativity," *Prespacetime Journal*, vol.6, no.8, pp.679-683 (2015).

[18] R. Bali and P. Kumari, "Bianchi Type III Inflationary Universe with Flat Potential and Constant Deceleration Parameter in General Relativity," *Journal of Rajasthan Academy of Physical Sciences,* vol.14, no.4-3, pp. 281-288 (2015).

[19] S. Sharma and L. Poonia, "Cosmic acceleration in LRS Bianchi type I space-time with bulk viscosity in general relativity," In IOP Conference Series: Materials Science and Engineering, vol.104, no.1, pp. 012025 (2021). IOP Publishing.

[20] S. Sharma and L. Poonia, "Cosmic inflation in Bianchi Type IX space with bulk viscosity," *Advances in Mathematics: Scientific Journal*, vol.10, no.1, pp.527-534 (2021).

[21] L. Poonia and S. Sharma, "Inflationary Scenario in Bianchi Type II Space with Bulk Viscosity in General Relativity," *Annals of the Romanian Society for Cell Biology,* vol.25, no.2, pp. 1223-1229 (2021).

[22] S. Sen, G.S. Khadekar and S. Samdurkar, "Bulk Viscous Bianchi Type-III Cosmological Model with Modified Takabayasi String in the Presence of Varying Λ," *Journal of Dynamical Systems and Geometric Theories,* vol.19, no.1, pp.95-112 (2021).

[23] A.S. Nimkar, J.S Wath and V.M. Wankhade, "Bianchi Type-IX Cosmological Model in Self-Creation Theory of Gravitation," *Journal of Dynamical Systems and Geometric Theories*, vol.19, no.1, pp.25-34 (2021).

[24] V. Maheshwari and L. Poonia, "Bianchi Type V Inflationary Cosmological Model with Bulk Viscosity in General Relativity," *Journal of Nano and Electronic Physics,* vol.15, no.4, pp.04029 (2023).

[25] D. Chhajed and A. Tyagi, "Bianchi Type-VIII Inflationary Cosmological Models with Flat Potential and Stiff Perfect Fluid Distribution in General Relativity," *Journal of Ultra Scientist of Physical Sciences-Section A* (Mathematics), vol.35, no.7, pp.68-78 (2023).

[26] V. Maheshwari, L. Poonia and R. Mathur, "Bianchi type III inflationary universe with flat potential and stiff fluid distribution in general relativity," *Journal of Interdisciplinary Mathematics*, vol.27, no.2, pp. 299–306 (2024).

[27] K.P. Singh and A.J. Meitei, "Evolution of viscous fluid string universe in Bianchi type-III metric with Λ term," *Palestine Journal of Mathematics,* vol.14, no.1 (2025).

Received December, 2024