

Investigating fuzzy topological indices of linear and multi-cyclic Naphthalene

Mehri Hasani [†]

Masoud Ghods ^{*}

Department of Mathematics, Statistics and Computer Science

Semnan University

Semnan 35131-19111

Iran

Abstract

This research study focuses on the theoretical examination of the fuzzy graph of aromatic hydrocarbon naphthalene. It introduces new definitions for the first Zagreb fuzzy index, forgotten index, and Y-index, and computes several fuzzy-based degree topological indices. The study provides a general formula for the fuzzy topology indices of naphthalene hydrocarbon by taking into account the degree of each edge. Additionally, a formula is presented to determine the specific topology index of naphthalene based on the number of rows and columns. The topological indices of linear naphthalene were compared, and it was found that the Randic and harmonic indices exhibited the highest values.

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Keywords: Fuzzy graph, Topological index, Zagreb index, Naphthalene, Molecular structure, Vertex degree.

1. Introduction

Graph theory is a type of mathematics that plays a significant role in fields such as physics, chemistry, and engineering [7]. Chemical graph theory is a field of chemical mathematical topology that combines chemistry and graph theory to represent chemical structures as

[†] ORCID-ID: 0000-0002-3970-2339

^{*} ORCID-ID: 0000-0002-5006-5107

[†] E-mail: hasani.mehri@semnan.ac.ir

^{*} E-mail: mghods@semnan.ac.ir (Corresponding Author)

mathematical models. This theory uses mathematical theorems and chemical methods to solve chemistry problems. In molecular graphs, atoms and bonds are represented as vertices and edges. Graph theory is used in chemistry to count isomers and generate molecular structures. The concept of membership in a set is based on the binary evaluation of connection or non-connection. However, in fuzzy sets, elements have membership degrees in the interval $[0, 1]$. Membership values of vertices and edges are not certain in fuzzy graph theory, which differs from a crisp approach. Fuzzy graphs are widely utilized in various scientific fields, such as mathematics and chemistry, due to their superior accuracy and flexibility compared to traditional models. A. Rosenfeld first introduced the theory of fuzzy graphs, and Zadeh introduced fuzzy sets in graph theory in 1965 [28]. In 1997, Xu mentioned the first usage of fuzzy graphs in chemistry [26]. Bhattacharya introduced the idea of using fuzzy graphs to create fuzzy groups [4]. In 2022, Mofti et al. studied the fuzzy Zagreb topological indices on linear hydrocarbon and multi-acyclic Benzene [19]. Inspired by these studies, we are motivated to investigate several topology indices on naphthalene.

Topological indices, numerical values associated with a graph, can forecast the properties of chemical compounds without conducting experiments [5]. In 2015 Gutman and Fortula introduced the forgotten topological index [5]. A study conducted by Almoti et al. in 2019 focused on examining the thermodynamic properties of linear acene molecules by utilizing a hyper-Zagreb index [2]. In 2020, Binu et al. proposed the Wiener index for fuzzy graphs [3], and Poulik and Ghorai introduced a specific index for bipolar fuzzy graphs [21]. In the same year, Pal et al. analyzed modern trends in fuzzy graph theory [20]. The following year, Ghorai and Poulik presented the Wiener absolute index, while Poulik et al. introduced the Randic index for bipolar fuzzy graphs [22, 23]. Pal and Islam studied the first Zagreb index, F-index, hyper-Wiener index, and hyper-connectivity index for fuzzy graphs [14-17]. In 2021, J. B Veeragoudar et al. conducted a study on the rst Zagreb index and coindex of the block-transformation graphs G [25]. These indices are used in various fields such as mathematics, biology, and bioinformatics, but their primary use is in quantitative structure property relationship QSPR [12]. In 2023, Hasani and Ghods examined M-polynomials and topological indices of some dendrimers [8]. In 2023 and 2024, they analyzed the QSPR of drugs used

to treat heart diseases and parkinson's disease [9-11]. Naphthalene ($C_{10}H_8$) is one of the simplest cyclic hydrocarbons composed of two benzene rings and plays a significant role in chemistry. In the graphical representation of naphthalene, hexagons, and squares are connected in a way that each square is between two hexagons. In the naphthalene nanotube, the first row consists of a sequence of $C_6, C_6, C_4, C_6, C_6, C_4, \dots$, while the second row consists of a sequence of $C_6, C_8, C_6, C_8, \dots$, [27] as shown in Figure 3. In this study, different topological indices in fuzzy graphs are explored, such as the first and second Zagreb, Randic, harmonic, and Y-index on linear and multi-cyclic naphthalene. New definitions for the first Zagreb index, the Forgotten index, and Y-index are introduced. A general formula for the fuzzy topology indices of naphthalene hydrocarbon is presented, taking into account the degree of each edge.

2. Preliminaries

Some foundational definitions are needed to establish our findings, and we also introduce new definitions, which are detailed in the following section and Tables 1 and 2.

Definition 1: [13] Suppose X is a finite set. The graph fuzzy G with vertices set $V(G)$ and edges set $E(G) = \{\alpha\beta \mid \eta(\alpha, \beta) > 0\}$, is a triplet, $G = (V, \xi, \eta)$, together with a fuzzy set ξ on V and a fuzzy relation η on V such that

$$\xi: V \rightarrow [0, 1]$$

$$\eta: V \times V \rightarrow [0, 1] \quad \forall \alpha, \beta \in V(G)$$

satisfying

$$\eta(\alpha, \beta) \leq \xi(\alpha) \wedge \xi(\beta),$$

where $\wedge$ represents the minimum.

Definition 2: Suppose G is a fuzzy graph and $\alpha \in V$, then the degree of vertex α in a fuzzy graph is defined by

$$d_G(\alpha) = \sum_{\alpha\beta \in E(G)} \eta(\alpha\beta)$$

Table 1
Description of degree-based topological indices

Topological index	Mathematical Formula
First Zagreb index: [6]	$M_1(G) = \sum_{i=1}^n [d(\alpha_i)]^2 = \sum_{\alpha_i, \alpha_j \in E(G)} [d(\alpha_i) + d(\alpha_j)]$
Second Zagreb index: [6]	$M_2(G) = \sum_{\alpha_i, \alpha_j \in E(G)} d(\alpha_i) d(\alpha_j)$
Forgotten index: [5]	$F(G) = \sum_{i=1}^n [d(\alpha_i)]^3 = \sum_{\alpha_i, \alpha_j \in E(G)} [d(\alpha_i)^2 + d(\alpha_j)^2]$
Y-index: [1]	$Y(G) = \sum_{i=1}^n [d(\alpha_i)]^4 = \sum_{\alpha_i, \alpha_j \in E(G)} [d(\alpha_i)^3 + d(\alpha_j)^3]$
Randic index: [24]	$R(G) = \sum_{\alpha_i, \alpha_j \in E(G)} \frac{1}{\sqrt{d(\alpha_i) \times d(\alpha_j)}}$
Harmonic index: [24]	$H(G) = \sum_{\alpha_i, \alpha_j \in E(G)} \frac{2}{(d(\alpha_i) + d(\alpha_j))}$

Table 2
Description of degree-based topological indices for fuzzy graphs

Topological index	Mathematical Formula
First Zagreb index: [19]	$M_{1_f}(G) = \sum_{i=1}^n \xi(\alpha_i) [d(\alpha_i)]^2$
Second Zagreb index: [19]	$M_{2_f}(G) = \frac{1}{2} \sum_{\alpha_i, \alpha_j \in E(G)} \xi(\alpha_i) d(\alpha_i) \xi(\alpha_j) d(\alpha_j) \quad \forall i \neq j$
Forgotten index: [19]	$F_f(G) = \sum_{i=1}^n \xi(\alpha_i) [d(\alpha_i)]^3$
Y-index: [18]	$Y_f(G) = \sum_{i=1}^n \xi(\alpha_i) [d(\alpha_i)]^4$
Randic index: [19]	$R_f(G) = \frac{1}{2} \sum_{\alpha_i, \alpha_j \in E(G)} [(\xi(\alpha_i) d(\alpha_i) \xi(\alpha_j) d(\alpha_j))]^{\frac{-1}{2}} \quad \forall i \neq j$
Harmonic index: [19]	$H_f(G) = \frac{1}{2} \sum_{\alpha_i, \alpha_j \in E(G)} [\xi(\alpha_i) d(\alpha_i) + \xi(\alpha_j) d(\alpha_j)]^{-1} \quad \forall i \neq j$

The new definitions for the first Zagreb index, F-index, and Y-index in fuzzy graphs are presented in Table 3.

Table 3
Introducing novel definitions of topological Indices in fuzzy graphs

Topological index	Mathematical Formula
First Zagreb index	$Z_{hg}(G) = \sum_{\alpha_i, \alpha_j \in E(G)} \eta(\alpha_i, \alpha_j) [\xi(\alpha_i)d(\alpha_i) + \xi(\alpha_j)d(\alpha_j)]$
Forgotten index	$F_{hg}(G) = \sum_{\alpha_i, \alpha_j \in E(G)} \eta(\alpha_i, \alpha_j) [\xi(\alpha_i)d(\alpha_i)^2 + \xi(\alpha_j)d(\alpha_j)^2]$
Y-index	$Y_{hg}(G) = \sum_{\alpha_i, \alpha_j \in E(G)} \eta(\alpha_i, \alpha_j) [\xi(\alpha_i)d(\alpha_i)^3 + \xi(\alpha_j)d(\alpha_j)^3]$

It is simply proved that $Z_{hg}(G) = M_{1f}(G)$, $F_{hg}(G) = F_f(G)$, and $Y_{hg}(G) = Y_f(G)$.

3. Main results

In this section, using Tables 2 and 3 some topological indices of multi-cyclic naphthalene in fuzzy graphs are computed, and a general formula based on the number of their rows and columns is presented for its extension. According to Figures 1 and 2, and based on the structure of linear naphthalene, the total number of vertices and edges are $10n$, and $13n - 2$, respectively. The set of all vertices and edges is divided into weight categories, as shown in Tables 4 and 5. Using Table 4, the vertex set with weight 0.2 has a total number of $3n$, the vertex set with weight 0.3 has a total number of $4n$, and the vertex set with weight 0.4 has a total number of $3n$. According to Figures 1, 2, 3 and 4, and based on the structure of multi-naphthalene, the total number of vertices and edges are

Table 4
Partition of vertices for fuzzy graphs of linear naphthalene

Weight	Degree	The total number of vertices
(0.2)	(0.3)	2
	(0.4)	$3n - 2$
(0.3)	(0.2)	$2n$
	(0.6)	$2n$
(0.4)	(0.5)	2
	(0.6)	$2n - 2$
	(0.8)	n

$10mn$, and $15mn - 2m - 2n$, respectively. The set of all vertices and edges is divided into weight categories, as shown in Tables 6 and 7. According Table 6, the vertex set with weight 0.2 has a total number of $3mn$, the vertex set with weight 0.3 has a total number of $4mn$, and the vertex set with weight 0.4 has a total number of $3mn$.

Table 5
Partition of edges for fuzzy graphs of linear naphthalene

Weight	Degree	The total number of edges
(0.3,0.4)	(0.6,0.5)	2
	(0.6,0.8)	$2n$
	(0.6,0.6)	$2n - 2$
(0.3,0.2)	(0.2,0.3)	2
	(0.2,0.4)	$4n - 2$
(0.2,0.4)	(0.3,0.5)	2
	(0.4,0.8)	n
	(0.4,0.6)	$2n - 2$
(0.4,0.4)	(0.6,0.6)	$n - 1$
(0.2,0.2)	(0.4,0.4)	$n - 1$

Table 6
Partition of vertices in fuzzy graphs of multi-naphthalene

Weight	Degree	The total number of vertices
(0.2)	(0.3)	$2m$
	(0.4)	$m(3n - 2)$
(0.3)	(0.2)	$2n$
	(0.4)	$2n(m - 1)$
	(0.6)	$2n$
	(0.8)	$2n(m - 1)$
(0.4)	(0.5)	$2m$
	(0.6)	$2m(n - 1)$
	(0.8)	mn

Table 7
Partition of edges in fuzzy graphs of multi-naphthalene

Weight	Degree	The total number of edges
(0.3,0.4)	(0.6,0.5)	2
	(0.6,0.8)	$2n$
	(0.6,0.6)	$2n - 2$
	(0.8,0.5)	$2m - 2$
	(0.8,0.8)	$2n(m - 1)$
	(0.8,0.6)	$(m - 1)(2n - 2)$
(0.4,0.2)	(0.5,0.3)	$2m$
	(0.8,0.4)	mn
	(0.6,0.4)	$2m(n - 1)$
(0.2,0.3)	(0.3,0.4)	$2m - 2$
	(0.4,0.4)	$(m - 1)(2n - 2)$
	(0.3,0.2)	2
	(0.4,0.2)	$4n - 2$
(0.4,0.4)	(0.6,0.6)	$m(n - 1)$
(0.2,0.2)	(0.4,0.4)	$m(n - 1)$
(0.3,0.3)	(0.4,0.8)	$2n(m - 1)$

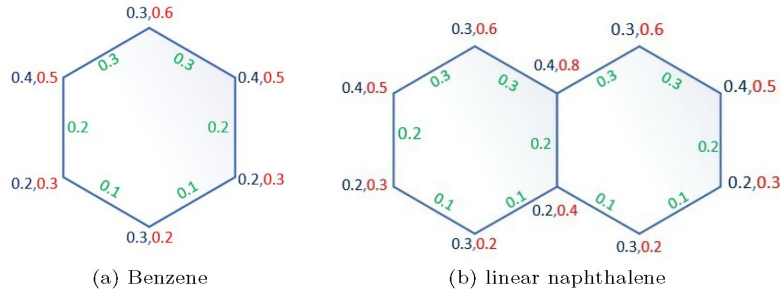


Figure 1
(1,1) unit of the graph Benzene and linear naphthalene

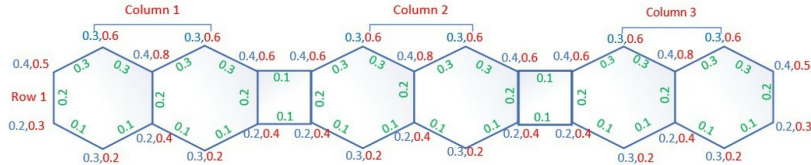


Figure 2
(1,3) unit of the graph of linear naphthalene

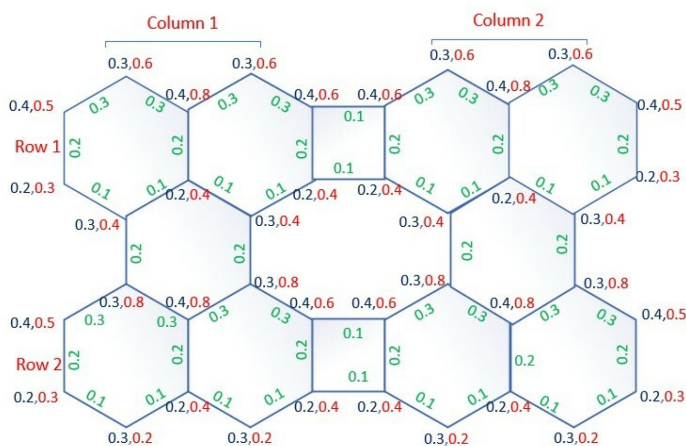


Figure 3
(2,2) unit of the graph of naphthalene

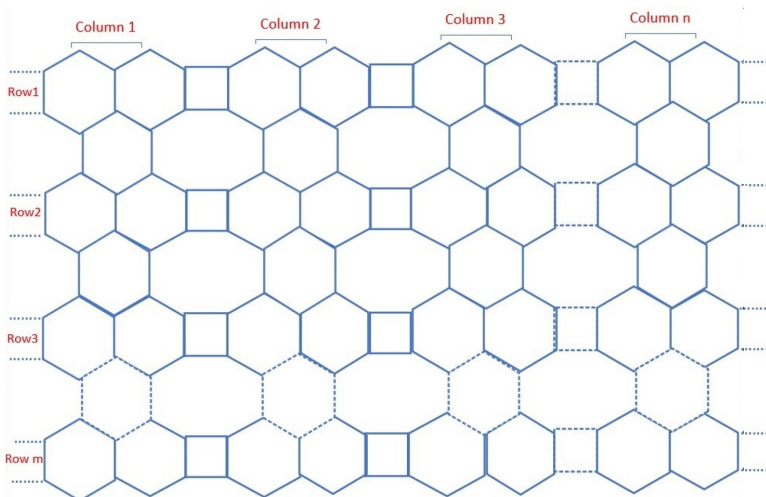


Figure 4
(m,n) unit of fuzzy graphs of multi-cyclic naphthalene

Theorem 1: Suppose N_l and N_m are linear naphthalene and multi-naphthalene fuzzy graphs, respectively. Then, the first Zagreb index of N_l and N_m is:

1. $Z_{hg}(N_m) = 1.12mn - 0.24n - 0.116m$
2. $Z_{hg}(N_l) = 0.88n - 0.116$

Proof: Using Table and , we have

$$\begin{aligned}
 Z_{hg}(N_m) &= \sum_{\alpha_i, \alpha_j \in E(G)} \eta(\alpha_i, \alpha_j) [\xi(\alpha_i) d(\alpha_i) + \xi(\alpha_j) d(\alpha_j)] \\
 &= 2(0.3)[(0.3)(0.6) + (0.4)(0.5)] + (2n)(0.3)[(0.3)(0.6) + (0.4)(0.8)] \\
 &\quad + (2n-2)(0.3)[(0.3)(0.6) + (0.4)(0.6)] + (2m-2)(0.3)[(0.3)(0.8) + (0.4)(0.5)] \\
 &\quad + (2n)(m-1)(0.3)[(0.3)(0.8) + (0.4)(0.8)] \\
 &\quad + (m-1)(2n-2)(0.3)[(0.3)(0.8) + (0.4)(0.6)] \\
 &\quad + (2m)(0.2)[(0.4)(0.5) + (0.2)(0.3)] + mn(0.2)[(0.4)(0.8) + (0.2)(0.4)] \\
 &\quad + (0.2)(2m)(n-1)[(0.4)(0.6) + (0.2)(0.4)] + (2m-2)(0.1)[(0.2)(0.3) + (0.3)(0.4)] \\
 &\quad + (0.1)(m-1)(4n-2)[(0.2)(0.4) + (0.3)(0.4)] + 2(0.1)[(0.2)(0.3) + (0.3)(0.2)] \\
 &\quad + (0.1)(4n-2)[(0.2)(0.4) + (0.3)(0.2)] + (0.1)m(n-1)[(0.4)(0.6) + (0.4)(0.6)] \\
 &\quad + (0.1)m(n-1)[(0.2)(0.4) + (0.2)(0.4)] + (0.2)2n(m-1)[(0.3)(0.4) + (0.3)(0.8)] \\
 &= 1.12mn - 0.24n - 0.116m
 \end{aligned}$$

The second part of the theorem is valid for $m = 1$.

Theorem 2: Suppose N_l and N_m are linear naphthalene and multi-naphthalene fuzzy graphs, respectively. Then, the second Zagreb index of N_l and N_m is:

1. $M_{2f}(N_m) = 0.2464mn - 0.072n - 0.0944m + 0.468$
2. $M_{2f}(N_l) = 0.1744n - 0.0476$

Proof: Using Table 2 and 7, we have

$$\begin{aligned}
 M_{2f}(N_m) &= \frac{1}{2} \sum_{\alpha_i, \alpha_j \in E(G)} \xi(\alpha_i) d(\alpha_i) \xi(\alpha_j) d(\alpha_j) \\
 &= \frac{1}{2} [2(0.3)(0.6)(0.4)(0.5) + (2n)(0.3)(0.6)(0.4)(0.8) \\
 &\quad + (2n-2)(0.3)(0.6)(0.4)(0.6) + (2m-2)(0.3)(0.8)(0.4)(0.5) \\
 &\quad + (2n)(m-1)(0.3)(0.8)(0.4)(0.8) + (m-1)(2n-2)(0.3)(0.8)(0.4)(0.6) \\
 &\quad + (2m)(0.4)(0.5)(0.2)(0.3) + mn(0.4)(0.8)(0.2)(0.4) \\
 &\quad + 2m(n-1)(0.4)(0.6)(0.2)(0.4) + (2m-2)(0.2)(0.3)(0.3)(0.4) \\
 &\quad + (m-1)(4n-2)(0.2)(0.4)(0.3)(0.4) + 2(0.2)(0.3)(0.3)(0.2) \\
 &\quad + (4n-2)(0.2)(0.4)(0.3)(0.2) + m(n-1)(0.4)(0.6)(0.4)(0.6) \\
 &\quad + m(n-1)(0.2)(0.4)(0.2)(0.4) + 2n(m-1)(0.3)(0.4)(0.3)(0.8)] \\
 &= 0.2464mn - 0.072n - 0.0944m + 0.468
 \end{aligned}$$

The second part of the theorem is valid for $m = 1$.

Theorem 3: Suppose N_l and N_m are linear naphthalene and multi-naphthalene fuzzy graphs, respectively. Then, the Randic index of N_l and N_m is:

1. $R_f(N_m) = 52.7499mn - 16.2296n - 4.5704m + 10.8411$
2. $R_f(N_l) = 56.5203n - 3.7293$

Proof. Using Tables 2 and 7, we have

$$\begin{aligned}
 R_f(N_m) &= \frac{1}{2} \sum_{\alpha_i, \alpha_j \in E(G)} [(\xi(\alpha_i)d(\alpha_i)\xi(\alpha_j)d(\alpha_j))]^{\frac{-1}{2}} \\
 &= \frac{1}{2}(2)[(0.3)(0.6)(0.4)(0.5)]^{\frac{-1}{2}} + \frac{1}{2}(2n)[(0.3)(0.6)(0.4)(0.8)]^{\frac{-1}{2}} \\
 &\quad + \frac{1}{2}(2n-2)[(0.3)(0.6)(0.4)(0.6)]^{\frac{-1}{2}} + \frac{1}{2}(2m-2)[(0.3)(0.8)(0.4)(0.5)]^{\frac{-1}{2}} \\
 &\quad + \frac{1}{2}(4n-2)(m-1)[(0.3)(0.8)(0.4)(0.8)]^{\frac{-1}{2}} \\
 &\quad + \frac{1}{2}(m-1)(2n-2)[(0.3)(0.8)(0.4)(0.6)]^{\frac{-1}{2}} \\
 &\quad + \frac{1}{2}(2m)[(0.4)(0.5)(0.2)(0.3)]^{\frac{-1}{2}} + \frac{1}{2}mn[(0.4)(0.8)(0.2)(0.4)]^{\frac{-1}{2}} \\
 &\quad + \frac{1}{2}2m(n-1)[(0.4)(0.6)(0.2)(0.4)]^{\frac{-1}{2}} + \frac{1}{2}(2m-2)[(0.2)(0.3)(0.3)(0.4)]^{\frac{-1}{2}} \\
 &\quad + \frac{1}{2}(m-1)(4n-2)[(0.2)(0.4)(0.3)(0.4)]^{\frac{-1}{2}} + \frac{1}{2}(2)[(0.2)(0.3)(0.3)(0.2)]^{\frac{-1}{2}} \\
 &\quad + \frac{1}{2}(4n-2)[(0.2)(0.4)(0.3)(0.2)]^{\frac{-1}{2}} + \frac{1}{2}m(n-1)[(0.4)(0.6)(0.4)(0.6)]^{\frac{-1}{2}} \\
 &\quad + \frac{1}{2}m(n-1)[(0.2)(0.4)(0.2)(0.4)]^{\frac{-1}{2}} + \frac{1}{2}2n(m-1)[(0.3)(0.4)(0.3)(0.8)]^{\frac{-1}{2}} \\
 &= 52.7499mn - 16.2296n - 4.5704m + 10.8411
 \end{aligned}$$

The second part of the theorem is valid for $m = 1$.

Theorem 4: Suppose N_l and N_m are linear naphthalene and multi-naphthalene fuzzy graphs, respectively. Then, the harmonic index of N_l and N_m is:

1. $H_f(N_m) = 22.4156mn - 9.4931n - 2.7056m + 7.844$
2. $H_f(N_l) = 27.2081n - 4.6359$

Proof: Using Tables 2 and 7, we have

$$\begin{aligned}
 H_f(N_m) &= \frac{1}{2} \sum_{\alpha_i, \alpha_j \in E(G)} [\xi(\alpha_i)d(\alpha_i) + \xi(\alpha_j)d(\alpha_j)]^{-1} \\
 &= \frac{1}{2} \left[\frac{2}{(0.3)(0.6) + (0.4)(0.5)} \right] + \frac{1}{2} \left[\frac{(2n)}{(0.3)(0.6) + (0.4)(0.8)} \right] \\
 &\quad + \frac{1}{2} \left[\frac{2n-2}{(0.3)(0.6) + (0.4)(0.6)} \right] + \frac{1}{2} \left[\frac{2m-2}{(0.3)(0.8) + (0.4)(0.5)} \right] \\
 &\quad + \frac{1}{2} \left[\frac{(2n)(m-1)}{(0.3)(0.8) + (0.4)(0.8)} \right] + \frac{1}{2} \left[\frac{(m-1)(2n-2)}{(0.3)(0.8) + (0.4)(0.6)} \right] \\
 &\quad + \frac{1}{2} \left[\frac{2m}{(0.4)(0.5) + (0.2)(0.3)} \right] + \frac{1}{2} \left[\frac{mn}{(0.4)(0.8) + (0.2)(0.4)} \right] \\
 &\quad + \frac{1}{2} \left[\frac{m(2n-2)}{(0.4)(0.6) + (0.2)(0.4)} \right] + \frac{1}{2} \left[\frac{2m-2}{(0.2)(0.3) + (0.3)(0.4)} \right] \\
 &\quad + \frac{1}{2} \left[\frac{(m-1)(4n-2)}{(0.2)(0.4) + (0.3)(0.4)} \right] + \frac{1}{2} \left[\frac{2}{(0.2)(0.3) + (0.3)(0.2)} \right] \\
 &\quad + \frac{1}{2} \left[\frac{(4n-2)}{(0.2)(0.4) + (0.3)(0.2)} \right] + \frac{1}{2} \left[\frac{m(n-1)}{(0.4)(0.6) + (0.4)(0.6)} \right] \\
 &\quad + \frac{1}{2} \left[\frac{m(n-1)}{(0.2)(0.4) + (0.2)(0.4)} \right] + \frac{1}{2} \left[\frac{2n(m-1)}{(0.3)(0.4) + (0.3)(0.8)} \right] \\
 &= 22.4156mn - 9.4931n - 2.7056m + 7.844
 \end{aligned}$$

The second part of the theorem is valid for $m = 1$.

Theorem 5: Suppose N_l and N_m are linear naphthalene and multi-naphthalene fuzzy graphs, respectively. Then, the Y -index of N_l and N_m is:

1. $Y_{hg}(N_m) = 0.5437mn - 0.1825n - 0.06064m + 0.00004$
2. $Y_{hg}(N_l) = 0.3612n - 0.0606$

Proof: Using Tables 3 and 7, we have

$$\begin{aligned}
 Y_{hg}(N_m) &= \sum_{\alpha_i, \alpha_j \in E(G)} \eta(\alpha_i, \alpha_j) [\xi(\alpha_i)d(\alpha_i)^3 + \xi(\alpha_j)d(\alpha_j)^3] \\
 &= 2(0.3)[(0.3)(0.6)^3 + (0.4)(0.5)^3] + (2n)(0.3)[(0.3)(0.6)^3 + (0.4)(0.8)^3] \\
 &\quad + (2n-2)(0.3)[(0.3)(0.6)^3 + (0.4)(0.6)^3] \\
 &\quad + (2m-2)(0.3)[(0.3)(0.8)^3 + (0.4)(0.5)^3] \\
 &\quad + (2n)(m-1)(0.3)[(0.3)(0.8)^3 + (0.4)(0.8)^3] \\
 &\quad + (m-1)(2n-2)(0.3)[(0.3)(0.8)^3 + (0.4)(0.6)^3] \\
 &\quad + (2m)(0.2)[(0.4)(0.5)^3 + (0.2)(0.3)^3] + mn(0.2)[(0.4)(0.8)^3 + (0.2)(0.4)^3] \\
 &\quad + (0.2)(2m)(n-1)[(0.4)(0.6)^3 + (0.2)(0.4)^3] \\
 &\quad + (2m-2)(0.1)[(0.2)(0.3)^3 + (0.3)(0.4)^3] \\
 &\quad + (0.1)(m-1)(4n-2)[(0.2)(0.4)^3 + (0.3)(0.4)^3]
 \end{aligned}$$

$$\begin{aligned}
& +2(0.1)[(0.2)(0.3)^3 + (0.3)(0.2)^3] \\
& + (0.1)(4n-2)[(0.2)(0.4)^3 + (0.3)(0.2)^3] \\
& + (0.1)m(n-1)[(0.4)(0.6)^3 + (0.4)(0.6)^3] \\
& + (0.1)m(n-1)[(0.2)(0.4)^3 + (0.2)(0.4)^3] \\
& + (0.2)2n(m-1)[(0.3)(0.4)^3 + (0.3)(0.8)^3] \\
& = 0.5437mn - 0.1825n - 0.06064m + 0.00004
\end{aligned}$$

The second part of the theorem is valid for $m = 1$.

The topological indices of linear naphthalene are compared in Figure 5.

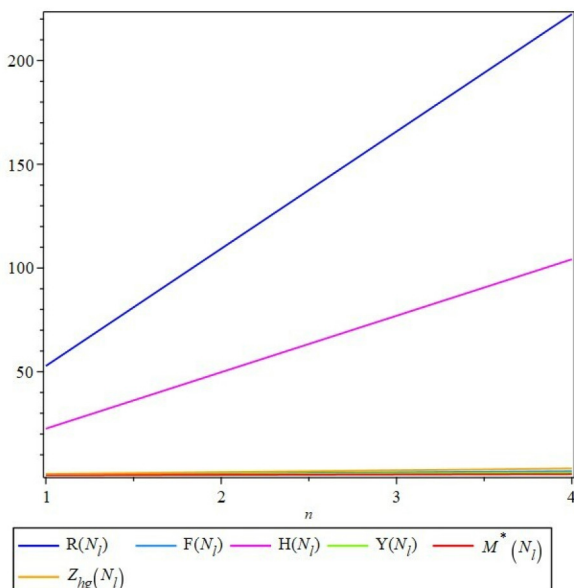


Figure 5
Comparison of topological indices of linear naphthalene

4. Conclusion

This study calculated various linear and multi-cyclic naphthalene hydrocarbon topological indices. A general formula for the fuzzy topological indices of naphthalene hydrocarbons was also developed, incorporating the degree of each edge. As illustrated in Figure 5, a comparison of the topological indices of linear naphthalene indicated that the Randic and harmonic indices exhibited the highest values. The findings of this research provide valuable insights that can aid scientists in

predicting or estimating the physicochemical properties of molecules by accurately determining their bond lengths and atomic masses.

Declarations

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- **Availability of data and materials**

This article includes all data generated or analyzed during the study.

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