

Eulerian integrals of I-function

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Abstract

A set of Eulerian integrals associated with I-function (modified) has been established in this paper. Furthermore, we will conclude with an examination of several specific instances of these integrals, accompanied by insightful remarks that underscore their relevance and utility. This work not only expands the theoretical framework surrounding Eulerian integrals but also paves the way for future research and applications in related areas.

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1. Introduction

Building upon the foundation laid by Prasad and Singh [5], it is essential to explore the implications of I-function having multiple variables with a modified version (MMIF) [4] in the context of fractional calculus. Recent studies have shown that the integration of these functions can lead to more generalized results in various applications, particularly in mathematical modelling and computational methods. For instance, the establishment of new Eulerian integrals involving the product of these functions not only enhances our understanding of their properties but also provides a framework for unifying existing results in the field of multivariable calculus. Furthermore, the analysis of these integrals are crucial for practical applications in engineering and physics. $N_0 = N \cup \{0\}$, where N is a set of positive integers. $I =$

$$I_{\substack{0, n_2; 0, n_3; \dots; 0, n_r; [R^1; m^1, n^1; \dots; m^{(r)}, n^{(r)}] \\ p_2, q_2; p_3, q_3; \dots; p_r, n_r; [R; p^1, q^1; \dots; p^{(r)}, q^{(r)}]}} \left[\begin{array}{l} z_1 \left| (a_{2j}; \alpha_{2j}^1, \alpha_{2j}^{11})_{1, p_2}; (\alpha_{3j}; \alpha_{3j}^1, \alpha_{3j}^{11}, \alpha_{3j}^{111})_{1, p_3}; \dots; (a_{rj}; \alpha_{rj}^1, \dots, \alpha_{rj}^{(r)})_{1, p_r} : \right. \\ z_r \left| (b_{2j}; \beta_{2j}^1, \beta_{2j}^{11})_{1, q_2}; (\beta_{3j}; \beta_{3j}^1, \beta_{3j}^{11}, \beta_{3j}^{111})_{1, q_3}; \dots; (b_{rj}; \beta_{rj}^1, \dots, \beta_{rj}^{(r)})_{1, q_r} : \right. \\ (e_j; g_j' u_j', \dots, g_j^{(r)} u_j^{(r)})_{1, R}; (a_j^{(r)}, \alpha_j^{(r)})_{1, p^{(r)}}, (a_j', \alpha_j')_{1, p^{(1)}} \left. \right] \\ (l_j; U_j' f_j', \dots, U_j^{(r)} f_j^{(r)})_{1, R}; (b_j', \beta_j')_{1, q^{(1)}}, (b_j^{(r)}, \beta_j^{(r)})_{1, q^{(r)}} \left. \right] \\ = \frac{1}{(2\pi w)^r} \int_{L_1} \dots \int_{L_r} \xi(s_1, \dots, s_r) \prod_{i=1}^r \varphi_i(s_i) z_i^{s_i} ds_1 \dots ds_r \quad (1) \end{array}$$

Where

$$\xi(s_1, \dots, s_r) = \frac{\prod_{j=1}^{n_2} \Gamma\left(-a_{2j} + \sum_{i=1}^2 \alpha_{2j}^{(i)} s_i + 1\right) \prod_{j=1}^{n_3} \Gamma\left(1 - a_{3j} + \sum_{i=1}^3 \alpha_{3j}^{(i)} s_i\right) \dots \prod_{j=1}^{n_r} \Gamma\left(1 - a_{rj} + \sum_{i=1}^r \alpha_{rj}^{(i)} s_i\right)}{\prod_{j=n_2+1}^{p_2} \Gamma\left(a_{2j} - \sum_{i=1}^2 \alpha_{2j}^{(i)} s_i\right) \prod_{j=n_3+1}^{p_3} \Gamma\left(a_{3j} - \sum_{i=1}^3 \alpha_{3j}^{(i)} s_i\right) \dots \prod_{j=n_r+1}^{p_r} \Gamma\left(a_{rj} - \sum_{i=1}^r \alpha_{rj}^{(i)} s_i\right)}$$

$$\frac{1}{\prod_{j=1 \text{ to } q_2} \Gamma\left(1 - b_{2j} + \sum_{i=1}^2 \beta_{2j}^{(i)} s_i\right) \prod_{j=1 \text{ to } q_3} \Gamma\left(1 - b_{3j} + \sum_{i=1}^3 \beta_{3j}^{(i)} s_i\right) \dots \prod_{j \text{ from } 1}^{q_r} \Gamma\left(1 - b_{rj} + \sum_{i=1}^r \beta_{rj}^{(i)} s_i\right)}$$

$$\times \frac{\prod_{j \text{ from } 1 \text{ to } R'} \Gamma\left(e_j + \sum_{i=1}^r u_j^{(i)} g_j^{(i)} s_i\right)}{\prod_{j \text{ from } 1 \text{ to } R} \Gamma\left(l_j + \sum_{i \text{ from } 1 \text{ to } r} u_j^{(i)} f_j^{(i)} s_i\right)} \quad (2)$$

$$\text{and} \quad \varphi(s_i; q) = \frac{\prod_{j=1}^{m^{(i)}} \Gamma(b_j^{(i)} - \beta_j^{(i)} s_i) \prod_{j=1}^{n^{(i)}} \Gamma(1 - a_j^{(i)} - \alpha_j^{(i)} s_i)}{\prod_{j=1+m^{(i)}}^{q^{(i)}} \Gamma(1 - b_j^{(i)} - \beta_j^{(i)} s_i) \prod_{j=1+n^{(i)}}^{p^{(i)}} \Gamma(a_j^{(i)} - \alpha_j^{(i)} s_i)} \quad (3)$$

The detailed representations of the above parameters, contour representation and convergence of the integral (1) has been discussed in [11]. The following result will facilitate a deeper understanding of the connections between the various mathematical techniques involved in this article.

Result 1: The following integral has been discussed by ([6], p.301 (2.2.6)).

$$\int_a^b (ut+v)^\gamma (-a+t)^{-1+\alpha} (-t+b)^{-1+\beta} dt =$$

$$\beta(\alpha, \beta)(au+v)^\gamma (b-a)^{\alpha+\beta-1} {}_2F_1\left(\begin{matrix} -\gamma+\beta, \alpha \\ -\frac{u(-a+b)}{v+ua} \end{matrix}\right) \quad (4)$$

Provided, $\text{Real}(\alpha)$, $\text{Real}(\beta)$ are greater than 0 and

$$\left| \arg\left(\frac{v+bu}{v+au}\right) \right| < \pi, a \neq b$$

Consider the following expansion:

Result 2:

$$(v + ut)^\gamma = (v + au)^\gamma \sum_{m=0}^{\infty} \frac{(-\gamma)_m \left(-\frac{(t-a)u}{au+v} \right)^m}{m!} \quad (5)$$

Provided, $\left| \frac{u(-a+t)}{v+ua} \right| < 1, t \in [a, b]$.

In the following sections, we will define the Eulerian integral of the Modified I-function and subsequently derive the corollaries as specific instances.

2. Main Integral

In this part, we will address the issue of obtaining a closed-form evaluation for the subsequent general Eulerian integral that pertains to the MMIF.

Here, we note $\frac{(-a+t)^\delta (-t+b)^\eta (v+tu)^{1-\delta-\eta}}{[D(ut+v) + (-D+C)(-a+t)]} = g(t)$

and $X_i = (-a+b)^{(\delta+\eta)v_i} (au+v)^{-(\delta+\eta)v_i} D^{-v_i} \forall (i=1, \dots, r)$

Theorem 1: *Prove that*

$$\int_a^b (ut+v)^\gamma (-a+t)^{\alpha-1} (-t+b)^{\beta-1} I[z_1(g(t))^{\alpha_1}, \dots, z_r(g(t))^{\alpha_r}] dt = (-a+b)^{\alpha+\beta-1} (au+v)^\gamma$$

$$\sum_{l,m=0}^{\infty} \frac{\left(-\frac{A}{B} + B \right)^m \left(\frac{-a+b}{au+v} \right)^m \left(-\frac{(-a+b)^u}{au+v} \right)^l}{l!m!} I_{U, p_r+4, q_r+3; R; Y}^{V, 0, n_r+4; R; X} \left[\begin{matrix} z_1 X_1 \\ z_r X_r \end{matrix} \middle| \begin{matrix} A; (1-m; a_1, \dots, a_r), \\ B; B, (1; v_1, \dots, v_r) \end{matrix} \right.$$

$$\left. (1+\gamma-l-m; (\delta+\eta)a_1, \dots, (\delta+\eta)a_r), (1-\beta; \eta a_1, \dots, \eta_r), (1-\alpha-l-m; \delta a_1, \dots, \delta a_r), A : E : \Im \right]$$

$$(1-\alpha-\beta-l-m; (\delta+\eta)a_1, \dots, (\delta+\eta)a_r), (1+\gamma-m; (\delta+\eta)a_1, \dots, (\delta+\eta)a_r) : L : \Re \left. \right]$$

(6)

Asymptotic property has observed in the form ([1], p.278).

$$a_i > 0; i = 1, \dots, r; \min[\operatorname{Re}(\alpha), \operatorname{Re}(\beta)] > 0; b \neq a, \max \left\{ \left| \frac{(D-C)(t-a)}{B(tu+v)} \right|, \left| \frac{(b-a)u}{au+v} \right| \right\} < 1, t \in [a, b]$$

$$\operatorname{Re}(\alpha) + \sum_{i=1}^r a_i \min_{1 \leq j \leq m^{(i)}} \operatorname{Re} \left[\frac{b_j^{(i)}}{\beta_j^{(i)}} \right] > 0; \operatorname{Re}(\beta) + \sum_{i=1}^r a_i \min_{1 \leq j \leq m^{(i)}} \operatorname{Re} \left[\frac{b_j^{(i)}}{\beta_j^{(i)}} \right] > 0 \quad \text{and}$$

$$\left| \arg(z_i(g(t))^{a_i}) \right| < \frac{1}{2} \Delta_i \pi; \quad (\text{Here } \Delta_i \text{ is taken from [9, 10, 11]}).$$

Proof: Expressing the MMIF in the following form to establish the proof of the theorem by (1), we obtain

$$\begin{aligned} & \int_a^b (ut+v)^\gamma (-a+t)^{\alpha-1} (-t+b)^{\beta-1} I[z_1(g(t))^{\alpha_1}, \dots, z_r(g(t))^{\alpha_r}] dt = \\ & \int_a^b (-a+t)^{\alpha-1} (-t+b)^{\beta-1} (v+ut)^\gamma \frac{1}{(2\pi w)^r} \int_{L_1} \dots \int_{L_r} \xi(s_1, \dots, s_r) \prod_{k=1}^r \varphi_k(s_k) z_k^{s_k} \\ & \left[\frac{(t-a)^\delta (b-t)^\eta (ut+v)^{1-\delta-\eta}}{D(ut+v) + (C-D)(t-a)} \right]^{\sum_{i=1}^r a_i s_i} ds_1 \dots ds_r dt \end{aligned} \quad (7)$$

Based on absolute convergence of integrals, exchanging the order of summation and integration in the above, we get

$$\begin{aligned} & \int_a^b (ut+v)^\gamma (-a+t)^{\alpha-1} (-t+b)^{\beta-1} I[z_1(g(t))^{\alpha_1}, \dots, z_r(g(t))^{\alpha_r}] dt = \\ & \frac{1}{(2\pi w)^r} \int_{L_1} \dots \int_{L_r} \xi(s_1, \dots, s_r) \prod_{k=1}^r \varphi_k(s_k) z_k^{s_k} \\ & \left[\int_a^b (t-a)^{\alpha+\delta \sum_{i=1}^r a_i s_i - 1} (b-t)^{\beta+\eta \sum_{i=1}^r a_i s_i - 1} (ut+v)^{\gamma - (\delta+\eta) \sum_{i=1}^r a_i s_i} \left\{ 1 - \frac{(D-C)((t-a))}{D(ut+v)} \right\}^{-\sum_{i=1}^r a_i s_i} dt \right] ds_1, \dots, ds_r \end{aligned} \quad (8)$$

using Result 2, we obtain

$$\int_a^b (ut+v)^\gamma (-a+t)^{\alpha-1} (-t+b)^{\beta-1} I[z_1(g(t))^{\alpha_1}, \dots, z_r(g(t))^{\alpha_r}] dt =$$

$$\sum_{m=0}^{\infty} \frac{\left(D - \frac{C}{D}\right)^m}{m!} \frac{1}{(2\pi w)^r} \int_{L_1} \dots \int_{L_r} \xi(s_1, \dots, s_r) \prod_{k=1}^r \varphi_k(s_k) z_k^{s_k} \frac{\Gamma(m + \sum_{i=1}^r a_i s_i)}{\Gamma(\sum_{i=1}^r a_i s_i)} \left[\int_a^b (-a+t)^{\alpha+m+\delta \sum_{i=1}^r a_i s_i - 1} (-t+b)^{\beta+\eta \sum_{i=1}^r a_i s_i - 1} (v+tu)^{\gamma-m-(\delta+\eta) \sum_{i=1}^r a_i s_i} dt \right] ds_1, \dots, ds_r \quad (9)$$

Now by using the Result.1 and the equation (1) to the resulting expression, we obtain the desired theorem result.

3. Special cases

Particular cases have been discussed in this section.

Case 1 : In (5), assuming $f(t) = ut + v = \lambda(-a+t) + \mu(-t+b) + (-a+b)$, $\gamma = -\alpha - \beta$, $u = -\lambda - \mu$ and $v = (1+\mu)b - (1+\lambda)a$

Also, $g(t) = \frac{(t-a)^\delta (b-t)^\eta (f(t))^{1-\delta-\eta}}{D(b-a) + (t-a)(C-D+D\lambda) + \mu D(b-t)}$ and replacing α, β by

$\alpha+1, \beta+1$ respectively, we come across with the following expression (considering the above Result. 1 yields [3].

$$\int_a^b \frac{(-a+t)^{\alpha-1} (-t+b)^{\beta-1}}{(-a+b) + \lambda(-a+t) + (b-t)^{\alpha+\beta} \mu} dt = \frac{(\lambda+1)^{-\alpha} (1+\mu)^{-\beta}}{(-a+b)} \beta(\alpha, \beta)$$

Provided $\text{Re}(\alpha), \text{Re}(\beta) > 0$; $-a + \lambda(-a+t) + \mu(-t+b) + b \neq 0$, $t \in [a, b]$, $a \neq b$, we arrive at the following case.

Case 2:

$$\int_a^b \frac{(t-a)^{\alpha-1} (b-t)}{[f(t)]^{\alpha+\beta+2}} I[z_1(g(t))^{\alpha_1}, \dots, z_r(g(t))^{\alpha_r}] dt = (-a+b)^{-1} (1+\lambda)^{-1-\alpha} (1+\mu)^{-1-\beta}$$

$$\sum_{m=0}^{\infty} \frac{\left(D - \frac{C}{D}\right) (1+\lambda)^m}{m!} I_{U;p_r+3,a_r+2|R;Y}^{V;0,n_r+3|R;X} \left[\begin{array}{c} z_1 D^{-a_1} ((1+\lambda)^{-\delta a_1} (1+\mu)^{-\eta a_1} \\ \cdot \\ z_1 D^{-a_r} ((1+\lambda)^{-\delta a_r} (1+\mu)^{-\eta a_r} \end{array} \left| \begin{array}{c} A; (1-m; a_1, \dots, a_r), \\ B; B, (1; v_1, \dots, v_r) \end{array} \right. \right]$$

$$\left. \begin{aligned} &(-a-m); \delta a_1, \dots, \delta a_r, (-\beta; \eta a_1, \dots, \eta_r), A : E : \mathfrak{S} \\ &(1-\alpha-\beta-m; (\delta+\eta)a_1, \dots, (\delta+\eta)a_r; 1) : L : \mathfrak{R} \end{aligned} \right] \quad (10)$$

In (10), taking $\lambda = \mu = 0$, $z_i = (b-a)^{\delta+\eta-1} v_i$; $(i=1, \dots, r)$ and $\delta = \eta = 1$, $\alpha = \beta = -\frac{1}{2}$, $A \rightarrow A$, $B \rightarrow B^2$, by applying the equation ([2], eq. (2.8)) and the duplication formula, we arrive at

Case 3:

$$\int_a^b (t-a)^{\alpha-1} (b-t)^{-\frac{1}{2}} I \left[\left[\frac{(t-a)(b-t)}{C^2(t-a)+D^2(b-t)} \right]^{\alpha_1}, \dots, \left[\frac{(t-a)(b-t)}{C^2(t-a)+D^2(b-t)} \right]^{\alpha_r} \right] dt = \sqrt{\pi}$$

$$I_{U; p_r+1, q_r+1 | R; Y}^{V; 0, n_r+1 | R; X} \left[\left(\frac{b-a}{(C+D)^2} \right)^{\alpha_1} \left| A; \left(\frac{1}{2}; a_1, \dots, a_r \right), A : E : \mathfrak{S} \right. \right. \\ \left. \left. \left(\frac{b-a}{(C+D)^2} \right)^{\alpha_r} \left| B; B, (0; v_1, \dots, v_r) : L : \mathfrak{R} \right. \right] \quad (11)$$

In case 1, taking $\lambda = \mu = 0$, $z_i = (b-a)^{(\delta+\eta-1)a_i}$; $(i=1, \dots, r)$ and $\delta = \eta = \frac{1}{2}$, $\beta = -\alpha - 2$, $a_i \rightarrow 2a_i$, $(i=1, \dots, r)$, by applying binomial expression and duplication formula; finally $a_i = 1 (i=1, \dots, r)$, $a = 0$, $b = 1$, we obtain the following relation.

Case 4:

$$\int_a^b t^\alpha (1-t)^{-\alpha-2} I \left[\left[\frac{(-t+1)t}{Ct+D(1-t)} \right]^2, \dots, \left[\frac{(-t+1)t}{Ct+D(1-t)} \right]^2 \right] dt = 2\sqrt{\pi} \left(\frac{D}{C} \right)^{\alpha+1}$$

$$I_{U; p_r+2, q_r+2 | R; Y}^{V; 0, n_r+2 | R; X} \left[(4CD)^{-1} \left| A; (-\alpha; a_1, \dots, a_r), (\alpha+2; a_1, \dots, a_r), A : E : \mathfrak{S} \right. \right. \\ \left. \left. (4CD)^{-1} \left| B; B, (1; 1, \dots, 1), \left(\frac{1}{2}; 1, \dots, 1 \right) : L : \mathfrak{R} \right. \right] \quad (12)$$

In Case 3, assuming $a_i = 1$ for all 'i', $b = 1$, $a = 0$, we arrive at the next case.

Case 5:

$$\int_a^b (1-t)^{\frac{1}{2}} t^\alpha I \left[\left[\frac{(-t+1)t}{C^2t + D^2(-t+1)} \right], \dots, \left[\frac{t(1-t)}{C^2t + D^2(1-t)} \right] \right] dt = \sqrt{\pi} \quad (13)$$

$$I_{U;p_r+1,q_r+1;|R:Y}^{V;0,n_r+1;|R':X} \left[\begin{matrix} (C+D)^{-2} \\ (C+D)^{-2} \end{matrix} \middle| \begin{matrix} A; \left(\frac{1}{2}; 1, \dots, 1 \right), A : E : \mathfrak{I} \\ B; B, (0; 1, \dots, 1) : L : \mathfrak{R} \end{matrix} \right]$$

5. Conclusion

The Eulerian integrals exhibit a remarkable duality in their generality concerning the variables involved. By strategically giving specific values to parameters and variables that comprise these integrals, one can adeptly apply these formulae to derive an extensive array of results. This encompasses a diverse range of functions, including but not limited to, the products of multiple such functions. These can be expressed through a variety of notations such as E, F, G, H, I as well as special functions that may involve either single or multiple variables [7, 8, 11, 12, 13]. Consequently, the formulae presented within this paper are characterized by their broad applicability and general nature. They hold the potential to be highly beneficial in a multitude of intriguing scenarios that are frequently encountered in the realms of both Pure and Applied Mathematics, as well as Mathematical Physics [14, 15, 16]. By leveraging the foundational principles underlying the Eulerian integrals, researchers and practitioners alike may uncover new insights and solutions to complex problems that permeate the literature in these fields.

References

- [1] B. L. J. Braaksma, "Asymptotic expansions and analytic continuations for a class of Barnes-integrals," *Compositio Mathematica*, vol. 15, pp. 239–341 (1962–1964).
- [2] A. Erdélyi, W. Magnus, F. Oberhettinger, and E. G. Tricomi, *Tables of Integral Transforms*, Vols. I and II, New York: McGraw-Hill (1954).
- [3] I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series, and Products*, Academic Press, New York (1980).

- [4] Y. N. Prasad, "Multivariable I-function," *Vijnana Parishad Anusandhan Patrika*, vol. 29, pp. 231–237 (1986).
- [5] Y. N. Prasad and A. K. Singh, "Basic properties of the transform involving an H-function of r variables as kernel," *Indian Acad. Math.*, no. 2, pp. 109–115 (1982).
- [6] A. P. Prudnikov, Y. A. Brychkov, and O. I. Marichev, *Integrals and Series of Elementary Functions* (in Russian), Nauka, Moscow (1981).
- [7] H. M. Srivastava and R. Panda, "Some expansion theorems and generating relations for the H-function of several complex variables," *Comment. Math. Univ. St. Pauli*, vol. 24, pp. 119–137 (1975).
- [8] H. M. Srivastava and R. Panda, "Some expansion theorems and generating relations for the H-function of several complex variables II," *Comment. Math. Univ. St. Pauli*, vol. 25, pp. 167–197 (1976).
- [9] H. S. P. Srivastava, "Certain class of Eulerian integrals of multivariable generalized hypergeometric function," *J. Indian Acad. Math.*, vol. 21, no. 2, pp. 255–270 (1999).
- [10] H. S. P. Srivastava, "On fractional derivative of certain generalized hypergeometric functions of several complex variables," *Vikram Math. J.*, vol. 15, pp. 41–56 (1995).
- [11] N. Srimannarayana, F. Ayant, Y. Pragathi Kumar, D. K. Pavan Kumar, and B. Satyanarayana, "Multiple Eulerian integrals with modified multivariable I-function having general arguments," *J. Interdiscip. Math.*, vol. 26, pp. 77–94 (2023).
- [12] B. Satyanarayana, D. K. Pavan Kumar, Y. Pragathi Kumar, and N. Srimannarayana, "Multiple integrals involving Pragathi–Satyanarayana's I-function, generalized gamma and generalized hypergeometric functions," *Commun. Appl. Nonlinear Anal.*, vol. 31, no. 2S, pp. 687–695 (2024).
- [13] B. Satyanarayana, D. K. Pavan Kumar, Y. Pragathi Kumar, F. Ayant, and N. Srimannarayana, "Solution of a boundary value problem involving general class of polynomial, Struve's function and PSI function of one variable," *Communications on Applied Nonlinear Analysis*, vol. 31, no. 7S, pp. 649–659 (2024).

- [14] B. Singh, N. Dhiman, S. Gupta, and A. Gupta, "Some new constructions of irreducible polynomials using composition of polynomials over finite fields," *Journal of Discrete Mathematical Sciences and Cryptography*, vol. 28, no. 6, pp. 2383–2396 (2025), doi: 10.47974/JDMSC-2308.
- [15] M. V. Santhi, V. U. M. Rao, and Y. Aditya, "Bianchi type-I bulk viscous string model in $f(R)$ gravity," *Journal of Dynamical Systems and Geometric Theories*, vol. 17, no. 1, pp. 23–38 (2019).
- [16] A. S. Nimkar, J. S. Wath, and V. M. Wankhade. "Bianchi Type-IX Cosmological Model in Self-Creation Theory of Gravitation." *Journal of Dynamical Systems and Geometric Theories*, vol. 19, no. 1, pp. 25-34, (2021).

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