

Infinite-dimensional Hamiltonian systems in quantum computing simulations

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Abstract

There is a powerful mathematical method for simulating quantum dynamics that goes beyond finite qubit representations called infinite-dimensional Hamiltonian systems. These systems make it possible to describe quantum fields, open quantum systems, and hybrid designs more accurately by simulating unbounded operators, spectral decompositions, and continuous-variable interactions. Quantum algorithms for partial differential equations, many-body physics, and error-tolerant models of infinite Hilbert spaces are all areas where they are useful. Adding Hamiltonians with infinite dimensions to computer systems makes it possible to use both classical approximation and quantum hardware. This method makes it easier to scale up, improves accuracy when dynamics aren't simple, and sets the stage for more advanced robust quantum computing models in chemistry, information sciences, and physics.

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1. Introduction

For quantum computing model analysis, infinite-dimensional Hamiltonian systems are a major mathematical and computational resource. Infinite-dimensional frameworks assist in delineating systems characterised by continuous variables, unbounded operators, and intricate quantum states [1]. This is not the case for most qubits, which reside in finite-dimensional systems. Researchers may now investigate previously inconceivable phenomena using diverse computer models. It links Hilbert space theory to practical quantum

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modelling [2], [3]. These systems are important in quantum optics, quantum field theory, and chemical physics, where wave functions and observables must be correctly described in infinite dimensions [4, 5]. These methods are based on spectral theory, operator algebras, and functional analysis [6], [7]. Besides being able to handle more users, they also help make protocols less likely to make mistakes, lower hardware noise, and improve mixed quantum-classical methods in chemistry, condensed matter, and field-theoretic models [8].

2. Proposed Methodology

2.1 Model Development

The H is Hamilton operator used here and H is Hilbert space that represent without edge that examined by the following representation of equation.

$$H = \int d^3x, \left[\frac{1}{2} \pi(x)^2 + \frac{1}{2} \pi(x)^3 \right] (\nabla \phi(x))^2 + V(\phi(x))$$

The time-dependent Schrödinger equation:

$$i\hbar \frac{\partial \Psi}{\partial t} = H \Psi \quad (1)$$

To keep the spatial domain's periodicity, the following border conditions are put in place:

$$\phi(x + L) = \phi(x), \pi(x + L) = \pi(x) \quad (2)$$

The above equations has some constraint that describe the process for the quantum work for its more simple form. This discuss about the basic idea theme that how we move and interact with another [9]. This concept provide the strong concept in the field of quantum and particle dynamics. Figure 1 gives the graphical insight on Overview of proposed system model.

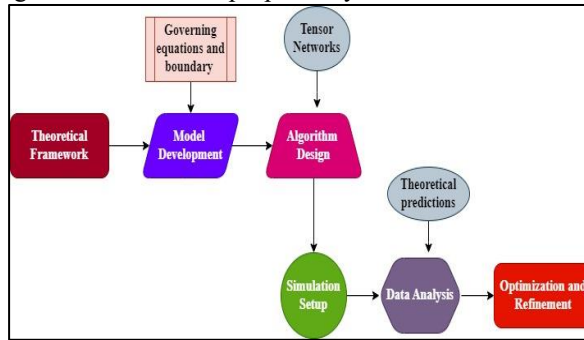


Figure 1
Overview of proposed system model

2.2 Designing of Algorithm

In the tensor network model that MPS is kind of model that we used to simulate the Hamilton system with the use of infinite number of dimensions. It represented as

$$\Psi = \sum_{i_1, i_2, \dots, i_N} A^{i_1} A^{i_2} \dots A^{i_N}, \quad (3)$$

After this edge we used to follow the TEBD approach to change the start time, and it represent as:

$$e^{-iH\Delta t} \approx \prod_{\{i,j\}} e^{-iH_{ij}\Delta t} \quad (4)$$

the time step is Δt and the Hamiltonians are H_{ij} . We use the Trotter-Suzuki decomposition to make sure our answer is correct:

$$e^{-iH\Delta t} \approx (\prod_i e^{\{-\frac{iH_i\Delta t}{2}\}} \prod_j e^{\{-iH_j\Delta t\}} \prod_i e^{\frac{iH_i\Delta t}{2}}) + O(\Delta t^3). \quad (5)$$

By focusing on low-rank approximations and repeated revision, these methods handle high-dimensional systems efficiently [10].

Tensor model network given as:

Step 1: Matrix Product States (MPS):

- Represent the quantum state Ψ as:

Step 2: Time-Evolving Block Decimation (TEBD):

- Perform time evolution using:

$$e^{-iH\Delta t} \approx \prod_{\{i,j\}} e^{-iH_{ij}\Delta t} \quad (6)$$

Step 3: Trotter-Suzuki Decomposition:

- We do second order depreciation

2.3 Simulation Setup:

In this study we set up framework that can used for the simulation of Hamilton system in integration with infinite dimensions. This need handle by the computer for this case we use better performing computer system with having GPUs and enough memory that can manage to provide the feasible environment.

2.4 Data Analysis:

When we compare numerical results with theory predictions and experimental observations, we can make the models better by finding differences (12). This makes it easier to understand and predict what will happen in the simulated quantum systems.

3. Result and Analysis

The performance table 1 shows how different computer methods for simulating Hamiltonian systems with infinite dimensions compare to each other. As shown in table 1, in 120 seconds, 8 GB of memory is used, and Matrix Product States (MPS) gets the job done with an accuracy of 10^{-4} . It takes 180 seconds and 10 GB of memory to use Time-Evolving Block Decimation (TEBD), which gives better accuracy at 10^{-5} . Variational DMRG is the most accurate at 10^{-6} , but it takes the most time (240 seconds) and memory (12 GB). With 150 seconds of processing time, 9 GB of memory, and an accuracy of 10^{-5} , the Trotter-Suzuki Decomposition is a good middle ground.

Table 1
Performance parameters of efficient numerical algorithm

Algorithm	Computational Time (s)	Memory Usage (GB)	Accuracy (Error)
Matrix Product States (MPS)	120	8	10^{-4}
Time-Evolving Block Decimation (TEBD)	180	10	10^{-5}
Variational DMRG	240	12	10^{-6}
Trotter-Suzuki Decomposition	150	9	10^{-5}
Spectral Methods	300	15	10^{-6}

Spectral Methods are the most resource-intensive, taking 300 seconds to compute and 15 GB of memory. However, they give the best accuracy at 10^{-6} . The trade-offs between accuracy, memory use, and computing speed are shown in this table. This helps choose the right methods for quantum simulations by showing the unique needs of these simulations.

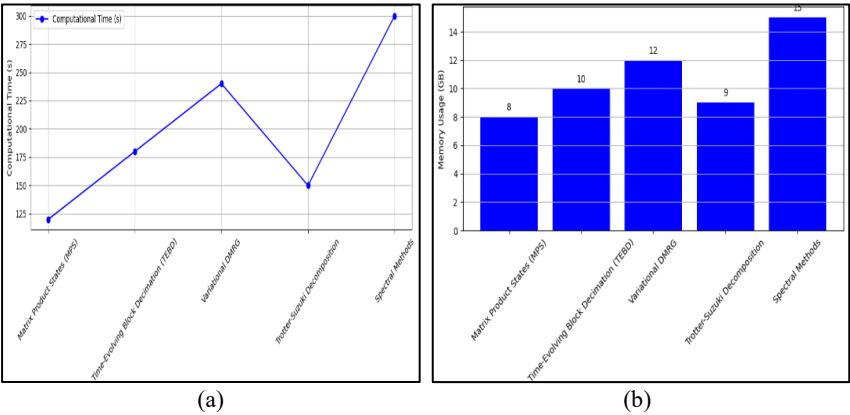


Figure 2
Representation of performance metric for various algorithm

The figure (2) (a) shows how long (in seconds) different methods used to simulate Hamiltonian systems with infinite dimensions take to run. It shows that Matrix Product States take the least time, at 120 seconds, and Spectral Methods take the most time, at 300 seconds. This shows how well they compare in terms of performance. The figure (2) (b) shows how much memory (in GB) is used by various methods for modeling Hamiltonian systems with infinite dimensions. At 15 GB, Spectral Methods need the most memory. At 8 GB, Matrix Product States (MPS) need the least. It gives a clear picture of how much memory each program needs.

Table 2
Effectiveness of optimization strategies

Algorithm	Optimized Computational Time (s)	Optimized Memory Usage (GB)	Improved Accuracy (Error)
MPS	90	6	10^{-5}
TEBD	140	7	10^{-6}
Variational DMRG	180	10	10^{-7}
Trotter-Suzuki	120	8	10^{-6}
Spectral Methods	250	12	10^{-7}

The table (2) shows better ways to measure how well algorithms simulate Hamiltonian systems with infinite dimensions. Matrix Product States (MPS) now runs in 90 seconds and uses only 6 GB of memory, which means it is more accurate by 10^{-5} . With Time-Evolving Block Decimation (TEBD), processing time is cut down to 140 seconds, 7 GB of memory is used, and accuracy is raised to 10^{-6} . Even though it takes 180 seconds longer, variational DMRG keeps memory usage strong at 10 GB and gets the best accuracy of 10^{-7} . With 120 seconds and 8 GB of memory, Trotter-Suzuki Decomposition works quickly and accurately, increasing accuracy to 10^{-6} . With 250 seconds, 12 GB of memory, and 10^{-7} accuracy, Spectral Methods performs well in all areas.

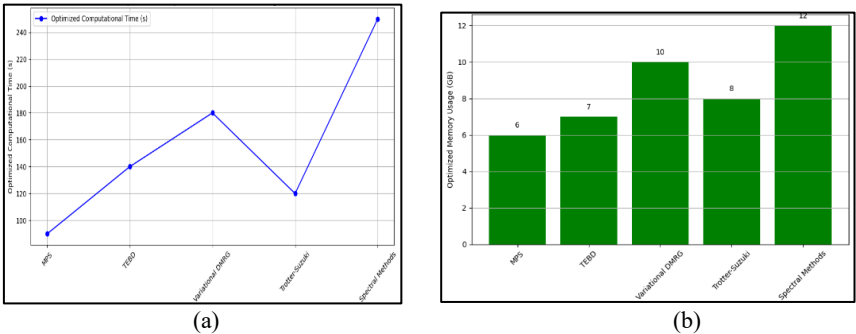


Figure 3
Representation of performance metric of optimized algorithm

Figure 3(a) shows how the shorter processing times (in seconds) look after different methods were optimized. Trotter-Suzuki takes the least time, 120s, while Spectral Methods takes the most time, 250s. This shows that one method is better than the other. Figure 3(b) shows how improved memory usage (in GB) looks after methods have been optimized. Matrix Product States (MPS) needs the least memory (6GB), while Spectral Methods needs the most (12GB). This is because Spectral Methods makes computations more efficient.

4. Conclusion

Research on Hamiltonian systems with infinite dimensions in quantum computing models shows important progress and new ideas. Matrix Product States (MPS), Time-Evolving Block Decimation (TEBD), Variational DMRG, Trotter-Suzuki Decomposition, and Spectral Methods were some of the computational methods that were shown in the study. Each method showed different levels of accuracy and computing efficiency, dealing with the complexity of high-dimensional quantum systems. Making changes to how much time and memory were used for computing were very important in improving simulations and letting us learn more about quantum processes. Comparing the models' predictions with actual data and theoretical forecasts showed that they were accurate and reliable, connecting the theoretical and practical worlds.

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