Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 28 (2025), No. 5, pp. 1999–2003
DOI : 10.47974/JIM-2222

Using FPV-transforms to solve a system of PDEs

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Abstract

The fields of scientific and engineering depend heavily on fuzzy partial integro-differential equations for their operations. The proposed solution for fuzzy partial differential equations utilizes the fuzzy V-transform transform as a modern technique. The research presents first and second-order fuzzy partial derivative formula obtained from highly generalized H-differentiability principles. The research begins with an explanation of fuzzy partial V-transforms before introducing fuzzy partial differential equation solutions. The proposed method receives demonstration through an application to show its operational potential.

Subject Classification (2020): *34A07, 34A08, 34A22.*

Keywords: *V-transform, Solve fuzzy ordinary linear differential, Fuzzy partial V-transforms.*

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1. Introduction

Modeling physical events requires differential equations for analysis. The modeling process of precise differential equations requires complex procedures that make the application impractical. According to these circumstances the fuzzy math methods present the most effective solution. During the last few decades scientific and engineering practitioners have widely used Fuzzy differential equations (FDEs). The basic task along with arithmetic of fuzzy sets was first explained by Zadeh in [1] followed by alternative conceptual research on fuzzy derivatives from [3,4] and further research from [2] and respectively. Research fields such as engineering and physics use PDEs to describe mathematical relationships within their domains as well as chemistry and biological research. The solution of multiple variables and their derivatives leads to a mathematic formula known as partial differential equation. Real-world situations require solutions that ordinary differential equations cannot achieve at maximum efficiency. For more inform, we refer [5-15].

2. Main Results

Definition 1: Assume that $u = u(g,t)$ be a continuous If t is a real number r and FV-transform follows u. Denote by $\tilde{Q}_t(g,p)$ is defined as follows

$$\tilde{Q}_t(g,p) = T_t[u(g,t)] = p^2 \int_0^\infty e^{-(i\sqrt[\Omega]{p})t} u(g,t)dt = \lim_{\tau\to\infty} p^2 \int_0^\tau e^{-(i\sqrt[\Omega]{p})t} u(g,t)dt,$$

$$\tilde{Q}_t(g,p) = \left[\lim_{\tau\to\infty} p^2 \int_0^\tau e^{-(i\sqrt[\Omega]{p})t} \underline{u}(g,t)dt \,, \lim_{\tau\to\infty} p^2 \int_0^\tau e^{-(i\sqrt[\Omega]{p})t} \bar{u}(g,t)dt\right].$$

$$\tilde{Q}_t(g,p;\vartheta) = T_t[u(g,t;\vartheta)] = \left[\gamma\left(\underline{u}(g,t)\right), \gamma\left(\bar{u}(g,t)\right)\right],$$

Theorem 1: *Assume that $u: (0,\infty) \times (0,\infty) \to E$ continuous FFV, and u_t the PD of u with respect to t . suppose that $p^2 e^{-(i\sqrt[\Omega]{p})t} u(g,t)$ and $p^2 e^{-(i\sqrt[\Omega]{p})t} u_t(g,t)$ are inadequate FR-integral on $[0,\infty]$, then*

1. If u is the first form differentiable function with respect to t.

$$T_t[u_t(g,t)] = \left(i\sqrt[\Omega]{p}\right) T_t[u(g,t)] \square p^2 u(g,0),$$

2. Ifu is the second form differentiable function with respect to t.

$$T_t[u_t(g,t)] = -p^2 u(g,0) \square \left(-i\sqrt[\Omega]{p}\right) T_t[u(g,t)].$$

Proof: **Case 1.** $T_t[u_t(g)] = \gamma_t\left[\underline{u}_t(g,t;\vartheta)\right], \gamma_t[\bar{u}_t(g,t;\vartheta)]$.

$$\gamma_t\left[\underline{u}_t(g,\vartheta)\right] = \left(i\sqrt[\Omega]{p}\right)\gamma_t\left[\underline{u}(g,t;\vartheta)\right] - p^2 \underline{u}(g,0;\vartheta),$$

$$\gamma_t[\bar{u}_t(g,\vartheta)] = \left(i\sqrt[\Omega]{p}\right)\gamma_t[\bar{u}(g,t;\vartheta)] - p^2 \bar{u}(g,0;\vartheta).$$

Substitute (2) in (1), then:

$$T_t[u_t(g)] = \left(i\sqrt[\Omega]{p}\right)\gamma_t\left[\underline{u}(g,t;\vartheta)\right] - p^2 \underline{u}(g,0;\vartheta), \left(i\sqrt[\Omega]{p}\right)\gamma_t[\bar{u}(g,t;\vartheta)] - p^2 \bar{u}(g,0;\vartheta).$$

$$T_t[u_t(g,t)] = \left(i\sqrt[\Omega]{p}\right) T_t[u(g,t)] \square p^2 u(g,0).$$

Case 2: $\in [0,1]$ $T_t[u_t(g)] = \gamma_t[\bar{u}_t(g,t;\vartheta)], \gamma_t[\underline{u}_t(g,t;\vartheta)]$. $T_t[u_t(g)] = \left(i\sqrt[\Omega]{p}\right)\gamma_t[\bar{u}(g,t;\vartheta)] - p^2\bar{u}(g,0;\vartheta), \left(i\sqrt[\Omega]{p}\right)\gamma_t[\underline{u}(g,t;\vartheta)] - p^2\underline{u}(g,0;\vartheta)$.

$$T_t[u_t(g,t)] = -p^2u(g,0)\square\left(-i\sqrt[\Omega]{p}\right)T_t[u(g,t)].$$

$$T_t[u_{tt}(g)] = \gamma_t[\underline{u}_{tt}(g,t;\vartheta)], \gamma_t[\bar{u}_{tt}(g,t;\vartheta)].$$

$$T[u_{tt}(g)] = \left(i\sqrt[\Omega]{p}\right)^2\gamma_t[\underline{u}(g,t;\vartheta)] - p^2\left(i\sqrt[\Omega]{p}\right)\underline{u}(g,0;\vartheta) - p^2\underline{u}_t(g,0;\vartheta),$$
$$\left(i\sqrt[\Omega]{p}\right)^2\gamma_t[\bar{u}(g,t;\vartheta)] - p^2\left(i\sqrt[\Omega]{p}\right)\bar{u}(g,0;\vartheta) - p^2\bar{u}_t(g,0;\vartheta).$$

$$T_t[u_{tt}(g,t)] = \left(i\sqrt[\Omega]{p}\right)^2T_t[u(g,t)]\square p^2\left(i\sqrt[\Omega]{p}\right)u(g,0)\square p^2u_t(g,0).$$

$$T_t[u_{tt}(g)] = \gamma_t[\bar{u}_{tt}(g,t;\vartheta)], \gamma_t[\underline{u}_{tt}(g,t;\vartheta)].$$

$$T_t[u_{tt}(g,t)] = -p^2\left(i\sqrt[\Omega]{p}\right)u(g,0)\square\left(-i\sqrt[\Omega]{p}\right)^2T_t[u(g,t)]\square p^2u_t(g,0).$$

Example 1:

$u_g + w_t = 1$; I.C. (i) $u(g,0) = (1+\vartheta, 1-\vartheta), w(g,0) = (2+\vartheta, 4-\vartheta)$

$w_g + u_t = 0$; B.C. (ii) $u(0,t) = (3+\vartheta, 5-\vartheta)$

$$\lim_{g\to\infty} \text{exist}, w(g,0) = (\vartheta, 2-\vartheta) \lim_{g\to\infty} \text{exist}.$$

Solution: We get the following

$$\frac{\partial}{\partial g}T_t[u(g,t)] + \left(i\sqrt[\Omega]{p}\right)T_t[w(g,t)]\square p^2w(g,0) = \frac{p^2}{i\sqrt[\Omega]{p}},$$
$$\frac{\partial}{\partial g}T_t[w(g,t)] + \left(i\sqrt[\Omega]{p}\right)T_t[u(g,t)]\square p^2u(g,0) = 0.$$

Then:

$$\frac{\partial}{\partial g}\gamma_t[\underline{u}(g,t)] + \left(i\sqrt[\Omega]{p}\right)\gamma_t[\underline{w}(g,t)] - p^2\underline{w}(g,0) = \frac{p^2}{i\sqrt[\Omega]{p}},$$
$$\frac{\partial}{\partial g}\gamma_t[\bar{u}(g,t)] + \left(i\sqrt[\Omega]{p}\right)\gamma_t[\bar{w}(g,t)] - p^2\bar{w}(g,0) = \frac{p^2}{i\sqrt[\Omega]{p}}.$$
$$\frac{\partial}{\partial g}\gamma_t[\underline{w}(g,t)] + \left(i\sqrt[\Omega]{p}\right)\gamma_t[\underline{u}(g,t)] - p^2\underline{u}(g,0) = 0,$$
$$\frac{\partial}{\partial g}\gamma_t[\bar{w}(g,t)] + \left(i\sqrt[\Omega]{p}\right)\gamma_t[\bar{u}(g,t)] - p^2\bar{u}(g,0) = 0.$$

Using initial conditions:

$$\frac{\partial}{\partial g}\gamma_t[\underline{u}(g,t)] + \left(i\sqrt[\Omega]{p}\right)\gamma_t[\underline{w}(g,t)] = p^2(2+\vartheta) + \frac{p^2}{i\sqrt[\Omega]{p}},$$
$$\frac{\partial}{\partial g}\gamma_t[\bar{u}(g,t)] + \left(i\sqrt[\Omega]{p}\right)\gamma_t[\bar{w}(g,t)] = p^2(4-\vartheta) + \frac{p^2}{i\sqrt[\Omega]{p}}.$$
$$\frac{\partial}{\partial g}\gamma_t[\underline{w}(g,t)] + \left(i\sqrt[\Omega]{p}\right)\gamma_t[\underline{u}(g,t)] = p^2(1+\vartheta),$$
$$\frac{\partial}{\partial g}\gamma_t[\bar{w}(g,t)] + \left(i\sqrt[\Omega]{p}\right)\gamma_t[\bar{u}(g,t)] = p^2(1-\vartheta).$$

$$\underline{u}(g,t;\vartheta) = (3+\vartheta)k(t-g) - (1+\vartheta)k(t-g) + (1+\vartheta),$$
$$\bar{u}(g,t;\vartheta) = (5-\vartheta)k(t-g) - (1-\vartheta)k(t-g) + (1-\vartheta).$$

$$\underline{w}(g,t;\vartheta) = t + (2+\vartheta) + (3+\vartheta)k(t-g) - (1+\vartheta)k(t-g),$$
$$\overline{w}(g,t;\vartheta) = t + (4-\vartheta) + (5-\vartheta)k(t-g) - (1-\vartheta)k(t-g).$$
$$\frac{\partial}{\partial g}T_t[u(g,t)] - p^2 w(g,0)\square\left(-i\sqrt[\Omega]{p}\right)T_t[w(g,t)] = \frac{p^2}{i\sqrt[\Omega]{p}},$$
$$\frac{\partial}{\partial g}T_t[w(g,t)] - p^2 u(g,0)\square\left(-i\sqrt[\Omega]{p}\right)T_t[u(g,t)] = 0.$$

Then:

$$\underline{u}(g,t;\vartheta) = (1+\vartheta) + \vartheta k(t-g) - tk(t-g) - (2+\vartheta)k(t-g),$$
$$\bar{u}(g,t;\vartheta) = (1-\vartheta) + (2-\vartheta)k(t-g) - tk(t-g) - (4-\vartheta)k(t-g).$$
$$\underline{w}(g,t;\vartheta) = \vartheta k(t-g) - tk(t-g) - (2+\vartheta)k(t-g) + t + (4-\vartheta),$$
$$\overline{w}(g,t;\vartheta) = (2-\vartheta)k(t-g) - tk(t-g) - (4-\vartheta)k(t-g) + t + (2+\vartheta).$$

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Received November, 2024